Non-perturbative Temperature Instabilities in N = 4 Strings

Costas Kounnas.

ENS-Paris and CERN-Geneva

Work in collaboration with:

Ignatios Antoniadis

and

J-Pierre Derendinger

Outline of this Talk

1. Non-perturbative Thermal Instabilities

2. $N = 4$ Thermal Effective Potential

$$V(s, t, u;\ H_h,\ H_{IIA},\ H_{IIB})$$

(i) D-1=4 thermal moduli s, t, u.
(ii) Non-perturbative thermal dyonic modes

$$H_h, \qquad H_{IIA}, \qquad H_{IIB}$$

which become tachyonic in a region of the thermal (s, t, u)–moduli space.

• $V(s, t, u; ...)$ **is common for all non-perturbative dual-equivalent strings with $N = 4$ SUSY in $D = 5$.**

• $V(s, t, u; ...)$ **describes the high-T instabilities of all known $N = 4$ superstrings.**

3. High-T Phase-Transitions.
Condensation of Euclidean 5-Branes at high-T.

Introduction

The partition function Z and the mean energy U develop power pole singularities if the density of states of a system grows exponentially with the energy,

$$\rho(E) \sim E^{-k} \; e^{bE} \, ,$$

$$Z(\beta) = \int dE \;\, \rho(E) \;\, e^{-\beta E} \sim \frac{1}{(\beta - b)^{(k-1)}} \, ,$$

$$U(\beta) = -\frac{\partial}{\partial \beta} \ln Z \sim (k-1) \frac{1}{\beta - b} + \ldots$$

- **Critical temperature,**

$\beta^{-1} \equiv T = T_H = b^{-1}$**,**

at which various thermodynamical quantities diverge.

[Hagedorn, Huang+Weinberg.]

Alternative interpretation of $U(\beta)$ pole singularity follows from the identification of the temperature with the inverse radius of a compactified euclidean time on S^1: $2\pi T = 1/R$

• **In this representation, the partition function is given by the (super)trace over the thermal spectrum of the theory in $(D-1)$ dimensions.**

$$\ln Z(\beta) = \mathrm{Str} \ln \mathcal{M}(\beta)$$

The pole singularity is a manifestation of a thermal state which becomes massless at the T_H, Hagedorn temperature.

$\rightarrow$ The knowledge of the thermal spectrum , $\mathcal{M}(\beta)$, as function of the S^1 radius $R = \beta/2\pi$ determines T_H.
[Sathiapalan, Kogan]

$* * *$ Perturbative string theories provide examples of exponentially growing density of states, with:

$k = D$, the dimension of space-time, and
$b^{-1} \sim \mathcal{O}(\alpha')^{-1/2}$.
[Axenides + S. Ellis + Kounnas]
[Antoniadis + J. Ellis + Nanopoulos]

The states which become tachyonic at T_H have necessarily a non-zero winding number n.
[Atick + Witten],
[Kounnas + Rostand],
[Antoniadis + Kounnas]

$\rightarrow$ In any perturbative field theory the winding sector is absent

→ there is no similar critical behaviour at high T in any field theory at the perturbative level.

•• **What about the thermal instabilities due to:**

i) Non-perturbative string states,

and/or

ii) Non-perturbative field theory states ?

Observations about the $N = 4$ BPS states

• **In all perturbative $N = 4$ strings the states which become tachyonic above T_H are BPS states preserving half of the supersymmetries ($N = 2$).**

• **The masses of the non-perturbative BPS states are known due to $N = 4$ supersymmetry.**

[Ferrara + Savoy + Zumino], [Sen],
[Schwarz + Sen], [Cvetic + Youm],
[Duff + Liu + Rahmfeld],
[Dijkraff + Verlinde + Verlinde],
[Kounnas + Kiritsis]

• **We can identify the non-perturbative BPS states able to induce high-T instabilities using the heterotic–type IIA–type IIB string dualities.**

[Hull + Townsend, Witten]

• In space-time dimensions $D \geq 6$ all BPS states preserve $N = 2$ (1/2–BPS).

