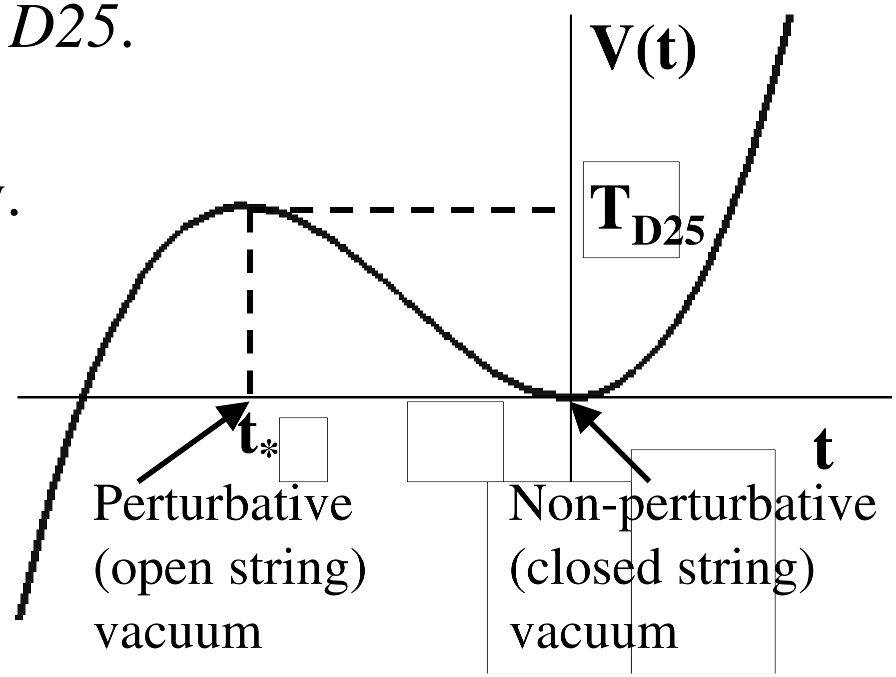
OUTLINE

- *Goal*: identify the D-branes as classical solutions in string field theory.
- *Method*: large non-commutativity is introduced as a tool that facilitates an explicit construction.
- *The result*: a soliton with tension *exactly* equal to the D-brane tension.
- They have the same fluctuation spectrum too; the non-abelian gauge symmetry of coincident branes is recovered.
- *Discussion*: lessons about the structure of the closed string vacuum as described by open string field theory.

INTRODUCTION

- Consider open bosonic string theory, *i.e.* bosonic string theory with a space-filling *D25*-brane.
- The open-string tachyon on the *D25* signals an instability.
- The true vacuum after condensation of the tachyon is the original closed string theory *without* the *D25*. Sen
- The energy liberated by the tachyon condensation exactly cancels the tension of the *D25*.
- This picture was tested in Open String Field Theory.

  Kostelecky+Samuel
  Sen+Zwiebach
  Taylor
  ...



APPROXIMATE LUMPS

- It is interesting to consider excitations of the non-perturbative vacuum.
- p-lumps are classical solutions with independence of p directions.
- The string field theory action is of the form:

$$S_I = T_{25}\int d^{26}x\sqrt{-g}\left[\frac{1}{2}f(t)g^{\mu\nu}\partial_\mu t\partial_\nu t + \ldots - V(t)\right]$$

- In the leading apx. $V(t)$ is a simple cubic, $f(t)=1$. The eom is:

$$\partial_\rho^2 t + \frac{24-p}{\rho}\partial_\rho t - V'(t) = 0$$

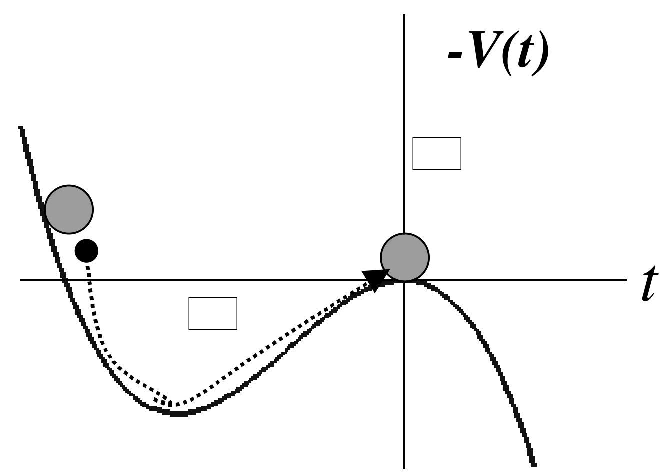
Harvey+
Kraus

- This is analogous to damped motion in the inverted cubic potential.
- Result for the energy of the lump:

p	T_p^0 / T_p
24	0.78
23	0.81
22	0.72
$\vdots$	$\vdots$

Tension of Dp-brane

p-lumps *are* Dp-branes



THE CHALLENGE

- The tachyon potential $V(t)$ is only known approximately except for specific features (like its height).
- The kinetic terms and numerous higher derivative terms are determined *in principle* by string field theory.
- In practice they are difficult to compute accurately; *e.g.* all massive fields must be taken into account.
- This is a problem because the unknown terms are of order string scale, as are the *Dp*-brane solutions we seek (in fact they are even smaller by a factor of g_s).

NON-COMMUTATIVITY

- B-fields are incorporated by replacing the metric and the coupling with

Schomerus
Seiberg
+Witten

$$G_{\mu\nu} = g_{\mu\nu} - (2\pi\alpha')^2 (Bg^{-1}B)_{\mu\nu} \qquad G_s = g_s \left(\frac{\det G}{\det(g + 2\pi\alpha' B)} \right)^{1/2}$$

- and multiplying fields using the non-commutative product

$$A * B = \exp\left(\frac{i}{2} \theta^{\mu\nu} \partial_\mu \partial_{\nu'} \right) A(x) B(x') \big|_{x=x'} \; ; \quad \theta^{\mu\nu} = -(2\pi\alpha')^2 \left(\frac{1}{g + 2\pi\alpha' B} B \frac{1}{g - 2\pi\alpha' B} \right)^{\mu\nu}$$

- The non-commutativity parameter θ provides a new scale for classical solutions; solitons simplify for large θ/α'.

Gopakumar+Minwalla+Strominger

- Complications of open string field theory are string scale w.r.t. the *open string metric*. They are much smaller than the soliton and therefore negligible.

See also Dasgupta+
Mukhi+Rajesh; Witten

- The soliton is string scale w.r.t. the *closed string metric*.

THE NC DECOUPLING LIMIT

- String theory with D-branes and B-fields reduce to NCYM in the low energy decoupling limit

  $$\alpha' \sim \varepsilon^{1/2} \to 0 \quad g_{ij} \sim \varepsilon \to 0 \quad \text{(in NC directions)}$$

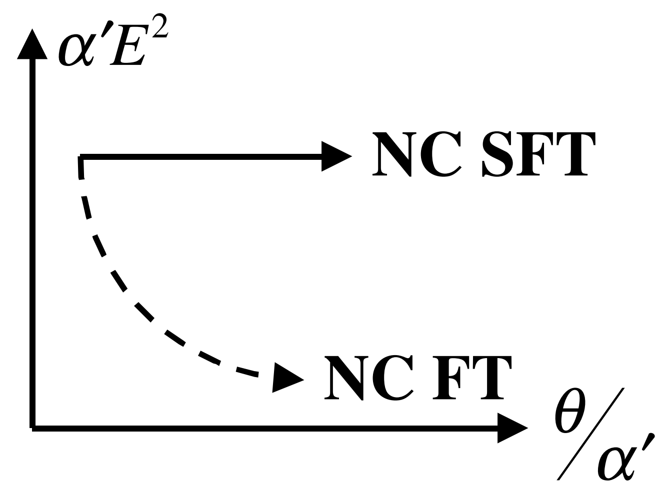
  Connes+Douglas++Schwarz; Seiberg+Witten

- This implies large non-commutativity

  $$\theta/\alpha' \sim \varepsilon^{-1/2} \to \infty$$

- We take $\theta/\alpha' \to \infty$ *without* taking a low energy limit; string excitations are therefore kept.

- Our limit is non-commutative String Field Theory (NCSFT).



