Renormalization group flows in disordered field theories

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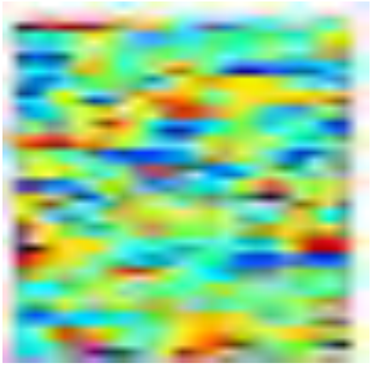

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Based on work in progress with V. Narovlansky, and on OA, Komargodski, Yankielowicz, 1509.02547

Disorder

- In QFT we like to assume that space-time is homogeneous. But in the real world this is never true !
- Lattices have impurities, background fields (magnetic, metric) are not constant (varying coupling constants), etc.
- Can ignore if scale of variation is much larger than scale of interesting physics. Usually true in particle physics, but often not true in condensed matter physics.

Motivations

- Near 2^{nd} order phase transitions have Euclidean CFTs + disorder ("classical disorder").
- Disordered materials may have space-dependent disorder ("quantum disorder"). Sometimes relativistic.
- Our goal is to try to understand the renormalization group flow in the presence of disorder (of both types) – which fixed points can disordered field theories (Euclidean or Lorentzian) flow to ? Do the disorder-averaged correlators obey a standard Callan-Symanzik equation ?

Quenched disorder

- Assume that the physical state of the system does not back-react on the disorder (e.g. cause impurities to come together): it is a non-dynamical background field = quenched disorder.
- So, we will take an ensemble of field theories with random background fields = coupling constants, compute something for each field theory and then average over the disorder (e.g. with the Gaussian distribution). Assume self-averaging – often (but not always) the case at long distances.

More simplifying assumptions

- Work with Euclidean/relativistic QFT (continuum limit).
- Take disorder to couple to a single scalar operator $\int d^d x\, h(x) O(x)$ or $\int d^d x\, dt\, h(x) O(x,t)$, generally most relevant operator.
- Disorder is (very) short-range. For simplicity take background fields / couplings *h* to vary independently and randomly at every point, e.g. Gaussian

$$\overline{h(x)} = 0, \quad \overline{h(x)h(y)} = c^2\, \delta(x-y)$$

Precise setup

- In order to obtain the desired distribution, average over *h(x).* For Gaussian use weight

$$\int [Dh] e^{-\frac{1}{2c^2}\int d^d x\, h^2(x)}$$

- Do <u>not</u> get a standard QFT w/correlators

$$\int [Dh] e^{-\frac{1}{2c^2}\int d^d x\, h^2(x)} \int [D\Phi] O_1(x_1) \ldots O_n(x_n) e^{-S[h]}$$

but rather disorder-averaged correlation functions are defined by

$$\overline{\langle O_1(x_1) \ldots O_n(x_n) \rangle} \equiv$$

$$\int [Dh] e^{-\frac{1}{2c^2}\int d^d x\, h^2(x)} \frac{\int [D\Phi] O_1(x_1) \ldots O_n(x_n) e^{-S[h]}}{\int [D\Phi] e^{-S[h]}}$$

Precise setup

- Usual definition of free energy with source :

$$e^{W[h]} = Z[h] = \int [D\Phi] e^{-S[h]}$$

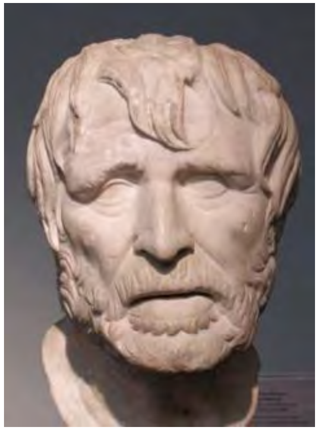
and then disordered free energy is

$$W_D = \int [Dh]\, W[h] e^{-\frac{1}{2c^2}\int d^d x\, h^2(x)}$$

- This governs the thermodynamical properties. Connected-disordered correlators are derivatives of this by other couplings $g_i(x)$.
- No good theoretical methods above $d=2$. In some cases can use perturbation theory (epsilon expansion). Often use Monte Carlo simulations, taking many random couplings and averaging. But no general RG analysis !