• In lower dimensions BPS states can be 1/4–BPS or even 1/8–BPS in D=3 and 2.

$\rightarrow$ In dimension six or higher all thermal instabilities are due to $N = 2$ BPS states. The entropy of those states grow linearly with the mass.

This behaviour of the entropy is similar to the non-zero winding sector of the perturbative strings

• In $D \leq 5$ the entropy grows faster than linear in mass due to the 1/4–BPS and 1/8–BPS states .

This behaviour is supported by the the area law grow of the black hole entropy in $D = 4, 5$

[Maldacena + Susskind],

[Dijkgraff + Verlinde + Verlinde]

In this case however, the temperature is fixed and the canonical ensemble does not exist.

Perturbative analysis

Thermal partition function:

Modification of the boundary conditions around the S^1-euclidean time [spin-statistics].

$$\Psi(t + 2L\pi R) = (-)^{La}\Psi(t)\,.$$

$a = 0 \rightarrow$ **Bosons.**
$a = 1 \rightarrow$ **Fermions.**

• **The generalization to string theory is dictated by modular invariance.**

[Atick + Witten], [Kounnas, Rostand]

$$(-1)^{La+nb+\delta Ln}$$

L **and** n **are the winding numbers along the two non-contractible loops on the world-sheet torus.**

a **and** b **denote the fermionic spin structures along the two cycles.**
$\delta = 1$ **for the heterotic string and**
$\delta = 0$ **zero for the type IIA and IIB strings.**

The consequence of this phase is to shift the lattice momenta of the S^1

$$P_{L,R} = \frac{1}{R}[\ m + \frac{a}{2} - \frac{n\delta}{2} \pm \frac{nR^2}{\alpha'}],$$

and reverse the GSO projection in the n**–odd winding sector.**

•• Theories with D**-dimensional space-time supersymmetry look at finite T as if SUSY is spontaneously broken in** $D-1$ **dimensions.**

A state with S^1 **charges** $(m,\ n,\ Q)$ **is mapped in the thermal case to:**

$$n \to n' = n,$$

$$m \to m' = m + \vec{e}\cdot\vec{Q} - \vec{e}\cdot\vec{e}\,\frac{n}{2}$$

$$\vec{Q} \to \vec{Q}' = \vec{Q} - \vec{e}\,n$$

$$Q' \equiv Q'_L + Q'_R$$

is the $D-1$ **helicity operator.**

[Rohm], [Kounnas + Porrati]
[Kounnas + Rostand], [Antoniadis + Kounnas]

The perturbative mass formula in SUSY theory:

$$\frac{1}{4}\alpha'\mathcal{M}^2 = \sum_{i=1}^{4} Q_i^2 - 1 + \frac{1}{2}\alpha' P_L^2 + \frac{1}{4}\alpha'\mathcal{M}^2_{rest},$$

$\mathcal{M}^2_{rest}$ **is the the contributions of the oscillator modes $\oplus$ the momenta of the remaining part of the other compactified dimensions.**

The four $U(1)$ charges Q_i are the four 10-D helicities $\rightarrow$ R-charges.

Q_i **are integers for space-time bosons (NS-states), half integers for space-time fermions (R-states).**

The SUSY GSO projection

$$\sum_{i=1}^{4} Q_i = \text{odd integer}, \qquad \sum_{i=1}^{4} Q_i^2 = \text{odd integer}$$

$\rightarrow$ **The lowest BPS states have:**

$$\mathcal{M}_{rest} = 0 \quad \text{and} \quad \sum_{i=1}^{4} Q_i = \pm 1, \qquad \sum_{i=1}^{4} Q_i^2 = 1$$

The mass formula for the lightest BPS fermions, gauge bosons and scalars have even windings n

