

Six Dimensional $\mathcal{N}=(2,0)$ SCFTs on $AdS_5 \times S^1$



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Exact Quantum Fields and
the Structure of M-theory



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Work in progress with Berkooz and Rey

Motivations

- No need to motivate here the study of the **6d $\mathcal{N}=(2,0)$ SCFTs** (many talks)...
- Studying supersymmetric theories on various curved spaces has already led to many interesting results, in particular with conserved supersymmetry; **$AdS_5 \times S^1$** preserves **16** supercharges, like **$S^5 \times S^1$**
- Can embed the **$\mathcal{N}=(2,0)$ SCFTs** on **$AdS_5 \times S^1$** into string theory and use the **AdS/CFT correspondence** to study them.
- Will find surprising results...

Review of A_{n-1} 6d $\mathcal{N}=(2,0)$ SCFTs

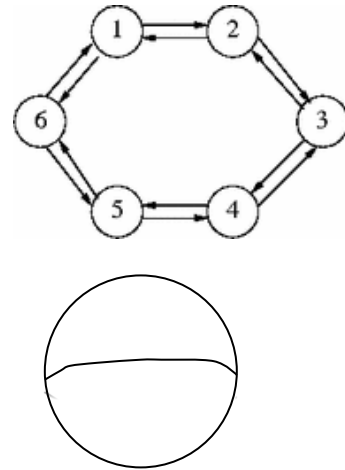
- Will focus on this case for simplicity.
- Arises from n M5-branes, n NS5-branes in type IIA, or a C^2/Z_n singularity in type IIB
- $\mathcal{N}=(2,0)$ tensor multiplet contains 2-form field with self-dual field strength, 5 scalars and fermions.
- Moduli space is $R^{5(n-1)}/S_n$ (ignoring center of mass), low-energy theory has $(n-1)$ tensor multiplets and BPS strings with tensions $|\Phi_i - \Phi_j|$; interacting SCFT at origin.

Compactification on a circle

- Leaves same moduli space $R^{5(n-1)}/S_n$
- At energies below $1/R$, get a 5d $\mathcal{N}=2$ SYM theory with gauge group $SU(n)$, $g_{\text{YM}}^2 \sim R$; moduli space described by VEVs of adjoint scalar fields. Wrapped BPS strings become BPS W -bosons; $SU(n)$ unbroken at origin.
- In type IIB on C^2/Z_n BPS strings come from D3-branes wrapped on 2-cycles. $3(n-1)$ scalars come from blow-up modes, $2(n-1)$ from B_2 and C_2 integrated on 2-cycles.

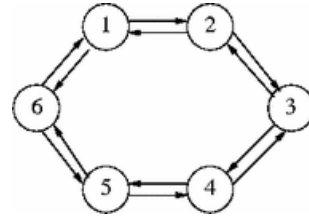
Embedding in string theory

- Consider type IIB string theory on $AdS_5 \times S^5 / Z_n$ = near-horizon limit of K D3-branes on C^2 / Z_n . Dual to 4d $\mathcal{N}=2$ $SU(K)^n$ elliptic quiver with bi-fundamental hypermultiplets (Kachru-Silverstein)
- Locally have C^2 / Z_n orbifold living on $AdS_5 \times S^1$ in $AdS_5 \times S^5 / Z_n$
- 4d $\mathcal{N}=2$ theory has n exactly marginal deformations – complex gauge couplings. One maps to type IIB dilaton-axion.



- Other $(n-1)$ to B_2 and C_2 fields on 2-cycles of singularity. Other blow-up modes tachyonic.
- At orbifold point have n equal gauge couplings, non-zero B_2 fields on 2-cycles.
- But can also take B_2 fields to zero – get at low energy 6d $\mathcal{N}=(2,0)$ A_{n-1} SCFT living on $AdS_5 \times S^1$ (coupling to rest of type IIB), with $R_{AdS}=R_S$ and specific boundary conditions
- Near this point “moduli space” (space of SUSY vacua on AdS_5) is C^{n-1}/S_n with A_{n-1} $(2,0)$ SCFT arising at the origin. Subspace of original moduli space (containing fields that remain massless on AdS_5).