EQUATIONS OF MOTION

- Take two directions non-commutative. Introduce *-product, open string metric G^{ij}, and open string coupling G_s in the Lagrangean

$$S_I = \frac{g_s T_{25}}{G_s} \int d^{26}x \sqrt{-G} \left[\frac{1}{2} f(t) G^{\mu\nu} \partial_\mu t \partial_\nu t + \ldots - V(t) \right]$$

- In the scaling limit the kinetic terms are negligible and the equations of motion become simply *V'(t)=0.*
- Goal: find lumps in this theory.
- Progress: the detailed form of the kinetic terms not needed.
- Problem: it is usually not possible to find *localized* solutions to the nonlinear equation *V'(t)=0*.

NON-COMMUTATIVE SOLITONS

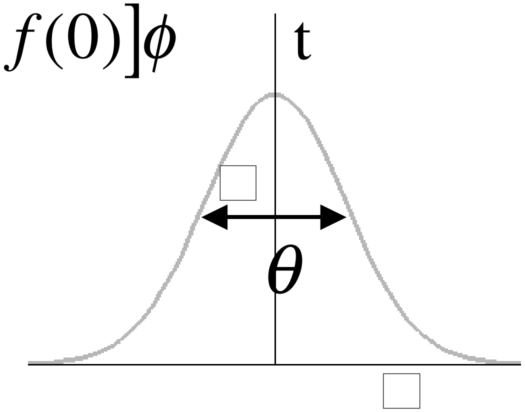
Gopakumar +Minwalla +Strominger

- Non-commutative multiplication allows nontrivial localized solutions to:

$$\phi * \phi = \phi \qquad (e.g. \quad \phi_0 = 2e^{-r^2/\theta})$$

- Functionals act in a simple way on such functions:

$$f(\lambda\phi) = \sum_{n=0} a_n \lambda^n \phi^n = f(0) + \left[f(\lambda) - f(0)\right]\phi$$



- If t_* is an extremum of $V(t)$ then

$$V'(t_*\phi_0) = V'(t_*)\phi_0 = 0$$

- so $t_*\phi_0$ solves the equations of motion.
- The corresponding energy is independent of the shape of $V(t)$:

$$E = \int d^2x V(t_*\phi_0) = \int d^2x V(t_*)\phi_0 = 2\pi\theta V_{\text{ext}}$$

D-BRANES AS NON-COMMUTATIVE SOLITONS

Harvey
+Kraus+FL
+Martinec

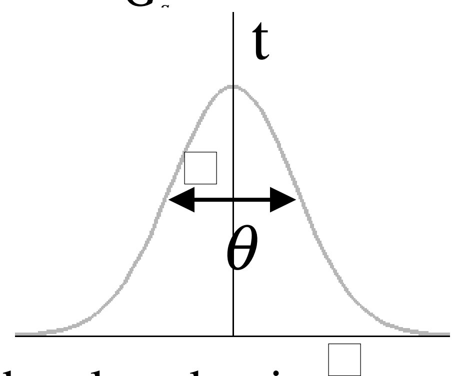
- Start with bosonic open string field theory with large non-commutativity.
- The effective action for the tachyon is

$$S = T_{25}\frac{g_s}{G_s}\int d^{26}x\sqrt{G}\left[\frac{1}{2}f(t)G^{\mu\nu}\partial_\mu t\partial_\nu t + \ldots - V(t)\right] \approx -T_{25}\frac{g_s}{G_s}\int d^{26}x\sqrt{G}\ V(t)$$

- Use the GMS soliton and find the energy

$$E = T_{25}\underbrace{\frac{g_s}{G_s}}_{2\pi\alpha'/\theta}\ 2\pi\theta\ \underbrace{V(t_*)}_{1} = T_{23}$$

exactly the right answer



- The tachyon vanishes far from the soliton; this is the closed string vacuum. Here the *B*-field is pure gauge so the lumps are equivalent to *D*-branes in the standard commutative vacuum.
- Construction generalizes trivially to even co-dimension branes.

