

Rigid Holography and 6d $N=(2,0)$ Theories on $AdS_5 \times S^1$



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Outline

- Why study field theories on $AdS_p \times M_q$?
- Rigid holography + simple examples.
- 6d A_{n-1} $\mathcal{N}=(2,0)$ SCFTs.
- How to study them on $AdS_5 \times S^1$ using rigid holography ?
- Naïve analysis \rightarrow a paradox \rightarrow a surprising duality.
- Summary and open questions.

Field Theories on $AdS_p \times M_q$ - why ?

- Field theories on **curved space** exhibit new features not visible in flat space.
- On **AdS** space have a new knob to turn : **boundary conditions**.
- Supersymmetric theories on $AdS_p \times M_q$ can preserve (all) **supersymmetry**. Hope to compute many things exactly. **Localization?**
- Can hope to learn more about mysterious theories (such as **6d $\mathcal{N}=(2,0)$ SCFTs**).
- Can use **AdS/CFT** !

Standard AdS/CFT

- A quantum gravity theory on AdS_{d+1} is equivalent to a CFT_d . Symmetries match.
- Need **boundary conditions** to define. At weak bulk coupling, fields in bulk map to operators (“**single-trace**”) in the CFT_d . Non-normalizable modes (fixed) map to **sources** for operators, normalizable modes (fluctuating) to their **VEVs**.
- Graviton maps to EM tensor T_{mn} .
- **Hilbert spaces** identical.

Rigid holography

- A quantum field theory on AdS_{d+1} still has same symmetries as a CFT_d .
- Bulk fields still sit in same representations as **local operators**, can identify source and VEV. Correlators of these operators obey usual conditions (unitarity, crossing, etc.).
- However, no T_{mn} , so cannot be equivalent to a **local CFT**.
- Example – free field in bulk = single operator and its products = “**generalized free field**”. Decoupled subsector of **CFT**.

Rigid holography

- Can we realize interacting theories as decoupled sector in a CFT ? Need to have a limit of the CFT where all correlators $\langle \mathcal{O}_{\text{QFT}} \dots \mathcal{O}_{\text{QFT}} \mathcal{O}_{\text{other}} \dots \mathcal{O}_{\text{other}} \rangle$ vanish, while some $\langle \mathcal{O}_{\text{QFT}} \dots \mathcal{O}_{\text{QFT}} \rangle$ remain finite.
- Vanishing of normalized $\langle \mathcal{O}_{\text{QFT}} \mathcal{O}_{\text{QFT}} T_{mn} \rangle$ requires $c \rightarrow \infty$, as in large N limit.
- But usually all correlators vanish, so need to have also some other parameters scaling with N to retain finite correlators.

How to find examples ?

- Can sometimes embed a field theory on $AdS_p \times M_q$ into string (M) theory on $AdS_m \times M_n$ which is dual to an $(m-1)$ dimensional CFT, and take a decoupling limit. So these QFTs are a subsector of $(m-1)$ dimensional CFTs (though not full local CFTs by themselves).
- In flat space string (M) theory with branes / defects, decouple low-energy field theory by taking M_s, M_p to ∞ keeping energies and couplings (g_{YM}) fixed.

How to find examples ?

- In string(M) theory on $AdS_m \times M_n$ with branes / defects filling $AdS_p \times M_q$, again take M_s and M_p to ∞ , but now need to keep R_{AdS} fixed. In dual CFT means taking $M_p R_{AdS} \sim N^\alpha$ to ∞ .
- Would like to keep QFT couplings fixed – may or may not be possible.
- Naturally keep SUSY.
- So field theory on $AdS_p \times M_q$ (with specific boundary conditions) = a subsector of the $(m-1)$ dimensional CFT. Rigid Holography

Caveats

- Is it an exact equivalence ? The dual subsector captures the response of the QFT on $AdS_p \times M_q$ to sources on the boundary.
- But this QFT also has local correlators in $AdS_p \times M_q$. Are these uniquely determined by the response to boundary sources ? (Are local QFT correlators in flat space determined by S-matrix ?)
- Get specific boundary conditions, not clear how to generalize.