- Theory has **16** conserved supercharges.
- **4d $\mathcal{N}=2$ SCFT** has a **global symmetry** $SU(2)_R \times U(1)_R \times U(1)^n$. The $U(1)^n$ is the flavor symmetry of the **n** bi-fundamental hypermultiplets. The lowest dimension operators charged under relative **$U(1)$** 's in $U(1)^n$ are di-baryons, with a classical dimension \sim **K** .



- In **string theory** : **D3-branes** wrapped around the **$(n-1)$ 2-cycles** and the **S^1** (in S^5/Z_n).
- A factor of $SU(2)_R \times U(1)_R \times U(1)$ is realized geometrically through isometries of S^5/Z_n . (**$SU(2)$** for **$n=2$**)

Naïve expectation

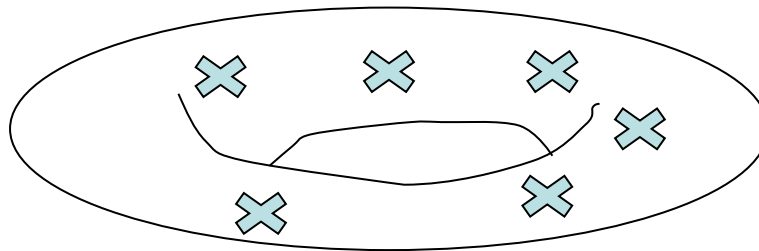
- Far from singular point get $(n-1)$ tensor multiplets on $AdS_5 \times S^1 \rightarrow U(1)^{n-1}$ gauge theory on $AdS_5 \rightarrow U(1)^{n-1}$ global symmetry of $4d \mathcal{N}=2$ SCFT (out of $U(1)^n$ acting on hypers; overall $U(1)$ is geometrical).
- When we go to singular point expect $\mathcal{N}=(2,0)$ theory on S^1 to give an $SU(n)$ gauge theory on AdS_5 . Not obvious since strongly coupled ($R_{AdS}=R_S$). Would mean global symmetry of $4d \mathcal{N}=2$ SCFT enhanced to $SU(n)$.

Failure of naïve expectation

- But can show that **global symmetries** in **4d $\mathcal{N}=2$ SCFTs** cannot be enhanced as a function of exactly marginal deformations (unlike in **4d $\mathcal{N}=1$**), except at free points.
- **Current multiplet** of **4d $\mathcal{N}=2$** cannot continuously become a **long multiplet**, except when have also conserved higher spin currents (as in free theories).
- Naïve expectation fails; note that **W-bosons** = wrapped **D3-branes** are not BPS. (Strings)
- What does happen in **4d $\mathcal{N}=2$ SCFT** ?

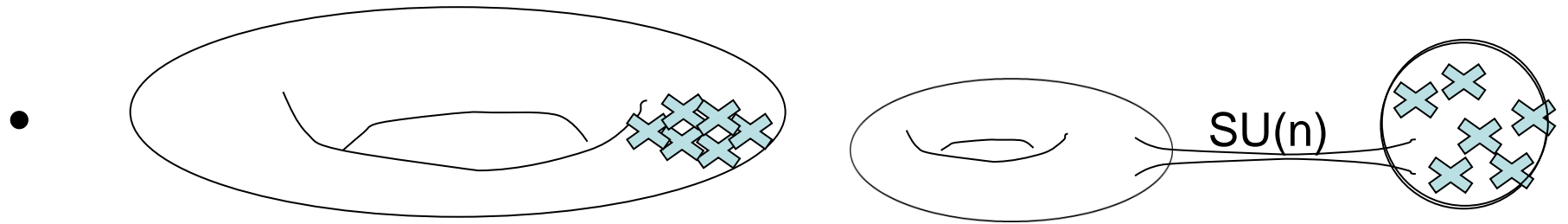
Singular limit in 4d $\mathcal{N}=2$ SCFT

- Space of couplings of $SU(K)^n$ quiver is moduli space of n marked points on a torus (Witten, based on brane construction). In modern language (Gaiotto et al) = A_{K-1} 6d $(2,0)$ theory ($K \neq n$) on a torus with n minimal $(U(1))$ punctures. Has a weakly coupled $SU(K)^n$ limit.



- Singular limit involves n punctures coming together – $(n-1)$ couplings go to infinity.