What can we do ?

- Hesiod (Ἡσίοδος) (~700 BC, near mount Helicon): "It is best to do things systematically, since we are only human, and disorder is our worst enemy."



How can we study this ?

- A general method is replica trick : recall

$$W_D = \int [Dh] \log(Z[h])\, e^{-\frac{1}{2c^2}\int d^dx\, h^2(x)} =$$

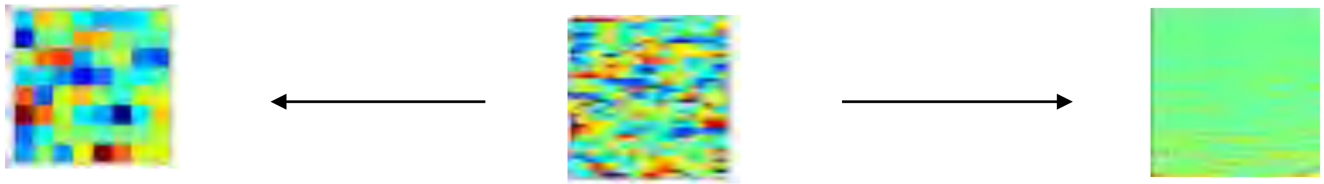
$$= \frac{d}{dn}|_{n=0} \int [Dh] Z^n[h]\, e^{-\frac{1}{2c^2}\int d^dx\, h^2(x)}$$

$$Z^n[h] = \int \prod_{A=1}^{n} [D\Phi_A]\, e^{-\sum_{A=1}^{n} S_A[h(x)]}$$

so a limit of standard field theories (*n* copies of original CFT all coupled to an extra non-dynamical "field" *h(x)*). General *n* and derivative can be non-trivial, but at least perturbatively (in any expansion) fine.

Classical disorder using replica

- We are interested in the RG flow of the disordered theory. For classical disorder it is a $n \to 0$ limit of standard RG flows, just with c^2 as an extra coupling constant (with dimension ($d-2\,\Delta_0$)), and starting with constant propagator for *h(x)*. So get standard RG (and standard Callan-Symanzik equation) ! At least perturbatively β, γ are polynomials in n.
- The new coupling c^2 may flow to zero and become irrelevant – end up in standard CFT – or flow to a constant value and flow to a disordered CFT (or gap)



Classical disorder using replica

- In Wilsonian RG can generate new couplings involving *h(x)* – change in disorder distribution. Can also generate disorder for other coupling constants.
- When disorder distribution is Gaussian, can also perform path integral over *h(x)* and rewrite replica theory as a completely standard field theory

$$S = \sum_{A=1}^{n} S_A + c^2 \sum_{A \neq B=1}^{n} \int d^d x \, O_A(x) O_B(x)$$

(generally no A=B term since short-distance limit is singular, except in free or large N theories).

- Same RG flow, here generate changes in disorder by extra terms coupling the two replicas.

Classical disorder using replica

- Interesting case is when disorder is almost marginal (c^2 is almost dimensionless). In this case we will only generate in the RG flow other operators which are close to being marginal, and *generally* there are no such operators (in conformal perturbation theory we would obtain all operators appearing in the OPE of $(O_A(x)\, O_B(x))$ with $(O_C(0)\, O_D(0))$). So the flow of c^2 will not mix with any other operators.
- Example : 3d Ising model with disorder for $\varepsilon(x)$ (random-bond), for which $\Delta \sim 1.41 \sim 1.5$. Flow to a weakly-disordered fixed point. (Komargodski, previous conference; Komargodski+Simmons-Duffin)

Classical disorder using replica

- The statement of a standard CS equation is not valid for high-dimension operators, since in the replica theory $\sum_A O_A$ can mix with $\sum_{A \neq B} O'_A \, O''_B$ if it has lower dimension. In the CS equation of the disordered theory, this mixes disorder-averaged connected correlators of O, $\overline{\langle O(x_1) O_1 \ldots O_n \rangle}$, and disorder-averaged products of connected correlators like $\overline{\langle O'(x_1) O_1 \ldots O_k \rangle \langle O''(x_1) O_{k+1} \ldots O_n \rangle}$. So we cannot diagonalize the anomalous dimension matrix of disconnected-connected correlators in general, and the same correlator will involve several different powers at an IR fixed point.