Examples in IIB on $AdS_5 \times S^5$

- NS5-branes on $AdS_4 \times S^2$ (6d SYM, LST) :
 $M_P R_{AdS} \rightarrow \infty$ requires $N \rightarrow \infty$. $g_6^2 \sim \alpha'$. Can take $M_s \rightarrow \infty$, get free 6d SYM on $AdS_4 \times S^2$. Or can keep M_s fixed ($g_s \sim 1/N$), and get UV completion : $\mathcal{N}=(1,1)$ LST on $AdS_4 \times S^2$ (non-local non-conformal example).
- D1-branes on AdS_2 (2d SYM) : Again need $N \rightarrow \infty$. Now $g_2^2 R_{AdS}^2 \sim (N g_s^3)^{1/2}$. So can take $g_s \sim 1/N$ and get free 2d SYM, or can keep $N g_s^3$ fixed and get interacting 2d SYM.

Our main example

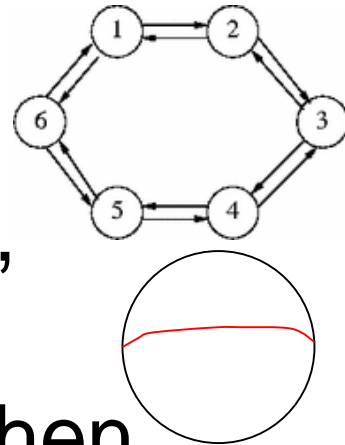
- 6d A_{n-1} $\mathcal{N}=(2,0)$ SCFT on $AdS_5 \times S^1$.
- It arises as the low-energy theory on n overlapping IIA NS5/M5-branes.
- The same theory arises in type IIB on a C^2/Z_n orbifold, at its singular point (related by string dualities).
- For a single M5-brane, low-energy theory is a free $\mathcal{N}=(2,0)$ tensor multiplet, containing 5 scalars (transverse motion), a self-dual 2-form ($dB = *dB$) and fermions.

Our main example

- For $n > 1$ get an interacting CFT whose only parameter is n . (Dual for $n \rightarrow \infty$.)
- M theory realization implies moduli space is R^{5n}/S_n (removing the center of mass R^5). At generic points get $(n-1)$ tensor multiplets.
- In IIB, it is realized by turning on blow-up modes for $(n-1)$ 2-cycles localized at the orbifold fixed point (3 scalars each), and two 2-form fields (B_2, C_2) on each of the 2-cycles.
- On $R^5 \times S^1$ at low energies get 5d $SU(n)$ SYM with $g_5^2 \sim R_S$, generally broken to $U(1)^{n-1}$.

$AdS_5 \times S^1$ embedded in string theory

- Consider **type IIB string theory** on $AdS_5 \times S^5 / Z_n$ = near-horizon limit of **K D3-branes** on C^2 / Z_n . Dual to **4d $\mathcal{N}=2$ $SU(K)^n$** elliptic quiver with bi-fundamental hypermultiplets (**Kachru-Silverstein**).
- Fixed points : $AdS_5 \times S^1$ in $AdS_5 \times S^5 / Z_n$, locally have a C^2 / Z_n orbifold there.
- At **orbifold** point **B_2 fields** non-zero. When vanish get **6d $\mathcal{N}=(2,0)$ A_{n-1} SCFT** on $AdS_5 \times S^1$ (coupled to rest of **type IIB**), with $R_{AdS} = R_S$ and specific **boundary conditions**.



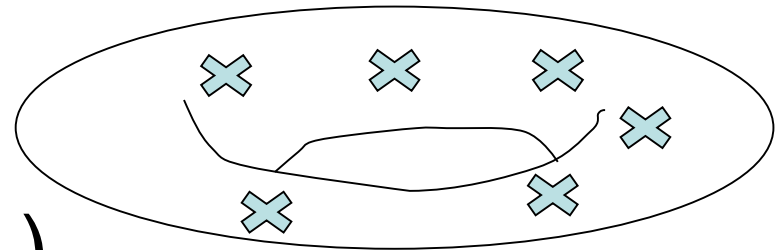
- 4d $\mathcal{N}=2$ SCFT has n exactly marginal deformations = complex gauge couplings.
- One maps to type IIB dilaton-axion.
- Other $(n-1)$ map to B_2 and C_2 on 2-cycles of singularity. Other blow-up modes tachyonic.
- Near this point “moduli space” (space of SUSY vacua on AdS_5) is C^{n-1}/S_n with A_{n-1} $(2,0)$ SCFT arising at the origin.
- Preserve 16 supercharges.
- At generic points on the “moduli space” have $(n-1)$ 6d 2-forms $\rightarrow U(1)^{n-1}$ gauge theory on AdS_5 . Dual to $U(1)^n$ global symmetry of hypermultiplets (diagonal $U(1)$ geometrical)₄