Singular limit in 4d $\mathcal{N}=2$ SCFT

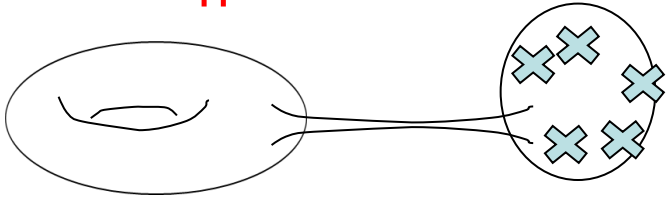


- Studied already (local on **Riemann surface**). In this limit develop a weakly coupled **SU(n)** gauge theory, with $g_{\text{SU}(n)}$ going to zero at singular point ! It is coupled to two different **4d $\mathcal{N}=2$ SCFTs** with an **SU(n)** global symmetry : A_{K-1} on a **torus** with a single **SU(n)** puncture ($Q_{K,n}$) and a **sphere** with one **SU(n)** puncture and n **U(1)** punctures (P_n).¹¹

Singular limit in 4d $\mathcal{N}=2$ SCFT

- The new $SU(n)$ is strong-weak dual to original $SU(K)^n$; similar to **Argyres-Seiberg** where $SU(2)$ arises at strong coupling in 4d $\mathcal{N}=2$ $SU(3)$ theory with $N_f=6$.
- Implies that 4d $\mathcal{N}=2$ SCFT has at singular point infinite number of **conserved high-spin currents** (instead of naïve expectation – new global $SU(n)$). These should somehow map to $\mathcal{N}=(2,0)$ theory on $AdS_5 \times S^1$.
- Does this theory develop **massless high-spin fields** ? Strange but not impossible on AdS_5^{12} .

Simpler interpretation

- Higher spins seem inevitable, but actually we propose a simpler picture. The new **4d** **SU(n)** and the **P_n** theory can live on the boundary of **AdS₅**; can have **4d** $\mathcal{N}=2$ theories living there. The **4d** **SU(n)** gauge theory couples to both **Q_{K,n}** and **P_n**, and has a vanishing **beta function**.
- Identify the bulk theory with  the **Q_{K,n}** theory. The **4d** **SU(n)** gauge theory must couple to **5d** **SU(n)** gauge fields on **AdS₅**, helping to cancel its **beta function**.

Simpler interpretation

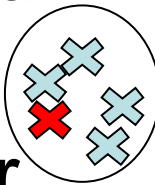
- This picture is related by extra AdS/CFT (for $SU(n) \times P_n$) to picture with high-spin fields in the bulk, but seems much simpler.
- Have $SU(n)$ in AdS_5 but no global symmetry. Usually say unique boundary condition for G gauge fields on AdS_5 !? When have global symmetry G can always gauge it = couple to 4d G gauge fields on boundary. When bulk theory is weakly coupled, get large (R_{AdS}/g_G^2) contribution to beta function of G , inconsistent with conformal symmetry.

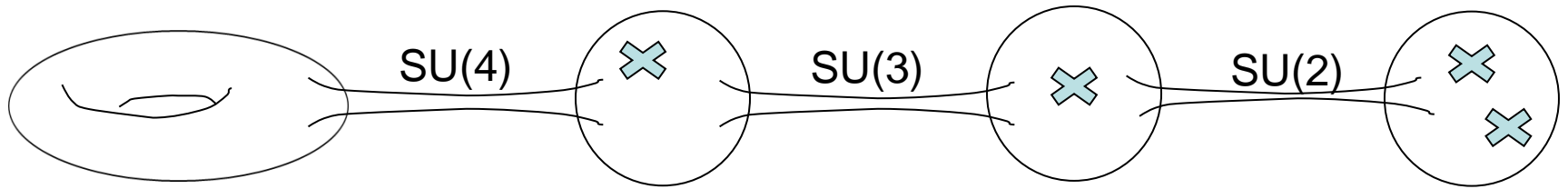
Simpler interpretation

- In our case we know contribution to **beta function**. Implies bulk **5d SU(n)** is strongly coupled at R_{AdS} . Thus, no contradiction with semi-classical analysis of allowed boundary conditions.
- Note that this **5d SU(n)** is different from the naïve one we expected; not broken when we go on the moduli space (**exactly marginal deformations** described by changing couplings of **SU(n)** and P_n on boundary; $U(1)^{n-1}$ acts on boundary P_n theory).