$\rightarrow$ $\frac{1}{2}\mathcal{M}^2 = P_L^2 \geq 0$, **with** $m \rightarrow' m' = m + \vec{e} \cdot \vec{Q} - \vec{e} \cdot \vec{e}\ \frac{n}{2}$
$\rightarrow$ **tachyonic states are not present if n is even.**

States with odd winding number n can become tachyonic:

$$\frac{1}{4}\alpha'\mathcal{M}^2 = \frac{1}{2}\alpha' P_L^2 + n(n - 2Q_L)$$

The only states which can become tachyonic have $n = \pm 1$ and $Q_L = \pm 1$ ($= -Q_R$ for type II). They are scalars in (D-1)-dimensions; the longitudinal components of the D dimensional gravitons.

The Hagedorn temperature corresponds to the critical value of the S^1 radius at which the first tachyonic state appears as $2\pi R = T^{-1}$ decreases.

- **Heterotic tachyon:**

$$(m, n, Q') = \pm(-1, 1, 0), \quad 2\pi T = \frac{2 - \sqrt{2}}{\sqrt{\alpha'_H}}$$

- **Type IIA tachyon:**

$$(m, \tilde{m}', Q') = \pm(0, 1, 0), \quad 2\pi T = \frac{1}{\sqrt{2\alpha'_{IIA}}}$$

- **The appearance of tachyons can never arise in any perturbative supersymmetric field theory, since**

it behaves like the zero-winding sector of strings.

• **In non-perturbative supersymmetric field theories such an instability can arise due to thermal dyonic modes which behave as the odd winding string states.**

• **Heterotic–Type II duality exchanges the winding number n with the magnetic charge ℓ**

$$\frac{1}{2}\mathcal{M}^2 = \left(\frac{m}{R} + \frac{nR}{\alpha'_H} + \frac{\ell R}{\lambda_H^2 \alpha'_H}\right)^2 ,$$

• **m and n are the S^1 momentum and winding numbers, ℓ is the non-perturbative wrapping number for the heterotic 5-brane around $T^4 \times S^1$**

⊗ **λ_H is the string coupling in $D = 6$**

⊗ **The tension of the 5-brane: $T_5 {=} \frac{1}{\lambda_H^2}$ in α'_h units**

⊗ **The string coupling in $D = 5$:**

$$g_5^2 = \frac{\alpha'_H}{R} \lambda_H^2$$

Using the $\mathcal{S}$–duality relations:

$$\lambda_H = \frac{1}{\lambda_{IIA}}, \qquad \lambda_H^2 \alpha'_H = \alpha'_{II},$$

$\rightarrow$ **the mass formula for type IIA strings:**

$$\frac{1}{2}\mathcal{M}^2 = \left(\frac{m}{R} + \frac{nR}{\alpha'_{II}\lambda_{IIA}^2} + \frac{\ell R}{\alpha'_{II}}\right)^2 .$$

- **The momentum and winding numbers are now** m **and** ℓ
- n **is now the wrapping number for the type IIA NS 5-brane around** $K_3 \times S^1$**.**

- **From the six-dimensional viewpoint,** m/R**, is the Kaluza-Klein momentum, while the last two terms correspond to BPS strings with tension:**

$$T_{p,q} = \frac{q}{\alpha'_H} + \frac{p}{\alpha'_{II}},$$

p, q **are relatively primes:** $(n, l) = k\ (q, p)$.

The common divisor k **defines the wrapping of the** $T_{p,q}$ **string around** S^1**.**

p **is the charge of the fundamental string and** q **the magnetic charge of the solitonic string ob-**

tained by wrapping the NS 5-brane around K_3.