Naïve expectation

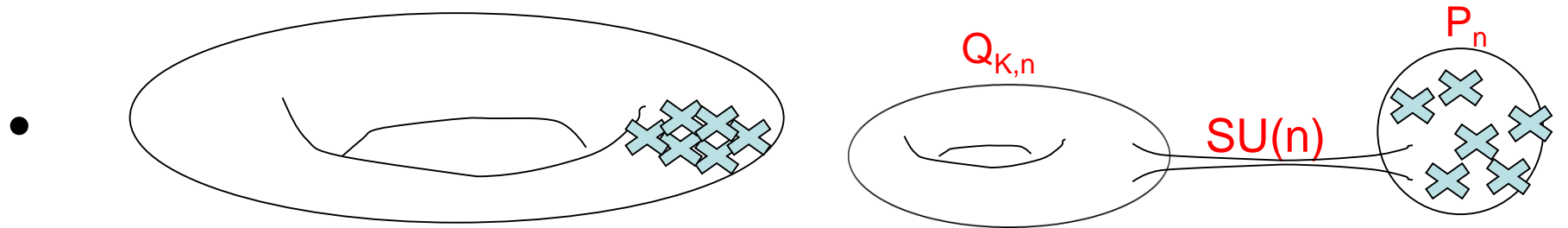
- At origin of “moduli space” expect $\mathcal{N}=(2,0)$ theory on S^1 to give an $SU(n)$ gauge theory on AdS_5 . Would mean global symmetry of 4d $\mathcal{N}=2$ SCFT enhanced to $SU(n)$.
- But can show from 4d $\mathcal{N}=2$ representations that global symmetries in 4d $\mathcal{N}=2$ SCFTs cannot be enhanced as a function of exactly marginal deformations (unlike in 4d $\mathcal{N}=1$), except at free points (high-spin currents).
- Consistent since W -bosons not BPS.
- What does happen in this 4d $\mathcal{N}=2$ SCFT ? ¹⁵

Singular limit in 4d $\mathcal{N}=2$ SCFT

- Space of couplings of $SU(K)^n$ quiver is the moduli space of n marked points on a torus (Witten). In Gaiotto language obtain this from A_{K-1} 6d $(2,0)$ theory on a torus with n minimal $(U(1))$ punctures. Has a weakly coupled $SU(K)^n$ limit. (Throats = weakly coupled 4d $\mathcal{N}=2$ theories.)
- Origin of “moduli space” : n punctures come together = $(n-1)$ couplings go to infinity.



Singular limit in 4d $\mathcal{N}=2$ SCFT



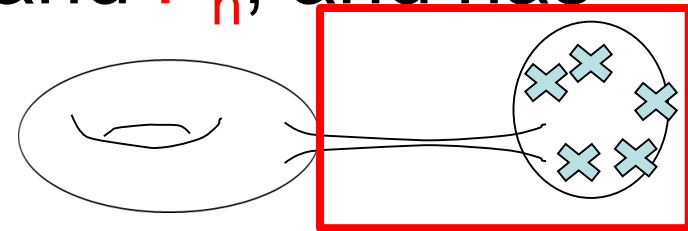
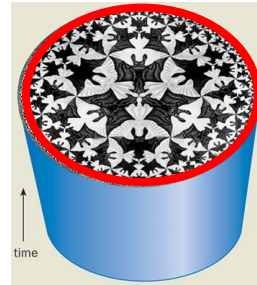
- Studied already (local on **Riemann surface**).
- **Global symmetry** not enhanced 😊, but get a **weakly coupled $SU(n)$ gauge theory**, with $g_{SU(n)}$ going to zero at origin, coupled to two different **4d $\mathcal{N}=2$ SCFTs** with **$SU(n)$ global symmetry** : A_{K-1} on a **torus** with a single **$SU(n)$ puncture ($Q_{K,n}$)** and a **sphere** with one **$SU(n)$ puncture** and **n $U(1)$ punctures (P_n)**.¹⁷