Quantum disorder

- Consider now the situation where we have a Lorentzian (relativistic) theory in which disorder is constant in time. We can still use the replica trick :

$$W_D = \frac{d}{dn}|_{n=0} \int [Dh] Z^n[h] \, e^{-\frac{1}{2c^2}\int d^dx \, h^2(x)}$$

$$Z^n[h] = \int \prod_{A=1}^{n} [D\Phi_A] \, e^{-\sum_{A=1}^{n} S_A[h(x)]}$$

But now the propagator of h is non-local in time so it is not a standard field theory. Naively (but not really in the disordered limit) a-causal.

Quantum disorder using replica

- When disorder distribution is Gaussian, can again perform path integral over h(x) but now we obtain explicitly a non-local (in time) theory :

$$S = \sum_{A=1}^{n} S_A + c^2 \sum_{A,B=1}^{n} \int d^d x \, dt \, dt' O_A(x,t) O_B(x,t')$$

(now we have also an A=B term since the operators are not at the same point).

- Naively Wilsonian RG flow makes no sense. But perturbatively (in any expansion) still get sensible results and can compute beta and gamma functions.

Quantum disorder using replica

- A big difference from the previous case is that now we always have a marginal operator $T_{00}(x)=-T_{ii}(x)$, and we expect it to be generated, namely

$$S \to S + g \sum_{A=1}^{n} \int d^d x \; dt \, T_{00}(x,t)$$

- In fact OPE implies that it is always generated at leading order in conformal perturbation theory, from the limit where $O_A(x,t')$ approaches $O_A(x,t)$:

$$\beta(g) = 2\frac{c_{OOT}}{c_T}c^2 + \cdots$$

- When g is finite this is simply a rescaling of the time direction (background metric g_{00}).

Lifshitz scaling in Quantum disorder

- However, in an RG flow we expect that the operator could acquire a non-zero anomalous dimension. This implies that the time direction acquires some anomalous dimension, and the theory ends up being invariant under a Lifshitz scaling transformation:

$$t \to \lambda^z t, \qquad x \to \lambda x$$

- Such fixed points are common in non-relativistic theories, and we see here that they generically arise also in relativistic disordered theories. And we see that in the renormalization group analysis z is an anomalous dimension just like any other ! Analysis should be relevant also in non-relativistic RG.

Lifshitz in holographic disorder

- Our analysis was motivated by an analysis of Hartnoll+Santos of a quantum-disordered relativistic holographic large N theory, where they discovered Lifshitz scaling for dimensionless c^2, with a Lifshitz parameter $z(c^2)$. (Holographic disorder was studied also in many other papers.) They did this perturbatively in the disorder c^2, and also numerically, averaging over different realizations.
- Can do same analysis directly in 1/N expansion – leading order identical to solving classical holographic equations of motion.

Disordered large N theories

- The beta function they find for g is the same as the general one we computed. They find the full flow leading to a fixed point with any c^2 and a non-zero anomalous dimension for g → Lifshitz scaling.
- Large N (or free) theories are actually more complicated, because there is a "double-trace" operator $O^2(x)$ that can mix in and is generated.
- Moreover, degenerate perturbation theory in the large N case with marginal disorder leads to a logarithmic CFT, in which correlators have extra logs (in position space), obey modified CS equation. Also found in not-connected correlation functions in general disordered theories (Cardy).

Summary

- We discussed renormalization group flows in two cases of “quenched disorder”.
- For classical disorder have standard flow including disorder parameter (and generally corrections to disorder distribution). At large N (or free) can get logarithmic CFTs.
- For quantum disorder naturally generate Lifshitz scaling as an anomalous dimension.
- Future : Hyperscaling violation ? OPE ? Number of degrees of freedom – is there a c-theorem ?
- Large N SYK-like theories (work in progress) ? (Random couplings lead to completely non-local analysis; may still self-average at large N.)