1. INTRODUCTION

THE STRING THEORY PROGRAMME

- Describe the elementary particles and their interactions
- Unify all known interactions (gravity included)

Heterotic $SO(32)$ and $E_8 \times E_8$ and type I the candidates for this target (they have gauge bosons in their spectrum).

D-branes (type II string-theory solitons) have also gauge fields on their world-volume.

Renewed interest in some old ideas and speculations that we live in a domain wall (brane in modern terminology).

Regge & Teitelboim
Rubakov & Shaposhnikov

$$\boxed{\text{Target: obtain the Standard Model in low energies.}}$$

Up-down approach:

D-brane configurations in D=10

$\Downarrow$

low-energy physics (SM?)

Down-up approach:

D-brane configurations in D=10

$\Uparrow$

Standard Model (+ ...)

Another place where string theory can be put to a test is Cosmology. Questions like: inflation, dark matter, flatness, ... should be answered.

Tseytlin, Vafa, Antoniadis, Bachas, Olive,
Veneziano, Kounnas, Kiritsis, Lüst, Rey
Brustein, Vasquez-Mozo, Barrow, ...

String-theory branes do not have gravity on their world-volume. Gravity exists on the bulk. How can we discuss cosmological questions in this framework?

• Generate a massless graviton on the brane (Randall-Sundrum).

Binetruy, Deffayet, Ellwanger, Langlois,
Lukas, Ovrut, Waldram, Kanti, Kogan,
Olive, Pospelov, Csaki, Erlich,
Hollowood, Terning, Abel, ...

• The motion of a D-brane in a gravitational background generates an effective cosmological evolution on the D-brane.

2. THE MODEL

Modeling our "Universe"
as a brane moving
in some background

10D gravitational background with spherical symmetry produced by macroscopic D-brane configuration

$$\boxed{ds^2 = \underbrace{-g_{00}(r)dt^2 + g_{ii}(r)d\vec{x}^2}_{\text{longitudinal}} + \underbrace{g_r(r)dr^2 + g_S(r)d\Omega^2}_{\text{transverse}}}$$

$+$ RR "electric" fields C_{p+1} $+$ dilaton Φ

The "Universe" as a microscopic brane probe in the above background

Brane (inter)actions

The brane dynamics is determined (in the IR) by the Born-Infeld action. We denote local coordinates on the brane by ξ^α, $\alpha = 0, ..., p$.

$$\boxed{S_p = \text{brane-tension} \times \text{“Volume”} + \text{brane-charge} \times \text{RR-potential}}$$

$$\boxed{S_p = T_p \int d^{p+1}\xi \; e^{-\Phi} \; \sqrt{\hat{g}_{\alpha\beta} + (F+B)_{\alpha\beta}} + \mu_p \int C_{p+1}}$$

If $X^M(\xi^\alpha)$ is the embedding of the Dp-brane in the target space, the induced metric on the brane $\hat{g}_{\alpha\beta}$ is

$$\boxed{\hat{g}_{\alpha\beta} = G_{MN}(X) \frac{\partial X^M}{\partial \xi^\alpha} \frac{\partial X^N}{\partial \xi^\beta}}$$

The potential C_{p+1} is the induced bulk RR-field, i.e.,

$$\boxed{C_3 = \frac{1}{3!} C_{M_1 M_2 M_3} \;\; \partial_{\alpha_0} X^{M_0} \; \partial_{\alpha_1} X^{M_1} \; \partial_{\alpha_3} X^{M_3} \;\; d\xi^{\alpha_0} \wedge d\xi^{\alpha_1} \wedge d\xi^{\alpha_3}}$$

We want to study the dynamics of a Dp-brane probe in a spherically symmetric background produced by a Dp′-brane

$$\boxed{\begin{array}{c} ds^2 = -g_{00}(r) dt^2 + g_{ii}(r) d\vec{x}^2 + g_r(r) dr^2 + g_S(r) d\Omega^2_{8-p'} \\ C_{01\ldots p} = C(r)\,, \qquad \Phi = \Phi(r) \end{array}}$$

The Lagrangian of our Dp-brane probe turns out to be (in the static gauge $\xi^0 = t,\ \xi^1 = x^1, ..., \ \xi^p = x^p$)

$$\boxed{L = \sqrt{g(r)^3\ [|g_{00}| - g_{rr}\ \dot{r}^2 - g_S(r)\ h_{ij}\ \dot{\varphi}^i\ \dot{\varphi}^j]} - C(r)}$$

where h_{ij} is the metric of the unit (8-p')-sphere parameterized by φ^i and

$$\boxed{r = r(t), \phi = \phi(t)}$$

The Lagrangian is most convenient written as

$$\boxed{\mathcal{L} = \sqrt{A(r) - B(r)\ \dot{r}^2 - D(r)\ h_{ij}\ \dot{\varphi}^i\ \dot{\varphi}^j} - C(r)}$$

$$A(r) = g^3(r)|g_{00}(r)|e^{-2\phi}\ ,\quad B(r) = g^3(r)g_{rr}(r)e^{-2\phi}\ ,$$
$$D(r) = g^3(r)g_S(r)e^{-2\phi}.$$