Singular limit in 4d $\mathcal{N}=2$ SCFT

- New $SU(n)$ is strong-weak dual to original $SU(K)^n$; similar to **Argyres-Seiberg**.
- Implies that 4d $\mathcal{N}=2$ SCFT has at singular point an infinite number of **conserved high-spin currents** (instead of naïve expectation – new global $SU(n)$). These should somehow be part of $\mathcal{N}=(2,0)$ theory on $AdS_5 \times S^1$.
- Does this local field theory develop **massless high-spin fields** ? Not impossible on AdS_5 , but very strange. Would like

Alternative description

- Can we get around inevitable conclusion ?
- We propose a simpler picture. The new **4d** **SU(n)** and the **P_n** theory can live on the boundary of **AdS₅**; can have **4d** $\mathcal{N}=2$ theories living there. The **4d** **SU(n)** theory couples to both **Q_{K,n}** and **P_n**, and has a vanishing **beta function**.
- Identify the bulk theory with the **Q_{K,n}** theory. The **4d** **SU(n)** gauge theory must couple to **5d** **SU(n)** gauge fields on **AdS₅**, helping to cancel its **beta function**.



Alternative description features

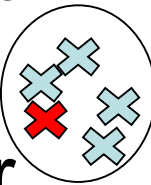
- Should be related by **duality** (extra **AdS/CFT** for **$SU(n) \times P_n$** ?) to the picture with **high-spin fields** in the bulk, but seems much simpler.
- Have **$SU(n)$** in **AdS_5** but no **global symmetry**. Usually say unique **boundary condition** for **G gauge fields** on **AdS_5** !? When have **global symmetry G** can always gauge it = couple to **4d G gauge fields** on boundary. When bulk theory is weakly coupled, get large **(R_{AdS}/g_G^2)** contribution to **beta function** of **4d G** , inconsistent with conformal symmetry. ²⁰

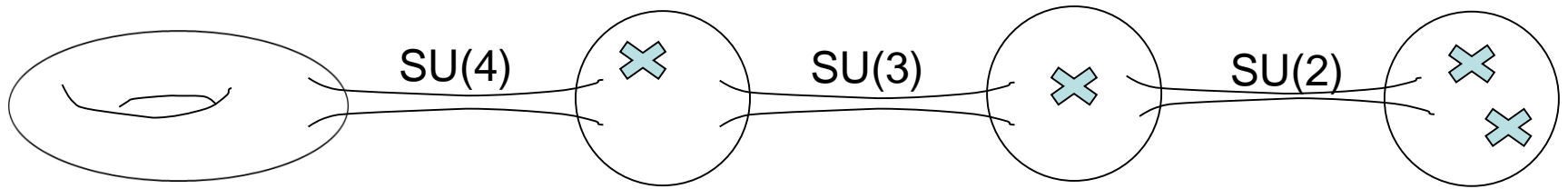
Alternative description consequences

- In our case we know contribution to **beta function**. Implies bulk **5d SU(n)** is **strongly coupled** at R_{AdS} . Thus, no contradiction with standard semi-classical analysis of allowed **boundary conditions**.
- On the “**moduli space**” **5d SU(n)** behaves very differently from the naïve expectation: not broken to $U(1)^{n-1}$ (**exactly marginal deformations** described by changing couplings of **SU(n)** and P_n on boundary; $U(1)^{n-1}$ acts on boundary P_n theory).

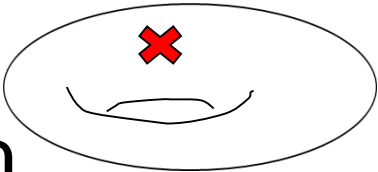
Moduli space of (2,0) on $AdS_5 \times S^1$

- At origin of “moduli space” coupling constant of 4d $SU(n)$ goes to zero – infinitely far away (in natural Zamolodchikov metric).
- Moreover, origin of “moduli space” is not just a point but an $(n-2)$ -dimensional space – space of moduli of P_n theory = a sphere with $(n+1)$ marked points. Big change...
- The P_n theory has a region in its parameter space where it becomes a weakly coupled 4d $SU(n-1) \times SU(n-2) \times \dots \times SU(2)$ theory with bi-fundamental hypers + $1+n$ fundamentals.²²





- Note all beta functions in this chain vanish.
 $Q_{K,n}$ (5d bulk) contributes to beta function of $SU(n)$ like $(n+1)$ fundamental hypers.
- In this region it is easy to compute how many d.o.f. we are adding on the boundary (say in sense of conformal anomalies) : $O(n^3)$. Amusing since bulk 6d (2,0) theory also has $O(n^3)$ d.o.f. But no clear relation – for instance, 6d d.o.f. and 4d d.o.f. lead to a different density of states as a function of temperature / energy.

