QUANTUM QUENCH AND HOLOGRAPHY

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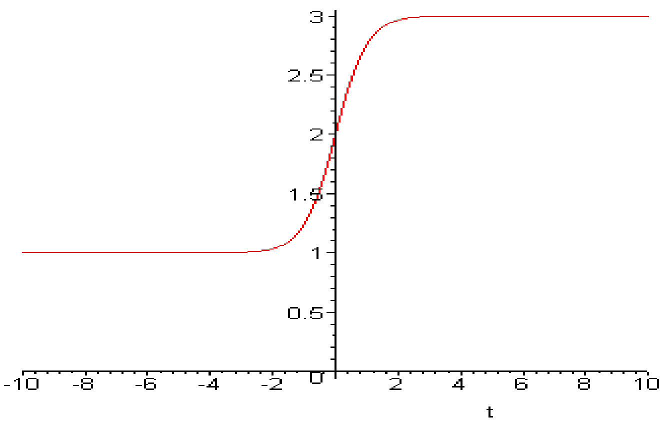
Quantum Quench

- Suppose we have a many-body system, described by a quantum field theory, whose parameters (like couplings or masses) are time dependent – e.g. going between constant values at early and late times



- If the system starts out in a ground state at early times what is the fate of the system at late times ?

- This is of course an old problem in quantum mechanics and in quantum field theory - which has applications to many areas of physics - cosmological particle production, black hole radiation …….
- In recent years this problem has attracted a lot of attention due to two related reasons.

- The first relates to the question of thermalization
- Does the system evolve into a steady state at late times ? If so does the state resemble a thermal state ?
- What is the characteristic time scale for this to happen ?

- The second issue relates to dynamics near quantum critical phase transitions.
- In this case, simple scaling arguments indicate that there are some properties of the excited state which reflect universal behavior characteristic of the critical point.
- Some of these arguments are adaptations of the classic arguments of Kibble and Zurek for thermal phase transitions.
- For example, suppose near a critical point the coupling changes in a linear fashion

$$g(t) - g_c \sim vt$$

- The energy gap Δ vanishes as

$$\Delta \sim |g - g_c|^{z\nu}$$

- z is the dynamical critical exponent and ν is the correlation length exponent.

- Then if one makes a scaling assumption one finds that for any operator with a scaling dimension x the one point function behaves as

$$< \hat{\mathcal{O}} > \sim (v)^{\frac{x\nu}{z\nu+1}}$$

- For example the density of defects is

$$< n >_{\mathrm{defect}} \sim (v)^{\frac{d\nu}{z\nu+1}}$$

- Unlike in equilibrium critical phenomena, there is no convincing conceptual framework which justifies this scaling assumption.

- For general two dimensional field theories which are suddenly brought to a critical point, *Calabrese and Cardy* used powerful techniques of 2d conformal field theories to show that, e.g.

