Generalized Two-Brane Models and the Resolution of the Hierarchy Problem

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Summary :

1. Introduction
2. Cosmological Evolution
3. Resolution of the Hierarchy Problem
4. Conclusions

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Introduction

• Extra space-like dimensions: An old idea : Kaluza-Klein theory, superstring theory...

... that was recently revived :

• A relatively large 11-th dimension in supergravity, felt only by gravity $\Rightarrow$ $M_{11} \sim M_{GUT}$ (Witten, Horava and Witten)

• d extra dimensions for gravity with size $L \leq 1$ mm $\Rightarrow$ $M_{4+d} \geq 1$ TeV (Arkani-Hamed, Dimopoulos and Dvali; Antoniadis, ADD)

• A number of extra dimensions for the Standard Model fields $\Rightarrow$ $M_{GUT} \ll 10^{16}$ GeV (Dienes, Dudas and Gherghetta)

• Brane-World Scenario : SM fields localized on a 3-brane, gravity propagates in the bulk

• The Hierarchy Problem : Choosing the position of our brane in the 5D spacetime $\Rightarrow$ $m_{EW}/M_P \simeq 10^{-16}$ (Randall and Sundrum)

• Unconventional Cosmology : For an empty bulk and $L = L_0$ (Kaloper and Linde; Lukas, Ovrut and Waldram; Binetruy, Deffayet and Langlois)

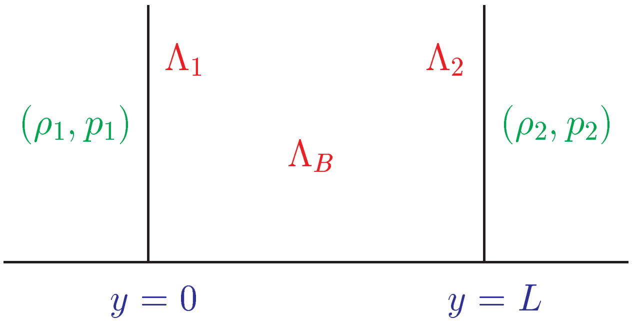
$$H^2 = \left(\frac{\dot{a}_0}{a_0}\right)^2 = \frac{\rho^2}{36 M_5^6} \neq \frac{\rho}{3 M_P^2}$$

(P. Kanti, I.I. Kogan, K.A. Olive and M. Pospelov : PLB, PRD, PLB, hep-ph/0005146)

Cosmological Evolution

- The Theoretical Framework: A 5D gravitational theory

$$S = -\int d^4x\, dy\, \sqrt{-\hat{g}}\,\Big\{\frac{\hat{R}}{2\hat{\kappa}^2} + \Lambda_B + \Lambda_1\,\delta(y) + \Lambda_2\,\delta(y-L) + \hat{\mathcal{L}}_o\Big\}$$



$$ds^2 = -n^2(t,y)\,dt^2 + a^2(t,y)\,\delta_{ij}\,dx^i dx^j + b^2(t,y)\,dy^2$$

- The Energy-Momentum Tensor:

$$\hat{T}_{MN} = \hat{T}^{(B)}_{MN} + \hat{T}^{(1)}_{MN} + \hat{T}^{(2)}_{MN}$$

where

$$\hat{T}^{M}_{(B)\,N} = \mathrm{diag}(-\Lambda_B, -\Lambda_B, -\Lambda_B, -\Lambda_B, \hat{T}^{5}_{(B)\,5})\,,$$

$$\hat{T}^{M}_{(i)\,N} = \mathrm{diag}\Big[\frac{\delta(y-y_i)}{b}\,(-\rho_i - \Lambda_i, p_i - \Lambda_i, p_i - \Lambda_i, p_i - \Lambda_i), 0\Big]$$

Cosmological Evolution

- The Equation for the Conservation of the EM tensor:

On the branes:

$$\frac{d\rho_i}{dt} + 3(\rho_i + p_i)\,\frac{\dot a_i}{a_i} + \rho_i\,\frac{\dot b}{b} = 0 \;\;\Rightarrow\;\; \boxed{b = const.}$$

In the bulk:

$$(\hat T^5_{(B)\,5})' + \hat T^5_{(B)\,5}\left(\frac{n'}{n} + 3\frac{a'}{a}\right) + \Lambda_B\left(\frac{n'}{n} + 3\frac{a'}{a}\right) = 0$$

If we write:

$$\hat T^5_{(B)\,5} = -\Lambda_B + \hat T^5_{(e)\,5} \;\;\Rightarrow\;\; \boxed{\hat T^5_{(e)\,5} = \frac{w(t)}{na^3}}$$

- Einstein's Equations in the Bulk: (Kanti, Kogan, Olive, Pospelov)

$$\hat G_{00} = \hat\kappa^2\,\hat T_{00} \quad\Rightarrow\quad f(a, a', a'', \dot a, n) = 0 \;\;(\surd)$$

$$\hat G_{ii} = \hat\kappa^2\,\hat T_{ii} \quad\Rightarrow\quad \text{trivial result} \;\;(\times)$$

$$\hat G_{05} = 0 \quad\Rightarrow\quad n(t,y) = \lambda(t)\,\dot a(t,y) \;\;(\surd)$$

$$\hat G_{55} = \hat\kappa^2\,\hat T_{55} \quad\Rightarrow\quad g(a, a', \dot a, \ddot a, n, n', \dot n, \hat T_{55}) = 0 \;\;(?)$$

Cosmological Evolution

- The Solution in the Bulk : If we assume that $\Lambda_B < 0$,

$$a^2(t,y) = d_1(t)\,\cosh(A|y|) + d_2(t)\,\sinh(A|y|) - \frac{2}{A^2\lambda^2(t)}$$

where $A^2 = 2\hat{\kappa}^2|\Lambda_B|/3$.

- Boundary conditions at $y=0$ and $y=L$:

$$(\mathbf{A}): \quad a^2(t,0) = a_0^2(t), \qquad a^2(t,L) = a_L^2(t)$$

$$(\mathbf{B}): \quad \frac{[a']}{a}\Big|_{y=y_i} = -\frac{\hat{\kappa}^2}{3}\,(\rho_i + \Lambda_i)$$

$$(\mathbf{C}): \quad \frac{[n']}{n}\Big|_{y=y_i} = \frac{\hat{\kappa}^2}{3}\,(3p_i + 2\rho_i - \Lambda_i)$$

$$\Downarrow$$

$$\boxed{\left(\frac{\dot{a}_0}{a_0}\right)^2 = \frac{\hat{\kappa}^2|\Lambda_B|}{3}\,\frac{\left[\frac{(\rho_1+\Lambda_1+\rho_2+\Lambda_2)}{\tanh(AL)} - \Lambda_{RS} - \frac{(\rho_1+\Lambda_1)(\rho_2+\Lambda_2)}{\Lambda_{RS}}\right]}{\Lambda_{RS} - (\rho_2+\Lambda_2)\,\tanh(\frac{AL}{2})}}$$

by using $n_0 = \lambda(t)\,\dot{a}_0 \equiv 1$ and $\Lambda_{RS} = \sqrt{6|\Lambda_B|/\hat{\kappa}^2}$.