- This is all for the specific **boundary condition** that we get from **type IIB**. Can also take a “**standard**” **boundary condition** for **5d SU(n)** gauge fields, and then the **(2,0)** theory is part of the gravitational dual to the $Q_{K,n}$ theory (which has an **SU(n)** **global symmetry**). In this case the **(2,0)** theory has no “**moduli space**”. How is this dual related to the previous one ?
- To decouple should take $K \rightarrow \infty$ with  couplings as above. Limit of **4d $\mathcal{N}=2$ SCFT** contains a subsector dual to **$\mathcal{N}=(2,0)$** theory on **$AdS_5 \times S^1$** .

Summary

- Introduced “rigid holography”, and used it to show that $A_{n-1} (2,0)$ theories on $AdS_5 \times S^1$ with $R_{AdS} = R_S$ and specific b.c. are different from expected – “moduli space” is singular near origin, have $SU(n)$ gauge fields on AdS_5 but with different behavior than in flat space.
- This theory appears as a decoupled sector in the large K , strong coupling limit of $4d \mathcal{N}=2$ $SU(K)^n$. Can get same theory also from IIA backgrounds with n NS5-branes on $AdS_5 \times S^1$, dual to other $4d \mathcal{N}=2$ quiver SCFTs.

Summary

- In retrospect, the behavior of the $A_{n-1} (2,0)$ theories on $AdS_5 \times S^1$ is not so surprising. They have a strongly coupled $SU(n)$ gauge theory on AdS_5 , as expected, and this theory does not have a “moduli space”, presumably because its’ scalars are tachyonic.
- Surprise is that when this theory is coupled to a 4d $SU(n) \times P_n$ theory on the boundary of AdS_5 , have a very different dual description with $U(1)^{n-1}$ gauge fields in the bulk, and nothing on the boundary.

Further questions

- What can we compute (16 supercharges)? Localization in 4d $\mathcal{N}=2$ SCFT ? Directly on $AdS_5 \times S^1$? (Work in progress)
- Gravity dual for (2,0) theory on $AdS_5 \times S^1$?
- Are “boundary correlators” (computable in principle) enough to characterize A_{n-1} (2,0) theory on $AdS_5 \times S^1$? (Is S-matrix enough?)
- Other boundary conditions? “Standard” with $SU(n)$ global symmetry for any R_{AdS}/R_S , for specific R_{AdS}/R_S can couple to 4d $\mathcal{N}=2$ $SU(n)$ theory on the boundary. Embed in string? ²⁷

Further questions

- Far on “moduli space”, got a description with $U(1)^{n-1}$ and “moduli” coming from the bulk; near the origin, have a description where they come from the boundary. What is relation between them ? AdS/CFT ? Strong-weak duality (similar to Gaiotto-Witten) ?
- Do other sets of punctures coming together on a Riemann surface also correspond to $(2,0)$ theories on $AdS_5 \times S^1$ (b.c.) ? For a torus it seems so. Can we bring together punctures+handles ?

Further questions

- Many possible generalizations. Simple to get generalization to $(2,0)$ LST on $AdS_5 \times S^1$.
- Other D_n and E_n $\mathcal{N}=(2,0)$ theories on $AdS_5 \times S^1$ can be similarly studied using other orbifolds of type IIB on $AdS_5 \times S^5$.
- Rigid holography should be useful for studying various $\mathcal{N}=(2,0)$ theories on $AdS_4 \times S^2$ and $AdS_3 \times S^3$, 6d $\mathcal{N}=(1,0)$ theories on $AdS_5 \times S^1$ and other manifolds, 5d theories on $AdS_4 \times S^1$, 4d $\mathcal{N}=4$ SYM on $AdS_3 \times S^1$, etc.