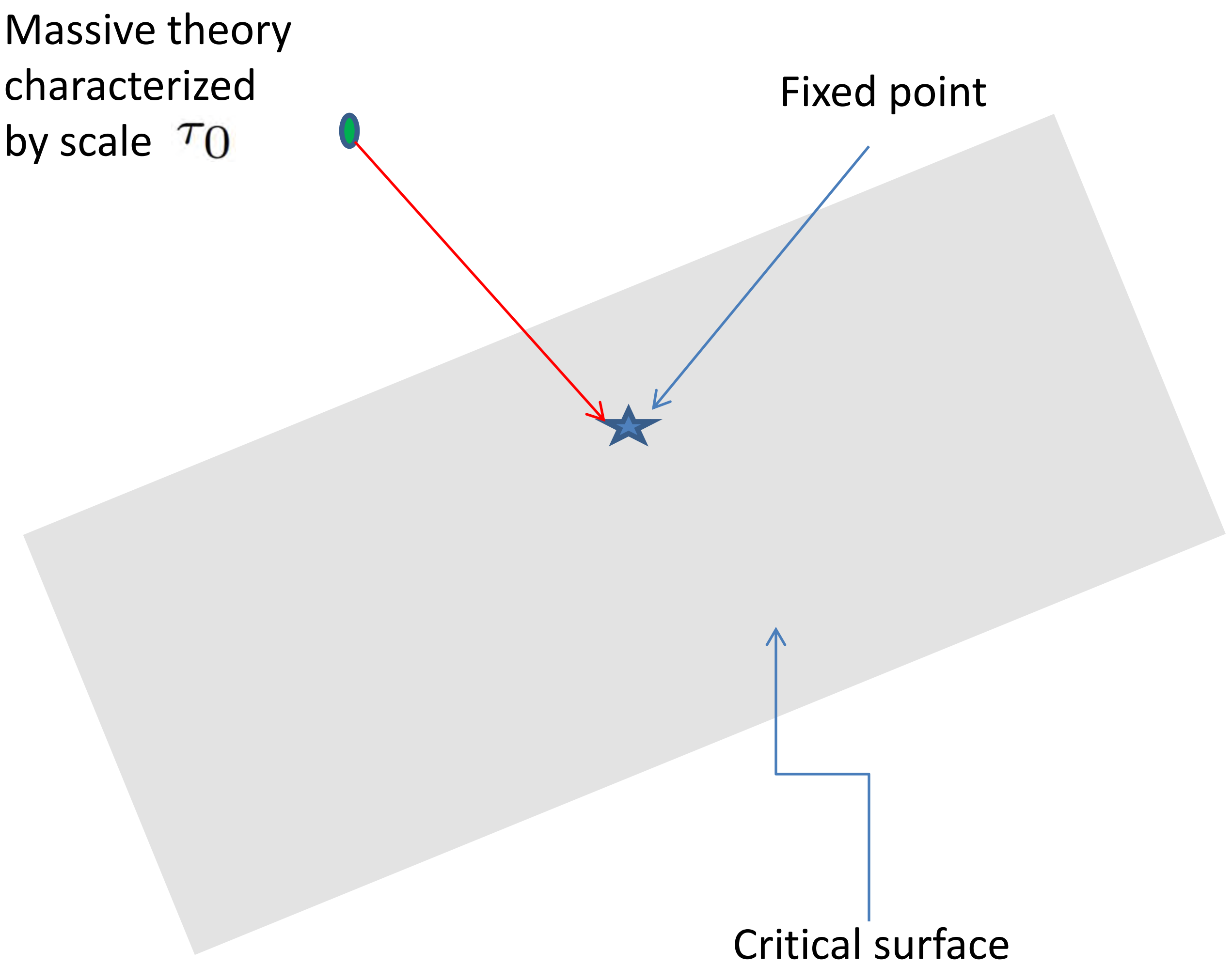
$$< \hat{\mathcal{O}}(t) > e^{-\frac{x\pi t}{2\tau_0}}$$

$$< \hat{\mathcal{O}}(r,t)\hat{\mathcal{O}}(0,t) >\sim \theta(2t-r)e^{-\frac{x\pi r}{2\tau_0}} + \theta(r-2t)e^{-\frac{x\pi t}{\tau_0}}$$

- Here τ_0 is a length scale which characterizes the non-critical theory from which we quench the system, x is the conformal dimension of the operator.
- The expressions show that ratios of relaxation times for different operators are universal.