(Kim; Lesgourgues, Pastor, Peloso, Sorbo)

Cosmological Evolution

And :

$$\frac{a_L^2}{a_0^2} = \frac{\Lambda_{RS} - (\rho_1 + \Lambda_1)\ \tanh(\frac{AL}{2})}{\Lambda_{RS} - (\rho_2 + \Lambda_2)\ \tanh(\frac{AL}{2})} \Rightarrow \frac{\dot{a}_L}{a_L} = \frac{\dot{a}_0}{a_0}\ f(\Lambda_{RS}, \Lambda_i, \rho_i, p_i)$$

- Standard Friedmann Equation : We define :

$$\alpha \equiv \frac{\Lambda_1}{\Lambda_{RS}}, \qquad \beta \equiv \frac{\Lambda_2}{\Lambda_{RS}}$$

(i) $\Lambda_{eff} = 0 \ \Rightarrow\ \Lambda_{RS}\left(-1 - \alpha\beta + \frac{\alpha+\beta}{\tanh(AL)}\right) = 0$

Special (RS) case : $\alpha = 1,\ \beta = -1,\ \forall\, L$

(ii) 4D $\kappa^2 \ \Rightarrow\ \kappa^2 = \frac{\hat{\kappa}^2\,|\Lambda_B|}{\Lambda_{RS}\tanh(AL)}\ \frac{1-\beta\tanh(AL)}{1-\beta\tanh(\frac{AL}{2})}$

(iii) $\mathcal{O}(\frac{\rho_i}{\Lambda_{RS}}) \ \Rightarrow\ \left(\frac{\dot{a}_0}{a_0}\right)^2 = \frac{\kappa^2}{3}\left(\rho_1 + \frac{\rho_2}{[\cosh(AL) - \beta\sinh(AL)]^2}\right)$

Note : Working in $\mathcal{O}(\frac{\rho_\mathrm{i}}{\Lambda_\mathrm{RS}})$, $\mathrm{H(0) = H(L)}$

Cosmological Evolution

- Stabilization of the extra dimension : Why $\hat{T}^5_5 \neq 0$? We set $b(t) = e^{\gamma(t)}$. Then:

$$\int dy \sqrt{-g}\, \frac{3}{\hat{\kappa}^2}\, D_\mu D^\mu \gamma = \int dy \sqrt{-g}\, e^{-\gamma}\, (\hat{T}^\mu_\mu - 2\hat{T}^5_5)$$

(Csaki, Graesser, Randall and Terning; Ellwanger)

- In this case : From (55)-component :

$$\hat{T}^5_{(e)\,5} = \frac{a_0^3\, |\Lambda_B|}{n a^3} \left\{ \alpha^2 + 1 - \frac{2\alpha}{\tanh(AL)} - \frac{2\beta}{\sinh(AL)}\, \frac{1-\alpha\, \tanh(\frac{AL}{2})}{1-\beta\, \tanh(\frac{AL}{2})} \right.$$

$$\left. - \frac{(\rho_1 - 3p_1)\,[1-\beta \tanh(\frac{AL}{2})] + (\rho_2 - 3p_2)\,[1+\beta \tanh(\frac{AL}{2})]}{2\Lambda_{RS}\, \sinh(AL)\, \cosh(AL)\, [1-\beta \tanh(AL)]} \right\}$$

- Special cases : We demand $\hat{T}^5_{(e)\,5} = 0$, and :

(A) If : $\Lambda_1 = \Lambda_2 = \Lambda_B = 0$ (Binetruy, Deffayet and Langlois)

$\rho_1 = -\rho_2$ and $p_1 = -p_2 \Rightarrow H^2 \sim (\rho_1 + \rho_2 + ...) \sim \mathcal{O}(\rho_i^2)$

(B) If : $\rho_i = p_i = 0$ (Randall and Sundrum)

$$\alpha = -\beta = 1, \quad \forall L$$

(C) If : $\rho_i = p_i = 0$ (Tye and Wasserman)

$$\alpha = \frac{\tanh(\frac{AL}{2}) - \beta}{1-\beta\, \tanh(\frac{AL}{2})}$$

Resolution of the Hierarchy Problem

- Observable Brane: $n_L(t) \neq 1 \;\Rightarrow\;$ Non-FRW spacetime $\;\Rightarrow\;$ FRW (?)