$T_{p,q}$**-strings cannot become tensioneless since there are never associated to vanishing cycles of the internal manifold.**

Thermal mass formula:

$$\frac{1}{2}\mathcal{M}_T^2 = \left(\frac{m+Q'+\frac{n}{2}}{R} + \frac{nR}{\alpha'_H} + \frac{\ell R}{\alpha'_{II}}\right)^2 - 2\left(\frac{\delta_{n,\pm 1}}{\alpha'_H} + \frac{\delta_{\ell,\pm 1}}{\alpha'_{II}}\right)\delta_{Q',0},$$

$\mathcal{M}_T^2$ **reproduces the perturbative result for both heterotic and type IIA string theories.**

$\otimes$ **In the heterotic perturbative limit** $\lambda_H \to 0$**, only the** $\ell = 0$ **states survive.**

$\otimes$ **In the type IIA perturbative limit** $\lambda_{II} \to 0$**, only the** $n = 0$ **states survive.**

$*\,*\,*$ **If the heterotic coupling** λ_H **is smaller than the critical value**

$$\lambda_H < \lambda_H^c = \frac{\sqrt{2}+1}{2}$$

the first tachyon appears at:

$$R = (\sqrt{2}+1)\sqrt{\alpha'_H/2}$$

$\to$ **heterotic Hagedorn temperature.**

$* * *$ **If the heterotic theory is strongly-coupled,**

$$\lambda_H > \lambda_H^c$$

the first tachyon appears at:

$$R = \sqrt{2\alpha'_H}\,\lambda_H = 2\sqrt{\alpha'_{II}/2}$$

$\rightarrow$ **type IIA Hagedorn temperature.**

5-dimensional theories at finite T

- **Includes type IIB strings**

IIA and IIB strings are related by T-duality:
The inversion of the compactification radius R_6 **from** $D = 6$ **to** $D = 5$

The thermal mass formula of the effective thermal theory in $D - 1 = 4$ **depends on three parameters:**

$\otimes$ **The string coupling** g_4

$\otimes$ **The temperature radius** R

$\otimes$ **The radius and** R_6 **from** $D = 6 \rightarrow D = 5$**.**

For convenience we introduce the combinations:

$$t = \frac{RR_6}{\alpha'_H} \quad u = \frac{R}{R_6} \quad s = \frac{1}{g_4^2} = \frac{t}{\lambda_H^2},$$

in terms of which the BPS mass formula is:

$$\mathcal{M}^2 = \frac{|m + ntu + im'u + in't + i\tilde{m}s + i\tilde{n}stu + \tilde{m}'su + \tilde{n}'st|^2}{stu}$$

m, n, m', n' **are the four** <u>**electric**</u> **momentum and winding numbers; the four** $U(1)^4$ **charges of the** T^2 **compactification.**

$\tilde{m}, \tilde{n}, \tilde{m}', \tilde{n}'$ **are their** <u>**magnetic**</u> **non-perturbative heterotic charges.**

$* * *$ **The 4-dimensional** <u>**thermal mass formula**</u> **is obtained by replacing** m **with** $m + Q' + n/2.$

The would be <u>**tachyons**</u> **in finite** T **corresponds to the states with** $m' = n' = \tilde{m} = \tilde{n} = 0.$

$$\frac{1}{2}\mathcal{M}_T^2 = \left(\frac{m + Q' + \frac{n}{2}}{R} + \frac{nR}{\alpha'_H} + \frac{\tilde{m}'R}{\alpha'_{IIA}} + \frac{\tilde{n}'R}{\alpha'_{IIB}}\right)^2$$

$$-2\left(\frac{\delta_{|n|,1}}{\alpha'_H} + \frac{\delta_{|\tilde{m}'|,1}}{\alpha'_{IIA}} + \frac{\delta_{|\tilde{n}'|,1}}{\alpha'_{IIB}}\right)\delta_{Q',0},$$

The various α' and the radius R are related with the duality invariant Planck scale.

$$\alpha'_H = 2\ \kappa^2\ s$$

$$\alpha'_{IIA} = 2\ \kappa^2\ t$$

$$\alpha'_{IIB} = 2\ \kappa^2\ u$$

$$R^2 = \alpha'_H\ t\ u = 2\ \kappa^2\ s\ t\ u$$

κ is the four dimensional gravity coupling:

$$\kappa = \sqrt{8\pi}\ M_P^{-1} = (2.4 \cdot 10^{18}\ \text{GeV})^{-1}.$$

- **R is identical for all perturbative string limits.**

$\tilde{m}'$ $\rightarrow$ wrapping number of the heterotic 5-brane around $T^4 \times S^1_R$ as in five dimensions.