$$\frac{\tau^{(1)}_{relax}}{\tau^{(2)}_{relax}} = \frac{x_2}{x_1}$$

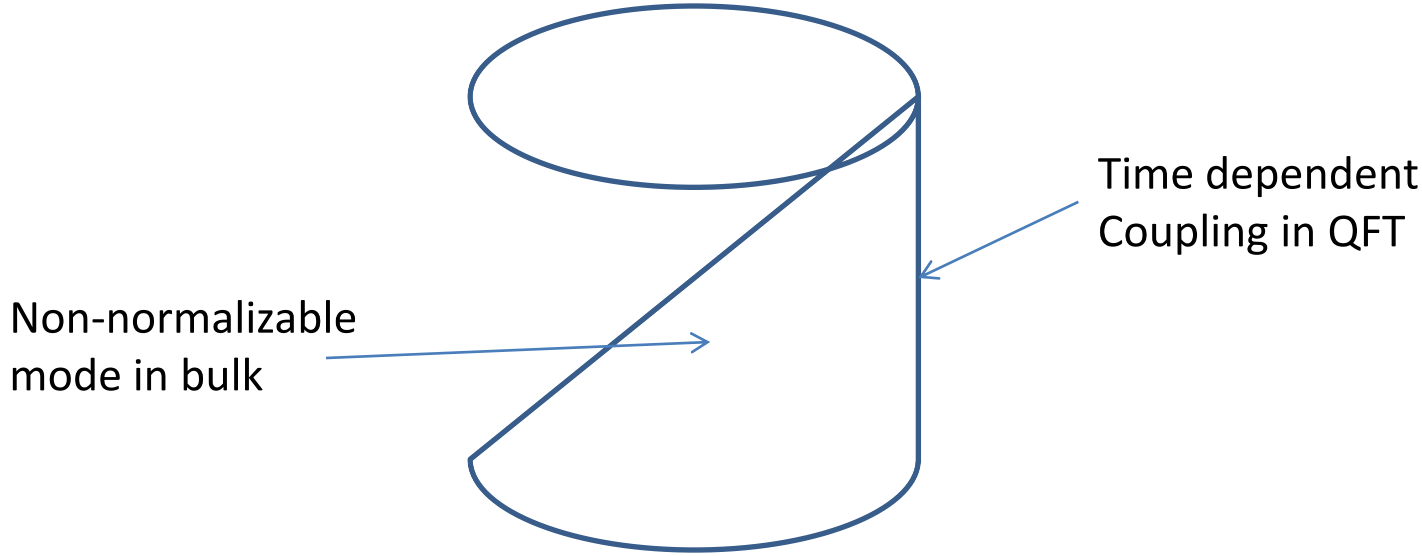
- Similar results for entanglement entropy.


Massive theory characterized by scale τ_0
Fixed point
Critical surface

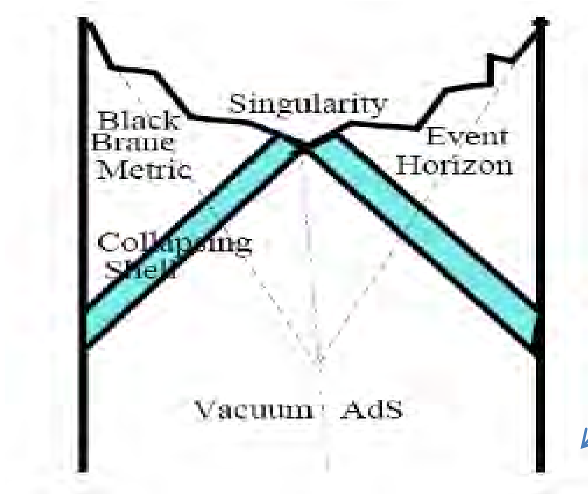
- Much of the recent interest in quantum quench comes from the fact that the response of systems to time dependent couplings can be now experimentally measured in cold atom systems.
- On the other hand there are very few theoretical tools to study this in strongly coupled systems, particularly near quantum critical points.
- Can we use AdS/CFT to understand this kind of phenomena ?
- While it is rather unlikely that one will be able to model real systems by gravity duals, one may be able to understand any universal behavior which underlie such phenomena.

Quench, Black Holes and Singularities

- In the AdS/CFT correspondence, boundary values of fields in the bulk of AdS are couplings of the dual field theory
- Thus quantum quench in a field theory would be represented by time dependent solutions in the bulk