Moduli space of (2,0) on $AdS_5 \times S^1$

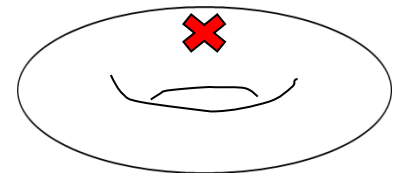
- At origin of “moduli space” coupling constant of $4d\ SU(n)$ goes to zero – infinitely far away (in natural **Zamolodchikov** metric).
- Moreover, origin of “moduli space” is not just a point but an $(n-2)$ -dimensional space – space of moduli of P_n theory = a sphere with $(n+1)$ marked points. Big change...
- The P_n theory has a region in its parameter space where it becomes a weakly coupled $4d\ SU(n-1) \times SU(n-2) \times \dots \times SU(2)$ theory with bi-fundamental hypers + $1+n$ fundamentals¹⁶





- Note all beta functions in this chain vanish.
 $Q_{K,n}$ contributes to beta function of $SU(n)$ like $(n+1)$ fundamental hypers.
- In this region it is easy to compute how many d.o.f. we are adding on the boundary (say in sense of conformal anomalies) : $O(n^3)$. Amusing since bulk $6d (2,0)$ theory also has $O(n^3)$ d.o.f. But no clear relation – for instance, $6d$ d.o.f. and $4d$ d.o.f. lead to a different density of states as a function of temperature / energy.

- This is all for the specific boundary condition that we get from **type IIB**. Can also take a “standard” boundary condition for **5d SU(n)** gauge fields, and then the **(2,0)** theory is part of the gravitational dual to the $Q_{K,n}$ theory (which has an **SU(n)** global symmetry). In this case the **(2,0)** theory has no “moduli space”. Does this make sense in full **type IIB** string theory ?



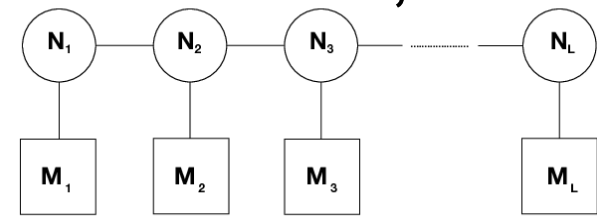
- Interesting to study other boundary conditions and other values of R_{AdS}/R_S ; generally no string theory embedding.

A decoupling limit

- Used **type IIB** to study A_{n-1} $\mathcal{N}=(2,0)$ theory on $AdS_5 \times S^1$, but, as in flat space, can also decouple it from rest of **type IIB**. Need to take $M_s \rightarrow \infty$, keeping n , g_s , R_{AdS} , tensions of wrapped **D3-branes** fixed (0). Bulk theory becomes free, $(2,0)$ remains interacting.
- In $SU(K)^n$ **SCFT** take $K \rightarrow \infty$ with couplings as above. Limit of **4d $\mathcal{N}=2$ SCFT** contains a sector dual to $\mathcal{N}=(2,0)$ theory on $AdS_5 \times S^1$. Not a **SCFT**. No local correlators. General : any branes or singularities on $AdS_p \times M$.

Universality of construction

- We embedded the $A_{n-1} (2,0)$ theory on $AdS_5 \times S^1$ in string theory in a specific way. But it also appears in many other places. For instance, linear 4d $\mathcal{N}=2$ quiver SCFTs, arising from D4-branes intersecting and ending on D6-branes and n NS5-branes, have type IIA string theory duals including n NS5-branes wrapping $AdS_5 \times S^1$. So they also include the same $A_{n-1} (2,0)$ theory on $AdS_5 \times S^1$ (and a corresponding decoupling limit).



Universality of construction

- The corresponding **4d $\mathcal{N}=2$** theories come from taking some **$A_{K-1} (2,0)$** theory on a different **Riemann surface**, but again with **n** minimal punctures. Again the limit where we get at low energies the **$A_{n-1} (2,0)$** theory involves bringing the **n** minimal punctures together. We get a weakly coupled **$SU(n)$** theory, coupled to **P_n** and to some other theory - can repeat the previous discussion.
- **A_{n-1}** on **$AdS_5 \times S^1$** (with same b.c.) a universal sector in the large **K** limit of all these **CFTs**.²¹

Universality of construction

- A special case of this is the **4d $\mathcal{N}=2$ $SU(K)$** gauge theory with **$N_f=2K$** , obtained from **K D4-branes** intersecting two **NS5-branes** and ending on **$2K$ D6-branes**. Here **$n=2$** so gravitational dual is highly curved, especially near the **NS5-branes** and the **D6-branes**.
- But can still claim that the **A_1 $(2,0)$ SCFT** on **$AdS_5 \times S^1$** with boundary conditions that couple it to **$SU(2)$** + a doublet hypermultiplet, is a decoupled sector in the large **K** strong coupling limit of this **SCFT**.