$\tilde{n}'$ $\rightarrow$ the same wrapping number after performing a T-duality along the $S^1_{R_6}$ direction.

Tachyonic states $\rightarrow$ $|n| = 1,\ |\tilde{m}'| = 1,\ |\tilde{n}'| = 1$

Thermal effective potential

We can construct the full T-dependent effective potential associated with the would-be tachyonic states.

Our procedure:

$\otimes$ $D=5$, $N=4$ **theories at finite T** $\to$ $D-1=4$ **effective SUGRA in which SUSY is spontaneously broken.**

$\otimes$ **For the description of the instabilities, it is sufficient to retain the potentially massless and tachyonic states.**

- **Spin-0 spin-1/2 states, the graviton and the one of the gravitinos.**

$\otimes$ **This sub–spectrum is described by:**
$N=1$ **SUGRA** $\oplus$ **chiral multiplets at finite-T.**

$\otimes$ **The scalar manifold of a generic** $N=4$ **in** $4D$

$$\left(\frac{Sl(2,R)}{U(1)}\right)_S \times \left(\frac{SO(6,r+n)}{SO(6)\times SO(r+n)}\right)_{T_I,U_I,Z_I}.$$

$\otimes$ **We keep only the six BPS states** $Z_A^{\pm}$**,** $A=1,2,3$ **which can generate thermal instabilities in heterotic, IIA and IIB string limits.**

$\otimes$ **For consistency, these states must be supplemented by the moduli S, T and U.**

$oplus$ **The truncated $N=1$ scalar manifold becomes:**

$$\left[\frac{Sl(2,R)}{U(1)}\right]_s \times \left[\frac{Sl(2,R)}{U(1)}\right]_T \times \left[\frac{Sl(2,R)}{U(1)}\right]_U$$

$$\times \left[\frac{SO(2,3)}{SO(2)\times SO(3)}\right]_{Z_A^+} \times \left[\frac{SO(2,3)}{SO(2)\times SO(3)}\right]_{Z_A^-}$$

Kähler potential:

$$K = -\log(S+S^*) - \log(T+T^*) - \log(U+U^*)$$
$$-\log Y^+(Z_A^+, Z_A^{+*}) - \log Y^-(Z_A^-, Z_A^{-*}),$$

with

$$Y^{\pm} = 1 - 2Z_A^{\pm} Z_A^{\pm *} + (Z_A^{\pm} Z_A^{\pm})(Z_B^{\pm *} Z_B^{\pm *}).$$

The Kähler function G which of the $N=1$ SUGRA

[Cremmer + Ferrara + Girardello + Van Proyen]

$$G = K + \log|W|^2$$

G **is determined by comparing the gravitino mass term of the truncated $N=4\to N=1$**

$$m_{3/2} = W\ exp[\ \frac{K}{2}\ ]$$

Before thermal corrections $\rightarrow$ superpotential W:

$$W_{SUSY} = 4\sqrt{2}\big[(\frac{1}{2}TU-\frac{3}{2})Z_1^+Z_1^- + (\frac{1}{2}SU-1)Z_2^+Z_2^- + (\frac{1}{2}ST-1)Z_3^+Z_3^-\big].$$

W_{SUSY} **does not break SUSY and it is not appropriate T-finite.**

$\otimes$ **Breaking of the $N=4$ SUSY requires a gauging with non-zero structure constants among the compensating fields which are used to define, G/H, the space of the scalar manifold.**

- **The finite-T gauging can be extracted from the thermal spectrum either in the $N=4$ field theory or in the heterotic thermal string spectrum**

$\rightarrow$ **Scherk-Schwarz gauging.**

[Kounnas + Porrati],
[Ferrara + Kounnas + Porrati + Zwirner]
[Antoniadis + Kounnas],
[Antoniadis + Derendinger +Kounnas]

- **Finite-T $\rightarrow$ add to W_{SUSY} the SUSY breaking term:**

$$W_T = W_{SUSY} + \sqrt{2}\ \Big[\ (1+Z_A^+Z_A^+)(1+Z_B^-Z_B^-)\ \Big]$$

.