What this implies for $a(t)$, Φ, m, ρ ?

$$S \;=\; \int d^4x\, dy\, \sqrt{\hat{g}}\, \Big[\, \hat{g}^{\mu\nu}\, \partial_\mu \Phi\, \partial_\nu \Phi + \lambda\, (\Phi^2 - v_0^2)^2 \Big] \delta(y - L)$$

$$= \int d^4x\, (a_L^3 n_L) \Big\{ [-\frac{1}{n_L^2}\, \dot{\Phi}^2 + \frac{1}{a_L^2}\, (\nabla \Phi)^2] + \lambda\, (\Phi^2 - v_0^2)^2 \Big\}$$

$$\tilde{g}_{\mu\nu} = \frac{1}{n_L^2}\, g_{L\mu\nu} \quad \Downarrow \quad d\tilde{s}^2 = -dt^2 + \tilde{a}^2(t)\, \delta_{ij} dx^i dx^j$$

$$= \int d^4x\, \sqrt{\tilde{g}}\, n_L^4 \Big[\frac{1}{n_L^2}\, \tilde{g}^{\mu\nu} \partial_\mu \Phi\, \partial_\nu \Phi + \lambda\, (\Phi^2 - v_0^2)^2 \Big]$$

$$\tilde{\phi} = n_L\, \phi \quad \Downarrow$$

$$= \int d^4x\, \sqrt{\tilde{g}}\, [\, \tilde{g}^{\mu\nu} \partial_\mu \tilde{\Phi}\, \partial_\nu \tilde{\Phi} + \lambda\, (\tilde{\Phi}^2 - \tilde{v}_0^2)^2]$$

Then :

$$\tilde{m} \;=\; n_L(t)\, m \qquad (\surd) \;-\; (?)$$

$$\tilde{a}(t) \;\equiv\; \frac{a_L(t)}{n_L(t)} \;\Rightarrow\; \frac{\dot{\tilde{a}}_L}{\tilde{a}_L} = \frac{\dot{a}_L}{a_L} - \frac{\dot{n}_L}{n_L} \qquad (?)$$

Resolution of the Hierarchy Problem

$$\frac{\dot{n}_L}{n_L} = \frac{\dot{a}_0}{a_0} - \frac{\dot{a}_L}{a_L} + \frac{(3w_1+1)\,\dot{\rho}_1\,\tanh(\frac{AL}{2})}{2\Lambda_{RS}\,[1-\alpha\tanh(\frac{AL}{2})]} - \frac{(3w_2+1)\,\dot{\rho}_2\,\tanh(\frac{AL}{2})}{2\Lambda_{RS}\,[1-\beta\tanh(\frac{AL}{2})]}$$

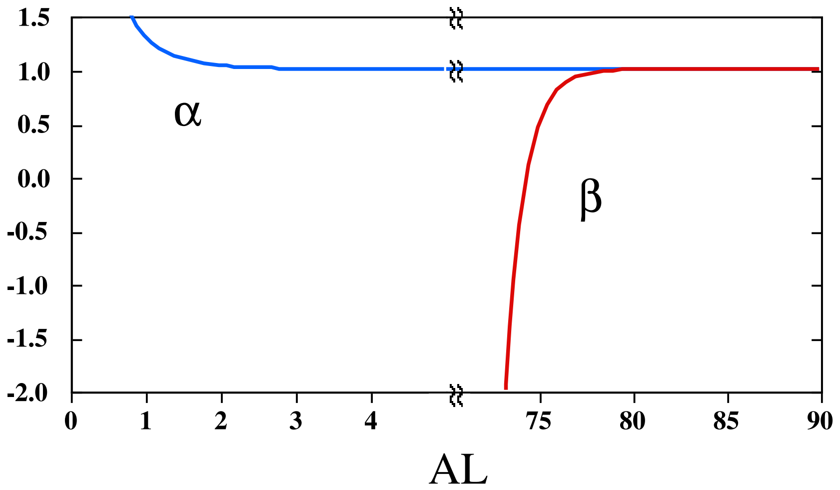
But :