First integrals

$$\boxed{E = \frac{A(r)}{\sqrt{A(r) - B(r)\dot{r}^2 - D(r)h_{ij}\dot{\varphi}^i\dot{\varphi}^j}} - C\quad , \qquad \text{energy}}$$

$$\boxed{h_{ij}\dot{\varphi}^i\dot{\varphi}^j = \frac{\ell^2(A(r) - B(r)\dot{r}^2)}{D(r)(D(r)+\ell^2)}\quad , \qquad \text{angular momentum}}$$

Solving the equations of motion for $r = r(t)\,, \quad \varphi^i = \varphi^i(t)$, we find the induced metric on the Dp-brane probe

$$\boxed{d\hat{s}^2 = \left(- g_{00}(r) + g_{rr}(r)\dot{r}^2 + g_S(r)h_{ij}\dot{\varphi}^i\dot{\varphi}^j\right) dt^2 + g(r)d\vec{x}^2}$$

Define cosmic time

$$d\eta = \frac{|g_{00}|g^3e^{-2\phi}}{|C+E|}dt\,,$$

the probe metric is written as

$$\boxed{d\hat{s}^2 = -d\eta^2 + g\left(r(\eta)\right)\, d\vec{x}^2\,.}$$

Standard form of a homogeneous and isotropic cosmological model with Robertson-Walker metric in comoving coordinates.

A NON-STATIC UNIVERSE MAY BE INTERPRETED AS A D-BRANE MOVING IN SOME BACKGROUND

Standard Cosmology in brief

Einstein equations

$$\boxed{\begin{array}{c} R_{MN} - \frac{1}{2} g_{MN}\ R + \Lambda\ g_{MN} = 8\pi T_{MN}, \\ \\ T_{\mu\nu} = \rho u_\mu u_\nu + p(g_{\mu\nu} + u_\mu u_\nu). \end{array}}$$

Energy-momentum tensor that of a perfect-fluid

$$\rho = \text{energy density}, \quad p = \text{pressure}, \quad u_\mu = \text{fluid 4-velocity}.$$

Homogeneity and isotropy specify the Robertson-Walker metric

$$ds^2 = -dt^2 + \alpha(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega_2 \right),$$

$k(>,=,<)0$ for spherical, flat and hyperbolic spatial sections, respectively.

$$\boxed{\text{Friedmann equations:} \begin{cases} \frac{\ddot{\alpha}}{\alpha} = -\frac{4\pi}{3}(\rho + 3p) + \frac{\Lambda}{3} \\ \left(\frac{\dot{\alpha}}{\alpha}\right)^2 = \frac{8\pi}{3}\rho + \frac{\Lambda}{3} - \frac{k}{\alpha^2} \end{cases}}$$

Friedmann equations relate geometric variables $\alpha(t)$, k to the matter variables ρ, p. These equations can be solved if a relations between ρ and p is provided, i.e., an equation of state.

For diffuse matter

$$\boxed{p = w\ \rho}$$

where w a proportionality factor with $|w| \leq 1$ (energy-conditions).
Most common values:

$$\boxed{w = \begin{cases} 0 & \text{Nonrelativistic matter (“dust”)} \\ +1/3 & \text{relativistic matter, radiation} \\ +1 & \text{massless scalar} \\ -1/3 & \text{Cosmic strings} \\ -2/3 & \text{domain walls} \\ -1 & \text{cosmological constant} \end{cases}}$$

Then we find for the energy density

$$\boxed{\rho \sim \alpha^{-3(w+1)}}$$

$$\boxed{\begin{array}{ll} \text{Radiation} & \rho \sim \dfrac{1}{\alpha^4} \\ \text{Non-relativistic matter} & \rho \sim \dfrac{1}{\alpha^3} \\ \text{Cosmological constant} & \rho \sim \text{const.} \end{array}}$$

We have seen that the metric of our Dp-brane-universe as it moves in the Dp′-background is

$$d\hat{s}^2 = -d\eta^2 + g(r(\eta))(d\vec{x})^2 .$$

By putting

$$\boxed{g(r(\eta)) = \alpha(\eta)\,,}$$

we may write the Friedmann equations and read off the effective "mirage" energy density

$$\boxed{\rho_{\text{effective}} = \frac{3}{8\pi}\left(\frac{\dot{\alpha}}{\alpha}\right)^2}$$

For the background metric

$$ds^2 = -g_{00}(r)dt^2 + g_{ii}(r)d\vec{x}^2 + g_r(r)dr^2 + g_S(r)d\Omega^2_{8-p'}\,,$$

we get the effective density

$$\boxed{\rho_{\text{effective}} = \frac{3}{4\pi}\frac{E^2 g_S e^{2\phi} - |g_{00}|(g_S g^p + \ell^2 e^{2\phi})}{4|g_{00}|g_{rr}g_S g^p}\left(\frac{g'}{g}\right)^2}$$