<u>Scalar potential</u>

• **The expression of the superpotential W_T, together with the Kähler potential K, defines <u>not only mass terms</u>, but the <u>full scalar sector</u> and its coupling to $N=1$ SUGRA.**

This allows to examine the vacuum structure from small or large values of $s = 1/g^2$.

$$V = \kappa^{-4} e^K \Big[\, (K^{-1})^i_j \, (W_i + W K_i) \, (\overline{W}^j + \overline{W} K^j) - 3|W|^2 \, \Big]$$

Important <u>simplifications</u> occur in field directions where tachyonic masses could arise.

$$s = \mathrm{Re}\, S, \quad t = \mathrm{Re}\, T, \quad u = \mathrm{Re}\, U,$$

$$\mathrm{Re}\, Z_A^+ = \mathrm{Re}\, Z_A^- \equiv z_A.$$

The $V(s,t,u;\ H_A$ becomes a simple 4^{th}–order polynomial in terms of H_A, with s,t,u-<u>moduli dependent</u> coefficients.

$$H_A = \frac{z_A}{1 - x^2}, \qquad x^2 = \sum_A z_A^2$$

$$\xi_1 = tu, \quad \xi_2 = su, \quad \xi_3 = st$$

$$V = V_1 + V_2 + V_3$$

$$\kappa^4 V_1 = \frac{4}{s}\Big[\ (\xi_1 + \xi_1^{-1})H_1^4 + \frac{1}{4}(\xi_1 - 6 + \xi_1^{-1})H_1^2\ \Big]$$

$$\kappa^4 V_2 = \frac{4}{t}\Big[\ \xi_2 H_2^4 + \frac{1}{4}(\xi_2 - 4)H_2^2\ \Big]$$

$$\kappa^4 V_3 = \frac{4}{u}\Big[\ \xi_3 H_3^4 + \frac{1}{4}(\xi_3 - 4)H_3^2\ \Big]$$

- **Thermal duality properties**

  $\xi_1 \ \rightarrow\ \xi_1^{-1}$: **heteroric finite-T duality.**

  $t \ \leftrightarrow\ u$, $H_2 \leftrightarrow H_3$: **IIA–IIB duality.**

The masse terms, $m_A^2 H_A^2$, determines the mass formulas around $H_A \sim 0$ for the heterotic, type IIA and type IIB tachyons:

$$m_1^2 = \frac{1}{2\kappa^2 s}\left[\xi_1^{-1} + \xi_1 - 6\right] = \frac{1}{\alpha'_H}\left[\frac{\alpha'_H}{2R^2} + \frac{2R^2}{\alpha'_H} - 6\right]$$

$$m_2^2 = \frac{1}{2\kappa^2 t}\left[\ \xi_2 - 4\ \right] = \frac{1}{\alpha'_{IIA}}\left[\ \frac{2R^2}{\alpha'_{IIA}} - 4\ \right]$$

$$m_3^2 = \frac{1}{2\kappa^2 u}\left[\, \xi_3 - 4 \,\right] = \frac{1}{\alpha'_{IIB}}\left[\, \frac{2R^2}{\alpha'_{IIB}} - 4 \,\right]$$

Vacuum structure of V.