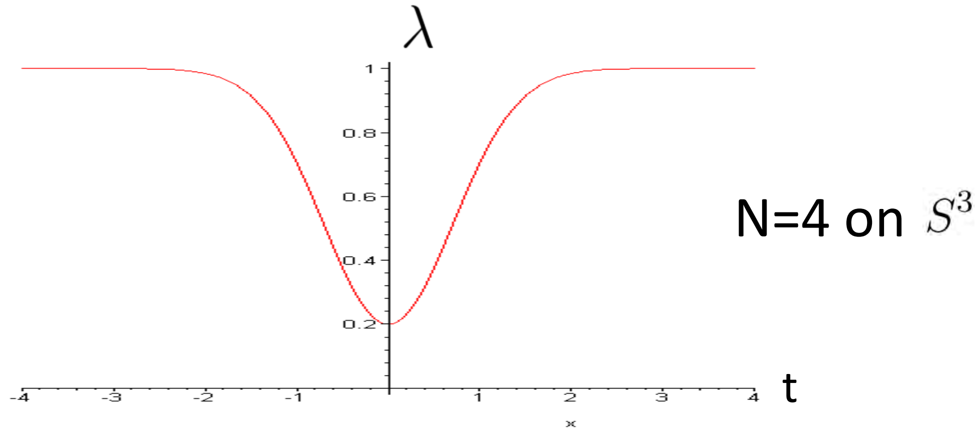
- When the rate of variation of the boundary coupling is large enough, one expects black holes to form.



Variation in this time range

- *Janik; Chesler and Yaffe*; *Bhattacharyya and Minwalla*; ....

- In other regimes one can form regions of large string frame curvature



- The ‘t Hooft coupling is large and constant at early and late times and dips to a small value at intermediate times
- The rate is slow enough that a black hole does not form so long as ‘t Hooft coupling is large.
- At intermediate times, curvatures are large – like in a space-like singularity.

- When the slowness parameter ϵ is much smaller than $1/N$, standard textbook adiabatic expansion works – the state is then basically the instantaneous ground state.
- The interesting case, however is $N\epsilon \gg 1$
- In this case while the standard adiabatic expansion fails, there is different kind of adiabatic expansion based on coherent states. This can be used to argue that there is a smooth passage through the "singularity".

(A. Awad, S.R.D., A. Ghosh, J.H. Oh and S. Trivedi)

- For issues of thermalization the scenarios of black hole formation are relevant.
- Boundary correlators at late times will become thermal – with a temperature equal to the Hawking temperature of the black hole being formed. The time scale for relaxation to this thermal state is larger for larger length scales, but faster than what one might have expected.

  (Abajo-Arrastia, Aparicio & Lopez; Ebrahim & Headrick; Balasubramanian et.al.)
- Thus, in this case, the quench leads to thermalization – this is manifested in the bulk as black hole formation.
- As is usual in applications of AdS/CFT, a difficult quantum problem in a strongly coupled field theory has been mapped to an easier classical problem in gravity.

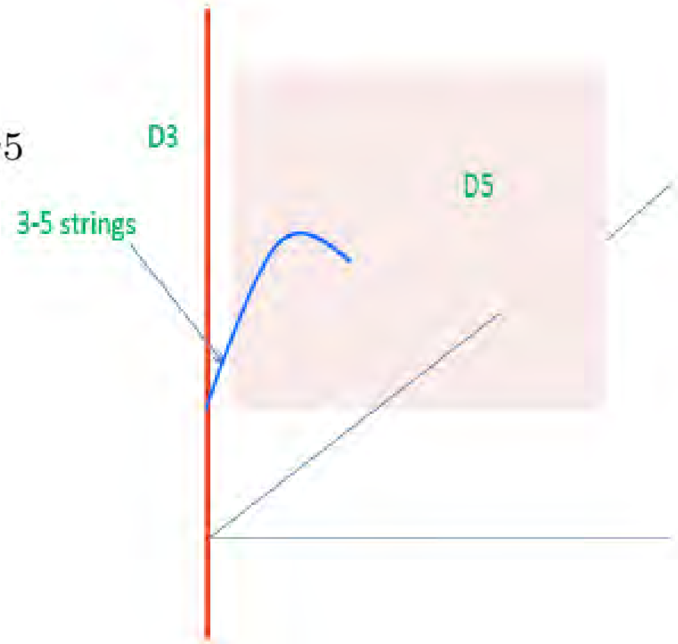
Probe Limits

- One of our motivations to study this problem is to understand dynamics near a critical point.
- Critical points at nonzero and zero temperatures have been extensively studied in AdS/CFT.
- A large and interesting class of such transitions can be studied in the probe approximation – this includes holographic superconductors, chiral symmetry breaking transitions etc.
- In this approximation, a subset of “bulk” fields can be decoupled from gravity in a consistent fashion – and their backreaction to gravity can be ignored.