Generalization to LSTs

- In our discussion we did not use the fact that the $(2,0)$ theory was conformal – can use same methods for non-conformal theories. In particular, can study “little string theories” (LSTs) arising from the decoupling limit of n NS5-branes, or a C^2/Z_n singularity, in type II, with $g_s \rightarrow 0$ and the string scale M_s fixed.
- We can obtain the $(2,0)$ LST on $AdS_5 \times S^1$ from the same starting point of type IIB on $AdS_5 \times S^5/Z_n$, by $K \rightarrow \infty$ with $g_s K$ fixed ($M_s R_{AdS}$ fixed). Similar but extra BPS string.

Summary

- Argued that A_{n-1} (2,0) theories on $AdS_5 \times S^1$ with $R_{AdS} = R_S$ and specific boundary conditions are different from expected – “moduli space” is singular near origin, have $SU(n)$ gauge fields on AdS_5 but different from naive ones arising in flat space.
- This theory appears as a decoupled sector in the large K , strong coupling limit of many 4d $\mathcal{N}=2$ quiver SCFTs.

Further questions

- What can we compute ? In **4d $\mathcal{N}=2$ SCFT** can compute many things using **localization**, and in particular correlation functions of some **Wilson/'t Hooft** lines on **S^4** . These map to **BPS strings** in **$(2,0)$** , so it should be possible to compute their tension as a function of the couplings of **$SU(n)$** and **P_n** , and some of their correlation functions.
- Are “boundary correlators” (computable in principle) enough to characterize **A_{n-1} $(2,0)$** theory on **$AdS_5 \times S^1$** ?

Further questions

- What other boundary conditions are allowed? Can we have $SU(n)$ global symmetry and supersymmetry for any R_{AdS}/R_S ? For specific values of R_{AdS}/R_S can we couple to $4d \mathcal{N}=2$ $SU(n)$ gauge fields+matter on the boundary? Can we embed these in string/M theory?
- Can we compute partition function on $AdS_5 \times S^1$ by some localization method? BPS states related to part of index of $4d \mathcal{N}=2$ SCFTs. (Work in progress Bae+Rey) ²⁶

Further questions

- Can we relate the $SU(n)$ we found to the $SU(n)$ appearing on $R^5 \times S^1$ (perhaps by taking R_{AdS}/R_S large) ?
- Far on moduli space, have a description with $U(1)^{n-1}$ and “moduli” coming from the bulk; near the origin, have a description where they come from the boundary. What is relation between them ? AdS/CFT ? Strong-weak duality (similar to $Gaiotto-Witten$) ?
- Gravity dual for $(2,0)$ theory on $AdS_5 \times S^1$?
- Any relation to infinite chiral algebras? AGT ?

Further questions

- Many possible generalizations...
- Other D_n and E_n $\mathcal{N}=(2,0)$ theories on $AdS_5 \times S^1$ can be similarly studied using type IIB on $AdS_5 \times S^5/\Gamma$, where Γ is the appropriate ADE-subgroup of $SU(2)$
- Similar methods should be useful for studying various $\mathcal{N}=(2,0)$ theories on $AdS_4 \times S^2$ and $AdS_3 \times S^3$, 6d $\mathcal{N}=(1,0)$ theories on $AdS_5 \times S^1$ and other manifolds, 5d theories on $AdS_4 \times S^1$, 4d $\mathcal{N}=4$ SYM on $AdS_3 \times S^1$, etc.

Further questions

- Which sets of punctures coming together on a **Riemann surface** correspond to **(2,0)** theories on **AdS₅xS¹** ? What happens otherwise ? What happens if you bring together punctures and handles on the **Riemann surface** ?
- When is there a **LST**-like limit, with another **BPS string** ? Seems to arise when bringing punctures together + a **1-cycle** in **Riemann surface** becoming small. New **(2,0)** **LSTs** ?