[A] Type II Runaway Phase

V **does not have a minimum when:**

$$\xi_2 < 4 \ \text{ or } \ \xi_3 < 4$$

Either H_2 or H_3 are tachyonic around $H_A = 0 \rightarrow$ Type II instabilities $\rightarrow$ phase transition to a phase with $H_A \neq 0$

$$H_2^2 = \frac{4-\xi_2}{8\xi_2}, \quad \kappa^4 V_2 = -\frac{1}{t}\frac{(4-\xi_2)^2}{16\xi_2},$$

$$H_3^2 = \frac{4-\xi_3}{8\xi_3}, \quad \kappa^4 V_3 = -\frac{1}{u}\frac{(4-\xi_3)^2}{16\xi_3},$$

$\rightarrow \xi_2$ or ξ_3 **slides to zero, and $|H_2|$ and/or $|H_3|$ run to infinity.**

The type II instabilities are characterized by the critical temperatures:

$$\xi_2 = 4, \qquad 2\pi T_A^c = \frac{R_6}{4\sqrt{2}\kappa^2}$$

$$\xi_3 = 4, \qquad 2\pi T_B^c = \frac{1}{2g^2\ R_6}$$

[B] **<u>Low–T Heterotic Phase</u>**

V **possesses** **<u>stable phases</u>** **in the domain:**

$$\xi_{2,3}\ >\ 4, \qquad \xi_1\ <\ \xi_1^c\ =\ (\sqrt{2}-1)^2$$

in which

$$H_1 = H_2 = H_3 = V_1 = V_2 = V_3 = 0.$$

• V **vanishes identically and** s, t **and** u **are** **<u>flat</u>** **directions.**

$[C]$ **<u>High–T Heterotic Phase</u>**

$$\xi_1^c\ <\ \xi_1\ <\ \frac{1}{\xi_1^c}$$

$$\kappa^4 V = -\frac{1}{s}\,\frac{(\ \xi_1 + \xi_1^{-1} - 6\ )^2}{16(\ \xi_1 + \xi_1^{-1}\ )}$$

• **For fixed** s, V **has** **<u>a stable minimum</u>:**

$$\xi_1 = 1, \quad H_1 = \frac{1}{2}, \quad H_2 = H_3 = 0, \quad \kappa^4 V = -\frac{1}{2s}.$$

• **The high-T heterotic phase cannot be reached for any value of the radius R_6 if**

$$g^2 \; > \; g_c^2 \; \equiv \; \frac{\sqrt{2}+1}{2\sqrt{2}} \sim .8536.$$

In this case, T_h^c:

$$2\pi T_h^c = \frac{g}{2^{1/4}\kappa}(\sqrt{2}-1),$$

always exceeds T_A^c and T_B^c

$\rightarrow$ **Only type IIA or IIB thermal instabilities exist in the heterotic strong coupling constant regime.**

• $R_6/\sqrt{\alpha'_H}$ **decides whether the type IIA or IIB instability will have the lowest critical–T.**

• **If the heterotic string is weakly-coupled,** $g^2 < g_c^2$ **the high-T heterotic phase is reached for values R_6:**

$$T_h^c \; < \; T_A^c \quad \text{and} \quad T_h^c \; < \; T_B^c$$

$$2\sqrt{2}g^2(\sqrt{2}-1) < \frac{R_6}{\sqrt{\alpha'_H}} < \frac{1}{2\sqrt{2}g^2(\sqrt{2}-1)}$$

The small and large R_6 limit, with fixed g, lead again to type IIA or type IIB instability.

In both 5-dimensional limits:

(i) Type IIA:

$$R_6 \to \infty, \quad g_5^2 = R_6\, g^2 = \text{fixed}, \quad R = \text{fixed}$$

and

(ii) Type IIB:

$$R_6 \to 0, \quad g_5^2 = \frac{\alpha'_H}{R_6}\, g^2 = \text{fixed}, \quad R = \text{fixed}$$

The inequality which separates heterotic and type II instabilities becomes:

$$g_5^2 \;<\; \sqrt{\frac{\alpha'_H}{2}}\, \frac{\sqrt{2}+1}{2}$$

The high-T Heterotic Phase Transition

$$\langle tu \rangle = 1, \quad \langle H_1 \rangle = \frac{1}{2}, \quad \langle H_2 \rangle = \langle H_3 \rangle = 0,$$