THE OPERATOR FORMALISM

- Non-commutative functions in *2D* are in *1-to-1* correspondence with the *1D quantum* phase space.

$$A * B \leftrightarrow \hat{A}\hat{B} \qquad \frac{1}{2\pi\theta}\int d^2x \leftrightarrow \mathrm{Tr}$$

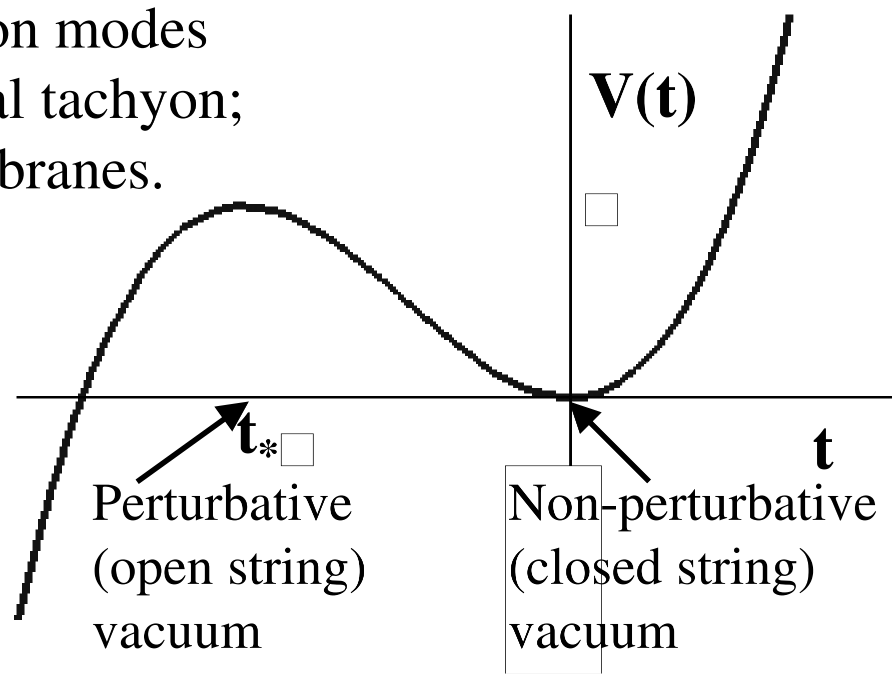
- Field configurations are thus interpreted as *operators*.
- Fields satisfying $\phi * \phi = \phi$ become *projection operators*.
- The simple Gaussian corresponds to projection onto the quantum ground state $t_0 = t_* \phi_{00} \qquad \phi_{00} \sim |0\rangle\langle 0|$
- Rank *k* projection operators are interpreted as *k* coincident *D*-branes $t_k = t_*(\phi_{00} + \phi_{11} + \ldots + \phi_{k-1,k-1}) \qquad \phi_{ij} \sim |i\rangle\langle j|$
- They have the right energy (the energy is proportional to the trace of the operator).

TACHYON FLUCTUATIONS

- It is interesting to explore excitations of the solitons.
- The simplest are fluctuations of the tachyon field.
- The spectrum of fluctuations are given by expanding the potential

$$t = t_k + \sum_{n,m=0}^{\infty} \delta t_{mn} \phi_{nm} \quad \Rightarrow \quad V(t) \approx V(t_k) + \frac{1}{2} V''(t_k) \sum_{n,m=0}^{k-1} \delta t_{mn} \delta t_{nm}$$

- The level k solution has k^2 tachyon modes with the same mass as the original tachyon; this is correct for k coincident D-branes.
- All that is needed is the potential and the kinetic terms near the *perturbative* vacuum.



THE GLOBAL $U(\infty)$ SYMMETRY

- The effective tachyon action in operator form is

$$S_I = T_{23} \int d^{24}x \, \mathrm{Tr}\left[\frac{1}{2} f(\hat{t}) G^{\mu\nu} \partial_\mu \hat{t} \partial_\nu \hat{t} + \ldots - V(\hat{t})\right]$$