Probe Branes

- One class of examples concern a small number N_f of D-branes in the bulk AdS (*Karch and Randall, Karch and Katz*).
- These branes introduce hypermultiplets in the boundary gauge theory, which in general live on a subspace of the original gauge theory – a defect QFT.
- For example a set of D5 branes wrapping $AdS_4 \times S^2$ in $AdS_5 \times S^5$ strings lead to hypers which live in 2+1 dimensions.
- Turning on a magnetic field along the D5 brane leads to a chiral symmetry breaking transition.

(Jensen,Karch,Son and Thompson)



- When the number of such branes is small, $N_f \ll N_c$ the dynamics of the brane can be treated separately, and their backreaction to the background metric can be ignored.
- In the dual field theory this means that hypermultiplet loops are suppressed – and one can – to the leading order - treat the 2+1 dimensional defect field theory in its own right.

Other examples in $AdS_5 \times S^5$

Probe	Wraps	Dual defect CFT dimensions
D1	AdS_2	0+1
D3	$AdS_3 \times S^1$	1+1
D5	$AdS_4 \times S^2$	2+1
D7	$AdS_5 \times S^3$	3+1

Probe Bulk Fields

- Another class of probe fields arise in more "*phenomenological*" models.
- An important case concerns holographic superconductors – charged scalar fields in the background of AdS black branes. When the charge is large, the backreaction of the scalar on the background metric can be ignored – under suitable conditions the field can condense, leading to superfluidity in the boundary field theory.

  (Gubser; Hartnoll, Herzog and Horowitz)
- We will deal with an even simpler example involving a neutral scalar later.

- In these situations we can ask what happens if we perform a quantum quench.
- For example, suppose we have a probe brane in pure AdS – initially in equilibrium. This corresponds to the vacuum state of the defect field theory.
- If we now start a quantum quench, viz. change a coupling of the theory – what happens in the future ? Does the system thermalize ?

- In our previous discussion (*not in probe approximation*) we found that under suitable circumstances a black hole is formed – this signals thermalization.
- In the probe approximation, the background geometry cannot change – so if the hypermultiplet sector of the field theory thermalizes, what does this mean in the bulk ?

Formation of Apparent Horizon

- We will show that in a large class of such quenches, there is a different mechanism which signals thermalization.
- In these examples, a time dependent coupling in the dual defect field theory leads to an induced metric which has an apparent horizon.
- Fluctuations of the brane feel this induced metric. An apparent horizon leads to Hawking radiation of these fluctuations.
- *(S.R.D., T. Nishioka and T. Takayanagi)*
- The phenomenon is similar to dumb holes – where fluctuations around a time dependent solution in a nonlinear field theory behave as if they are propagating in a metric with a horizon.

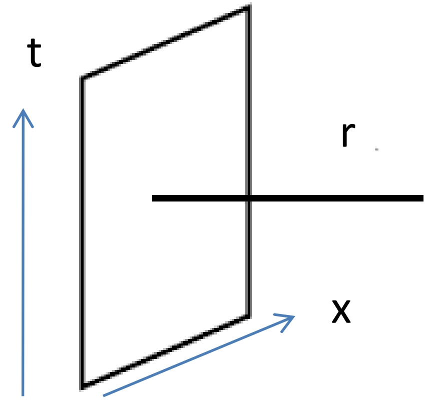
Rotating D1 Branes

- Consider the $AdS_5 \times S^5$ metric written in the form

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\vec{x}^2 + d\theta^2 + \sin^2\theta d\varphi^2 + \cos^2\theta d\Omega_3^2$$