COSMOLOGY OF A PROBE D3-BRANE IN VARIOUS BACKGROUNDS

- AdS_5 black hole

The near-horizon limit of a macroscopic D3-brane is $AdS_5 \times S^5$. One the brane is black, the metric of the associated solution is

$$ds^2 = \frac{r^2}{L^2}\left(-f dt^2 + d\vec{x}^2\right) + \frac{L^2}{r^2} f^{-1} dr^2 + L^2 d\Omega_5^2 \,,$$

$$f = 1 - \frac{r_0^4}{r^4}\,, \qquad C = \frac{r^4}{L^4} - \frac{r_0^4}{2L^4}\,, \qquad \Phi = \text{const.}$$

The effective density is

$$\boxed{\rho_{\text{effective}} = \frac{3}{8\pi L^2}\left[\left(1 + \frac{E}{\alpha^4}\right)^2 - \left(1 - \left(\frac{r_0}{L}\right)^4 \frac{1}{\alpha^4}\right)\left(1 + \frac{\ell^2}{L^2}\frac{1}{\alpha^6}\right)\right]}$$

$$\boxed{\begin{array}{lcccc} w = & \frac{1}{3} & 1 & \frac{5}{3} & \frac{7}{3} \\ & \downarrow & \downarrow & \downarrow & \downarrow \\ & \text{radiation} & \text{massless scalar} & ? & ? \end{array}}$$

- Near horizon of a Dp-brane ($p > 3$)

The effective density corresponding to the motion of a D3-brane universe moving in the near-horizon of a black Dp-brane is

$$\boxed{\rho_{\text{eff}} = \left(\frac{(21-3p)}{32\pi L^2}\right)\ \alpha^{\frac{2(3-p)}{7-p}}\ \left[\frac{E^2}{\alpha^{2(7-p)}} - \left(1 - \left(\frac{r_0}{L}\right)^{7-p}\frac{1}{\alpha^4}\right)\left(1 + \frac{\ell^2}{L^2}\frac{1}{\alpha^{2(5-p)+\frac{8}{7-p}}}\right)\right]}$$

Effective indices

$$\boxed{\begin{array}{llcccccc} p=4: & w & = & -7/9, & 5/9, & 11/9, & 7/9, & 19/9, \\ p=6: & w & = & 1, & 5/3, & 7/3, & 3, & 13/3, \end{array}}$$

In particular, for the motion of a D3 in a D5-brane background we get

$$\boxed{\rho_{\text{effective}} = \frac{3}{32\pi L^2}\ \frac{1}{\alpha^2}\ \left[\frac{E^2}{\alpha^4} - \left(1 - \left(\frac{r_0}{L}\right)^4\frac{1}{\alpha^4}\right)\left(1 + \frac{\ell^2}{L^2}\frac{1}{\alpha^4}\right)\right]}$$

Effective indices

$$\boxed{p=5: \quad w = \ 1,\ \frac{7}{3}}$$

The term $-\alpha^{-2}$ corresponds to a 'mirage" curvature. Produces an effect identical to a spatial positive curvature.

How matter is described in this context?

EXAMPLE: Electric field on the probe brane.

$$\boxed{\mathcal{L} = \sqrt{A - B\dot{r}^2 - \mathcal{E}^2 g^2} - C}$$

$$\mathcal{E}^2 = 2\pi\alpha' \ E_i E^i \, , \quad E_i = -\frac{\partial A_i(t)}{\partial t} \, .$$

We find that

$$E_i = \frac{\mu_i}{g}\sqrt{\frac{A - B\dot{r}^2}{\mu^2 + g^2}} \, , \quad \mu_i \text{are constants}$$

The effective energy density turns out to be

$$\rho_{\rm eff} = \frac{3}{8\pi}\frac{(C+E)^2(1+\mu^2 g^{-2}) - |g_{00}|g^3 e^{-2\phi})}{4|g_{00}|g_{rr}g^3 e^{-2\phi}}\left(\frac{g'}{g}\right)^2 \, .$$

For a D3-brane universe in the background of a near-extremal D3-brane

$$\boxed{\rho_{\rm eff} = \frac{3}{8\pi L^2}\left[\left(1+\frac{E}{\alpha^4}\right)^2\left(1+\frac{\mu^2}{\alpha^4}\right) - \left(1-\left(\frac{r_0}{L}\right)^4\frac{1}{\alpha^4}\right)\left(1+\frac{\ell^2}{L^2}\frac{1}{\alpha^6}\right)\right]}$$

CONCLUSIONS AND OPEN PROBLEMS

A MODEL OF OUR UNIVERSE

We live on a D-brane floating in the vacuum

vacuum stability=brane equilibrium

towards equilibrium = brane motion

$\Downarrow$

COSMOLOGICAL EVOLUTION ON THE BRANE.

INTERPRETATION: "MIRAGE" DENSITY ON THE BRANE

Mirage Cosmology : cosmological evolution on the brane is interpreted as brane motion in the bulk.

- Initial Singularity.
- Inflation.