• **It solve the equations of motion for all scalar fields with the exception of $s = \mathrm{Re}\, S$.**

⊗ **The dynamics of s and $g_{\mu\nu}$ is described by:**

$$\mathcal{L}_{bos.} = -\frac{1}{2\kappa^2} eR - \frac{e}{\kappa^2}(2s)^{-2}(\partial_\mu S^*)(\partial^\mu S) + \frac{e}{2\kappa^4 s}.$$

For all (fixed) values of s, the cosmological term is negative with and the geometry is anti-de-Sitter.

• **However the anti-de-Sitter background does not stabilize s.**

⊗ **$\mathcal{L}_{\text{bos.}}$ follows from a 2-D conformal field theory describing a non-critical string with central charge deficit:**

$$\delta \hat{c} = -4$$

• **Because of the runaway dilaton potential,**

$$V(\Phi) = -const.\, e^{-2\Phi}, \qquad s = e^{-2\Phi}$$

there is no static solution for Φ and/or $g_{\mu\nu}$.

$\rightarrow$ **The high-T heterotic phase has to correspond to a stable non-critical string solution.**

•• **The stability is ensured if there is a restoration of space-time SUSY defined around a non-trivial Dilaton and/or gravitational backgrounds.**

[Kounnas], [Antonadis + Kounnas]
[Antoniadis + Ferrara + Kounnas]
[Kiritsis + Kounnas + Luest]

$\rightarrow$ **The M_{boson}=M_{Fermion} degeneracy in high-T phase follows from the spectral flows a Superconformal Symmetry on the world-sheet $N_{sc} = 2, 4$.**

$\rightarrow$ **Euclidean 5-brane solution or Super-Liouville Strings in $5 + 1$ "dimensions".**

Candidate Backgrounds in High-T Phase:

[Kounnas], [Antonadis + Kounnas]
[Antoniadis + Ferrara + Kounnas]
[Kiritsis + Kounnas + Luest]

•• $5+1$ **Super-Liouville with** $\delta\hat{c} = -4$

with $2d$**-current algebra based on:**

$$U(1)_{Q^2=2} \times U(1)^3 \times U(1)_{R=1} \times U(1)_{R_6}$$

•• **Non-compact Parafermionic-spaces:**

$$\left[\frac{SL(2,R)}{U(1)_{V,A}}\right]_{k=4} \times \left[\frac{SL(2,R)}{U(1)_{V,A}}\right]_{k=4} \times U(1)_{R=1} \times U(1)_{R_6}$$

•• **Highly curved** $10D$ **Euclidean space,** $R_6 = 1$.

$$\left[\frac{SO(2,2)}{SO(2) \times SO(2)}\right]_{k=2} \times SO(4)_{k=1}$$

•• **$\mathcal{S}$- and $\mathcal{T}$- duals or marginal deformations of the above backgrounds.**

$\otimes$ **All the above high–T solutions are valid in the heterotic weak coupling regime in order:**

(i) to avoid the type II singularities

$$e^{2\Phi} << \frac{\sqrt{2}+1}{2\sqrt{2}} \sim .8536$$

and

(ii) to trust the perturbative string solution.

$\bullet\bullet\bullet$ **The weak coupling limitation can be achieved in the Axial Parafermionic space:**

$$e^{-2\Phi} = \frac{1}{g_0^2}\,(1+zz*)\,(1+ww*) \;>\; \frac{1}{g_0^2}$$

$$ds^2 = \frac{4dzdz^*}{1+zz^*} + \frac{4dwdw^*}{1+ww^*}$$

$$g^2 \;=\; e^{2\Phi} \;<\; g_0^2$$

$\Phi(z,w)$ **is bounded at any point,** (z, z^*, w, w^*), **of the four–dimensional parafermionic space.**