- The kinetic term is negligible for large non-commutativity; so the action is invariant under the $U(\infty)$ symmetry

$$\hat{t} \to U \hat{t} U^\dagger$$

- Origin of symmetry: area-preserving diffeomorphisms in transverse space (these are preserved by *-product).
- The symmetry gives a large degeneracy of solutions at any level (rank of projection operator).
- *E.g.* a single D-brane corresponds to any of the operators

$$t_0 = t_* \phi_{nn} \qquad \phi_{nn} \sim |n\rangle\langle n|$$

GAUGE SYMMETRY

- The original space-filling *D25*-brane is invariant under non-commutative *U(1)* gauge transformations

$$\delta_\lambda A_\mu = \partial_\mu \lambda - i[A_\mu, \lambda] \qquad \delta_\lambda t = -i[t, \lambda]$$

where $[a,b] = a * b - b * a$

- In the operator formalism the dependence on the non-commuting coordinates is replaced by infinite matrices.
- The $D=23+1$ theory thus has exact $U(\infty)$ gauge invariance.
- This is even before solitons are considered.
- The $U(\infty)$ relates projection operators of the same rank; so degenerate solitons are in fact *gauge equivalent* and not distinct states.

LOCAL VS. GLOBAL SYMMETRY

- Before introduction of gauge fields the $U(\infty)$ is *global* and *approximate* (broken by finite non-commutativity).
- The gauge fields give a $U(\infty)$ symmetry which is *local* and *exact* (even for finite non-commutativity).
- This is *not* the standard gauging of a global symmetry.
- The full invariance under area-preserving diffeomorphisms is not there before introduction of gauge fields.
- Important analogy: coordinate transformations in general relativity are gauged by coupling to metric.
- Gauge symmetry and spacetime symmetries are mixed up in the non-commutative setting!

NON-ABELIAN GAUGE SYMMETRY

- Insert operator-valued fluctuations of the soliton in the effective low energy action:

$$A_\mu(x) = \sum_{n,m=0}^{\infty} A_{\mu,mn}\phi_{nm} \qquad\qquad t(x) = \sum_{n,m=0}^{\infty} t_{mn}\phi_{nm}$$

$$S = T_{25}\frac{g_s}{G_s}\int d^{26}x\sqrt{G}\left[\frac{1}{2}f(t)D_\mu tD^\mu t - \frac{1}{4}h(t)F_{\mu\nu}F^{\mu\nu} - V(t) + \ldots\right]$$

- Result: The $U(\infty)$ is broken to $U(k)$ by a soliton at *level k*.
- The descendant $U(k)$ gauge symmetry has correct interactions

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i\left(A_\mu * A_\nu - A_\nu * A_\mu\right) \;\rightarrow\; F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i\left[A_\mu, A_\nu\right]$$

- From a theory with *abelian* gauge group *U(1)* and *non-commutative* spacetime we find a theory with *non-abelian* group *U(k)* and *commutative* spacetime!

THE BI-FUNDAMENTAL FIELDS

- The soliton also preserves a $U(\infty\text{-}k)$ subgroup of $U(\infty)$. This gives no physical excitations because the kinetic energy vanishes in the closed string vacuum $h(t_*)=0$.
- The "off-diagonal" components of the infinite matrix $A_{\mu,mn}$ give fields in the bi-fundamental of $U(k) \times U(\infty\text{-}k)$.
- These symmetries are spontaneously broken by the soliton so the gauge fields acquire mass by the Higgs mechanism (off-diagonal tachyons give the missing polarizations).
- The mass is estimated as string scale by keeping only quadratic terms. Higher orders are not negligible; they (should) remove the fields from the spectrum.

Kostelecky +Samuel

THE CLOSED STRING VACUUM

- Infinite non-commutativity is crucial for the analysis.
- The closed string vacuum is independent of the *B*-field; but this is not manifest from the open string point of view.
- *New variables* shed light on this problem (and more generally on the structure of the closed string vacuum).
- Example: in non-commutative geometry derivatives are algebraic ($\partial \to -[a^\dagger,\cdot]$ $\bar{\partial} \to [a,\cdot]$) so the field strength is

$$F = -iF_{z\bar{z}} = \left[C,\overline{C}\right] + \frac{1}{\theta} \quad \text{where } C = a^\dagger - iA \;,\; \overline{C} = a + i\overline{A}$$