(i) $\rho_i = \rho_c \; : \quad \dfrac{\dot{\rho}_i}{\Lambda_{RS}} \sim (\dfrac{\rho_c}{\Lambda_{RS}})^{3/2} M_P \sim 10^{-148}\, M_P$

(ii) $\rho_i = \rho_N \; : \quad \dfrac{\dot{\rho}_i}{\Lambda_{RS}} \sim (\dfrac{\rho_N}{\rho_c})^{3/2}\, 10^{-148}\, M_P \sim 10^{-90}\, M_P$

- Back to the Hierarchy Problem:

$$\frac{m_{EW}}{M_P} \simeq 10^{-16} = n_L \simeq \sqrt{\frac{1-\alpha\tanh(\frac{AL}{2})}{1-\beta\tanh(\frac{AL}{2})}}$$



Resolution of the Hierarchy Problem

- Asymptotic Behavior: In the limit $AL \to \infty$

$$\Big(\frac{\dot{a}_0}{a_0}\Big)^2 = \frac{\hat{\kappa}^2|\Lambda_B|}{3}\Big[-1+\frac{(\rho_1+\Lambda_1)}{\Lambda_{RS}}\Big] \;\Rightarrow\; \Lambda_{eff} = \Lambda_{RS}\,(a-1)$$

- Observed Energy Density: ρ_2 (non-FRW) $\Rightarrow$ (?) (FRW)

By using:

$$T^M_{\mu\nu} = \frac{2}{\sqrt{g_L}}\,\frac{\delta S^M}{\delta g_L^{\mu\nu}} \;\Rightarrow\; T^M_{\mu\nu} \to \tilde{T}^M_{\mu\nu} = n_L^2\,T^M_{\mu\nu}$$

Since: $\tilde{T}^\mu_\nu = (-\tilde{\rho}_2, \tilde{p}_2, \tilde{p}_2, \tilde{p}_2)$

$$\tilde{\rho}_2 = n_L^4\,\rho_2 \simeq \frac{\rho_2}{[\cosh(AL)-\beta\,\sinh(AL)]^2}$$

As a result,

$$\boxed{\Big(\frac{\dot{a}_0}{a_0}\Big)^2 \propto \Big(\frac{\dot{a}_L}{a_L}\Big)^2 \propto (\rho_1+\tilde{\rho}_2)}$$

Conclusions

- A 5D Two-Brane Cosmological Model : Contributions from a bulk cosmological constant, brane self-energies and time-dependent energy densities localized on the branes

- Restoration of the 4D Standard Cosmology: $H^2 \sim (\rho_1 + \tilde{\rho}_2)$

(i) The scale factor b should be constant

(ii) A (t, y)-dependent $\hat{T}^5_5$ should be present in the theory to create a unique minimum for b

(iii) The effective cosmological constant Λ_{eff} should be zero or very small $\Rightarrow$ a necessary fine-tuning between the parameters of the theory

(iv) The FRW character of the spacetime should be restored: $\tilde{\rho} = n_L^4\, \rho$

- Resolution of the Hierarchy Problem: A by-product of the restoration of the FRW character of spacetime

(i) Since: $\tilde{\rho} = n_L^4\, \rho \Rightarrow \tilde{m} = n_L\, m$

(ii) In every realistic cosmological model with $\rho_i(t) \neq 0 \Rightarrow n_L = n_L(t)$
For every $\rho_i \leq \rho_N \Rightarrow \dot{n}_L/n_L \ll \mathcal{O}(1)$

- Absence of Correlation:

(i) Arbitrary equations of state: $(\rho_1, p_1) \neq F(\rho_2, p_2)$

(ii) Two conditions imposed: $\Lambda_{eff} = 0,\ n_L \simeq 10^{-16} \Rightarrow (\alpha, \beta)$

(iii) The inter-brane distance L remains an independent parameter