September 3, 2021

It is a great honor for me to introduce Demetri Christodoulou – a laureate of the 2021 Poincaré Prize – awarded "for pathbreaking contributions to mathematical understanding of the Einstein equations, including fundamental results on black hole formation and the discovery of a nonlinear memory effect in the theory of gravitational radiation, and for introducing a powerful geometric point of view for the problem of shock formation for compressible fluids."

Demetri Christodoulou is a singular mathematician whose work has had a profound impact on the fields of General Relativity, hyperbolic partial differential equations and fluid dynamics.

Even though I described him as a mathematician, Demetri started his journey in the physics department. He received his PhD in physics in Princeton at the age of 19 under the direction of Johny Wheeler. His thesis showed existence of an *irreducible* mass of a black hole and played a key role in the development of black hole thermodynamics. It has been reported that Demetri's original thesis problem, which he was unable to solve at the time, was a bit more challenging and its solution required a 40 year detour.

This detour took Demetri first to mathematics and then to the point of view, his point of view, which sees general relativity as the arena of partial differential equations and geometry.

One of the early successes of this philosophy was his work throughout the 80's and the early 90's on the spherically symmetric Einstein-scalar field model. Here, Demetri proved a slew of remarkable results, giving an almost complete description of large data dynamics and establishing a very satisfying dichotomy: for generic initial data, gravitational and scalar waves either disperse and the spacetime converges to the flat Minkowski space, or a black hole with the exterior which converges to Schwarzschild forms. The generic caveat is crucial, for he also found exotic solutions containing so called naked singularities which, mercifully, turned out to be unstable. This circle of ideas was also an inspiration behind the discovery and the study of the so called *critical phenomena* in numerical relativity, in which one probes the *universality* of behavior on the boundary in transition from "regular" to "singular" regime.

A truly watershed moment was his proof, in 1993, jointly with Klainerman, of *stability of Minkowski space*. This was not just a fundamental result but it gave birth to the synthesis of Lorentzian geometry and hyperbolic PDE's. Its lasting impact is still felt today. This work, among other things, established the laws of gravitational radiation and led Demetri to the discovery of the *nonlinear gravitational memory effect* – a measurable phenomenon in which a wave train causes a permanent relative displacements of test masses. This is now known as the Christodoulou memory effect.

In 2009 Demetri returned to his original thesis problem, his pièce de résistance – the problem of black hole formation. Here, the story begins with the Penrose's incompleteness theorem from 1965 which guarantees geodesic incompleteness of any spacetime satisfying a null energy condition, possessing a non-compact Cauchy hypersurface, and also, crucially, a 2-d *trapped* surface. For all the incredible importance and influence of this result, it fails to address if this type of geodesic incompleteness (colloquially referred to as singularity) can develop in evolution. Yet, this result lies at the foundation of all of our current understanding of the predicted *theory of gravitational collapse* and, in particular, its mechanism of *black hole formation*. With trapped surfaces being a characteristic feature of black holes, this boils down to the question of evolutionary formation of trapped surfaces.

The problem lay dormant for 40 years until it was solved in 2009 by Christodoulou for the problem without matter (note that matter makes black hole formation easier) and without symmetry. It was a remarkable *tour de force.* The basis of it was an astonishing insight identifying a whole class of initial data which, on one hand are sufficiently large, since this phenomenon requires a strong gravitational field regime, while, on the other, still allows one to control the dynamics all the way to the eventual formation of a trapped surface. The appropriate data turned out to correspond to sharp directional bursts of curvature. It takes a painstaking analysis and a deep understannding algebraic structure of the Einstein equations to construct the necessary large enough portion of spacetime. It turned out to be an even more powerful idea when viewed in a more general PDE context

After that or, actually, already slightly before, Demetri turned to an even older subject – dynamics of 3-dimensional compressible fluids governed by the Euler equations. A fundamental feature of these equations is that its solutions develop shock singularities even when generated by smooth and, even more remarkably, small data. This is a classical phenomena much studied in the literature going back to Riemann and Stokes. The 20th century development of the subject was focused on creating a framework accommodating *global solutions containing shocks.* This became tractable only for the 1-dimensional equations and was based on analysis associated with the functions of bounded variation with the pioneering work by Glimm, Lax, Oleynik, Kruzkov and others. In higher dimensions, including the physical 3-dimensional case, conceptual and analytical difficulties were very high and progress very slow.

In 2007 Christodoulou published a monograph containing a proof of shock formation for the 3-dimensional relativistic Euler equations. There, he identified a precise class of initial data for which he constructed a *maximal Cauchy development* with a singular boundary and gave a complete geometric description of both the boundary and of the singular behavior of solutions. The result was a consequence of a completely novel *geometric* point of view on the problem. The corresponding results for the Newtonian case were given a self-contained adaption by Christodoulou-Miao in 2012.

It turns out, that part of the constructed Cauchy development becomes unphysical and has to be replaced by a physical solution containing a shock which emanates from the first singular surface. This is the problem of shock development, which, once again, was solved by Demetri in his monumental work in 2017, albeit in the restricted irrotational context.

These works signify the dawn of new era in the study of higher dimensional compressible fluid dynamics, or more general classes of equations admitting shocks, whose ultimate horizon is perhaps to supersede the global theory of weak shock admitting solutions, developed in the 1-dimensional case.

Let me conclude by saying that Christodoulou's impact on General Relativity, fluid dynamics and PDE's can not be overstated. It will be absorbed, internalized, and felt for many years to come. Demetri's career took him from Princeton, to Caltech, to Munich, to Syracuse, to New York, back to Princeton and then to Zurich. It was said about Feynman that the productivity of his colleagues was inversely proportionate to the distance from their offices to his. I think a bigger compliment might be the opposite statement. For Demetri, this can be attested by generations of mathematicians who have read and tried to digest Demetri's papers and books. I, personally, was fortunate enough to meet Demetri when I came to Princeton and then be influenced by his work over the years, so I was lucky to benefit from both. I would like to offer Demetri again my warmest congratulations on his outstanding achievement in winning the 2021 Henri Poincaré Prize.

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