Gopakumar+ Minwalla+ Strominger; Seiberg; Sen

- Proposal: the closed string vacuum is

$$t = t_* \qquad C = \overline{C} = 0 \qquad \left(\text{instead of} \quad t = t_* \qquad A = \overline{A} = 0\right)$$

Standard minimum

operator equations specifying the gauge field

SYMMETRY RESTORATION

- In the new variables vacuum has

$$\mathrm{Tr}\, F_{\mu\nu}F^{\mu\nu} = \mathrm{Tr}\, \frac{1}{\theta^2}\cdot I = \infty$$

- This is no problem because the corresponding energy is

$$\mathrm{Tr}\left[\mathrm{h}(\mathrm{t}_*)F_{\mu\nu}F^{\mu\nu}\right] = \mathrm{Tr}\left[0\cdot\frac{1}{\theta^2}\cdot I\right] = \infty$$

- The new variables restore $U(\infty)$ symmetry of vacuum

$$C_\mu \to UC_\mu U^\dagger \quad \text{leaves } \mathrm{C}_\mu = 0 \text{ invariant}$$

- A fundamental problem in string theory: what is the full symmetry (spontaneously broken by any perturbative string background)?

Recent discussion: Shenker

- The symmetry restoration in open string field theory may give a paradigm for the discussion.

THE VACUUM IN OPEN VARIABLES

- Non-commutative field strengths are related to ordinary ones:

$$F^{\mathrm{ord}} = F\frac{1}{1-F/\theta} = \infty$$

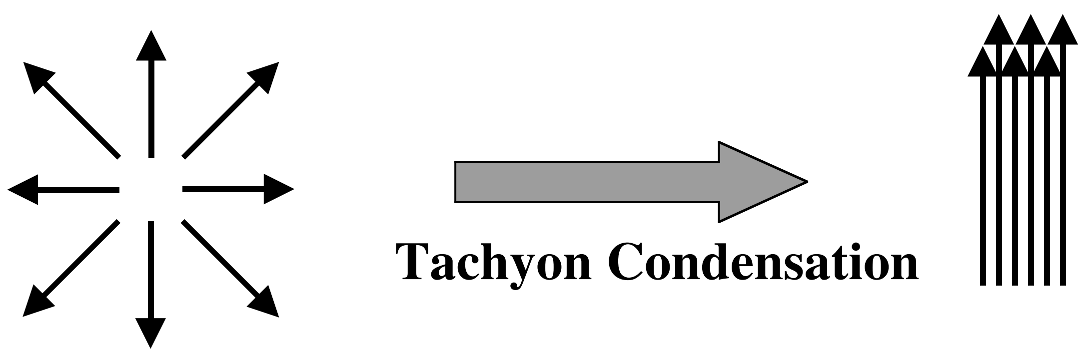
Seiberg+ Witten

- In ordinary variables only $F^{\mathrm{ord}}+B$ is physical so we may as well take $F^{\mathrm{ord}}=0, B=\infty$.
- The issue: infinite non-commutativity $B=\infty$ is determined *dynamically* (but the open variables do not describe how).
- A problem with symmetry restoration: infinite non-commutativity is required in all *Euclidean* directions; but then continuation to Minkowski signature is impossible.

OPEN STRING CONFINEMENT

- The closed string vacuum does not support open string degrees of freedom; they are *confined*.
- Understanding the detailed confinement mechanism is difficult, particularly in terms of open string variables.
- A true dynamical description is needed and is difficult. Symmetries are not sufficient.

Sen;Yi; Kraus+ Harvey; Bergman+ Hori+Yi; Harvey+ Kutasov+ Martinec



Kogut+ Susskind '74

Perturbative vacuum:
flux-lines spread

Non-Perturbative vacuum:
flux-lines align (confinement)

SUMMARY

- Excitations of the closed string vacuum resulting from tachyon condensation are interesting.
- Large non-commutativity facilitates explicit construction of solitons interpreted as D-branes.
- Well-known properties of D-branes can be recovered from this point of view. A highlight: non-abelian symmetry.
- The *world-volume* gauge symmetry of D-branes plays a central role; it realizes *spacetime* symmetries.
- A major goal is to understand the structure of the closed string vacuum. This involves describing the *confinement* mechanism of the open strings.