- $f(r) = r^2$ - Poincare Patch
- $f(r) = 1 + r^2$, $d\vec{x}^2 \to d\bar{\Omega}_3{}^2$ - Global
- The D1 brane wraps the (r, t) directions.
- Introduce Edddington-Finkelstein coordinate

$$u = t - \int \frac{dr}{f(r)} \, , \qquad v = t + \int \frac{dr}{f(r)}$$



- There are solutions $\varphi(u, v)$ with

$$\theta = \pi/2 \qquad \vec{x} = \Omega_3 = \text{constant}$$

Recall φ is an angle on S^5 - we have a rotating D1 brane

- The equation satisfied by $\varphi(u,v)$ is

$$\partial_u\partial_v\varphi + \frac{2}{L}\partial_v\varphi\partial_u\left(\frac{\partial_u\varphi\partial_v\varphi}{f(r)}\right) + \frac{2}{L}\partial_u\varphi\partial_v\left(\frac{\partial_u\varphi\partial_v\varphi}{f(r)}\right) = 0$$

- Where $L = 1 - \frac{4}{f(r)}\partial_u\varphi\partial_v\varphi$
- This looks rather complicated. However

$$\partial_u\varphi = 0 \text{ or } \partial_v\varphi = 0$$

are trivially solutions. We choose

$$\partial_u\varphi = 0$$

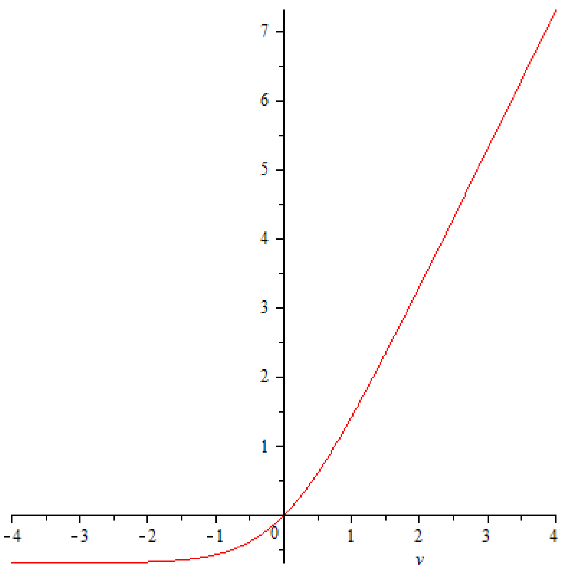
- The induced worldsheet metric for a solution which is an arbitrary function $\varphi(v)$ is

$$ds^2_{ind} = -f(r)dudv + (\partial_v\varphi)^2 dv^2 = 2drdv - [f(r) - (\partial_v\varphi)^2]dv^2$$

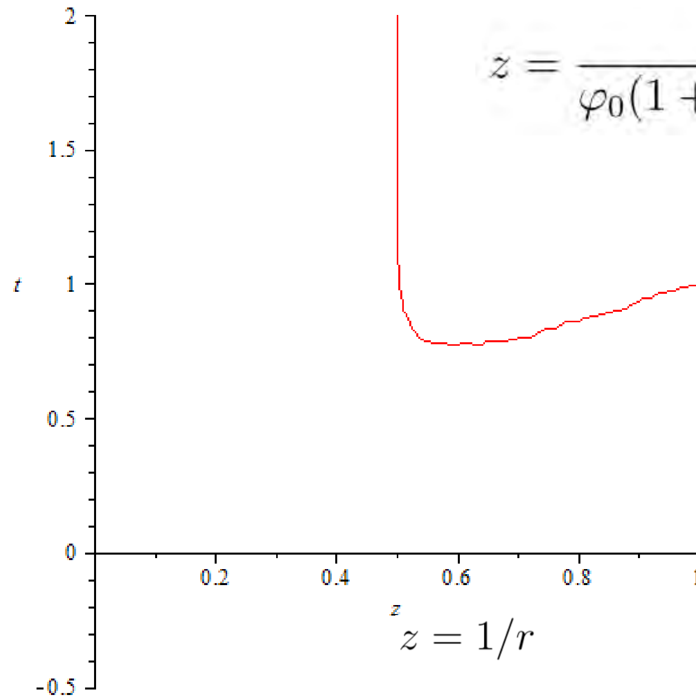
- This is Vaidya-type metric, with an apparent horizon at

$$f(r) = (\partial_v\varphi)^2$$

- For the Poincare patch, there is an apparent horizon for any $\varphi(v)$
- For global AdS, there is a threshold. For uniform rotation, there is a horizon only when $\omega > \omega_c$
- When $\varphi(v) \to \omega v$ at late retarded times, the apparent horizon asymptotes to an event horizon at $f(r) = \omega^2$



$$\varphi(v) = \varphi_0[v + \frac{1}{k}\log(\cosh(kv))]$$

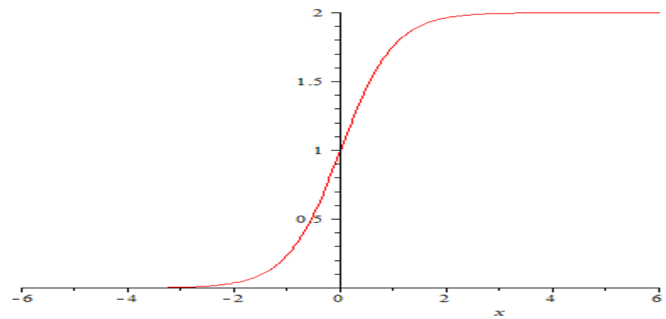


$$z = \frac{1}{\varphi_0(1+\tanh(k(t-z)))}$$

$z = 1/r$

- For a quench-like situation, we want a non-zero $\varphi'(v)$ for a relatively short period of time, e.g.

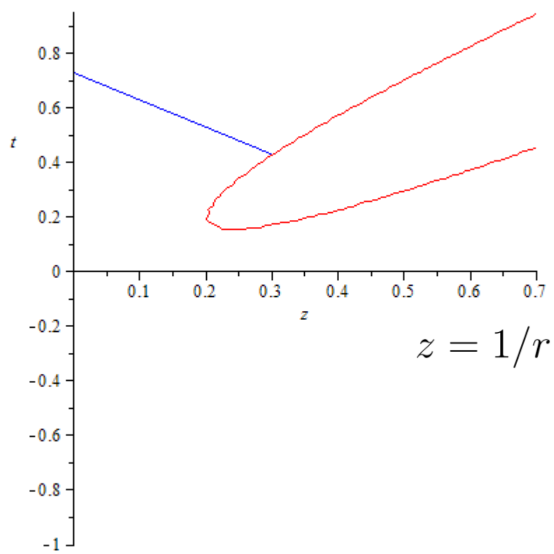
$$\varphi(v) = \varphi_0(1 + \tanh kv)$$



- There is an apparent horizon at

$$r = \varphi'(v) = \frac{k\varphi_0}{\cosh^2 kv}$$

but no event horizon.



Fluctuations around rotating D1

- The apparent horizon on the world-sheet is perceived by fluctuations around the rotating D1 solution.
- Denote the fluctuations by

$$y^I \equiv (\delta\varphi, \delta\theta, \delta\psi_i, \delta x^1, \delta x^2, \delta x^3)$$

- The quadratic action for the fluctuations is given by

$$S_2 = \frac{T_{D1}}{2} \int d^2\xi \sqrt{-\gamma_0}\gamma_0^{ab} G_{IJ}(\xi^a, x_0^I)\partial_a y^I \partial_b y^J$$

- Where $(\gamma_0)_{ab}$ is the induced metric on the world-sheet evaluated on the classical solution

$$(\gamma_0)_{ab} d\xi^a d\xi^b = 2drdv - [f(r) - (\partial_v\phi)^2]dv^2$$

- If the spin is constant, $\varphi(v) \to \omega v$ it is clear that the Unruh vacuum correlators will be thermal with a temperature

$$T = \frac{1}{\beta} = \frac{\omega}{2\pi}$$

(Lawrence & Martinec)

- The end-point of the string in fact performs characteristic Brownian motion. For $\delta\varphi$ fluctuations

Ballistic

$$\langle (\Delta y^\varphi(t-t'))^2 \rangle \sim \frac{\pi(t-t')^2}{12\beta^2}, \qquad \pi(t-t') \ll \beta ,$$

Diffusive

$$\langle (\Delta y^\varphi(t-t'))^2 \rangle \sim \frac{(t-t')}{2\beta} - \frac{1}{2\pi}\log[2\pi(t-t')/\beta] , \qquad \pi(t-t') \gg \beta$$

- For δx fluctuations, one needs a UV cutoff leading to a finite mass of the entire string, m - and one gets

$$\langle [\Delta y^i(t-t')]^2 \rangle \sim \frac{(t-t')^2}{m\beta}, \qquad t \ll m\beta^2$$

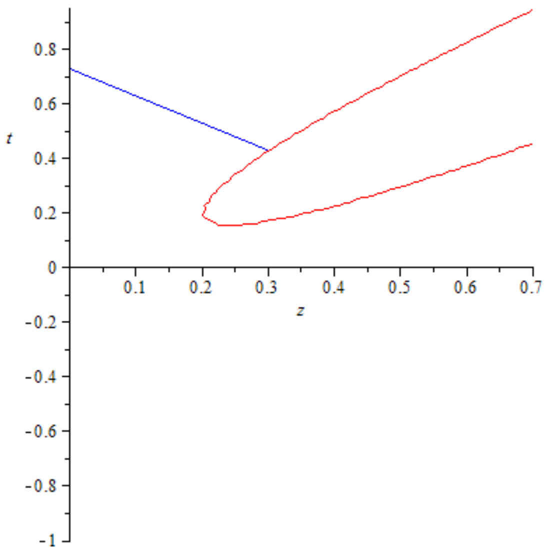
$$\sim \beta|t-t'| , \qquad t \gg m\beta^2$$

- Such Brownian motion has been shown for a string in the presence of a black brane by

  Son & Teaney;

  de Boer, Hubeny, Rangamani & Shigemori
- In their case the black brane background induces a non-trivial metric on the worldsheet – leading to this effect.
- In our case, we do not have a black brane – just pure $AdS_5 \times S^5$
- However, a similar induced metric appears because of the nontrivial solution of a nonlinear theory. *(cf. Acoustic Black Holes).*
- In fact for constant rotation of the D1 the induced metric is exactly that for a string in BTZ black hole background.
- In the dual theory, the time dependent coupling produces this thermal effect – though **only in the hypermultiplet sector.**

- Even if there is no event horizon, hawking pairs are created at the apparent horizon and arrive at the boundary at some later time. The temperature perceived in the boundary field theory is that characteristic of the apparent horizon at an earlier time.



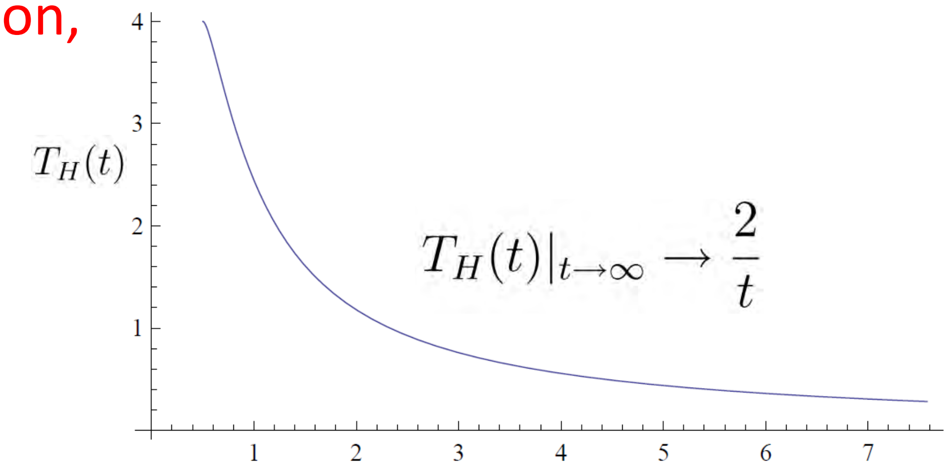
- Even if there is no event horizon, hawking pairs are created at the apparent horizon and arrive at the boundary at some later time. The temperature perceived in the boundary field theory is that characteristic of the apparent horizon at an earlier time.
- For a large k , this may be estimated by geometric optics
- The temperature on the boundary is given by

$$T_H(t) = \varphi'(v)$$

- Where v has to be obtained as a function of t by solving

$$t = v + \frac{2}{\varphi'(v)}$$

- At large times, the temperature goes to zero as expected in a scale invariant fashion,



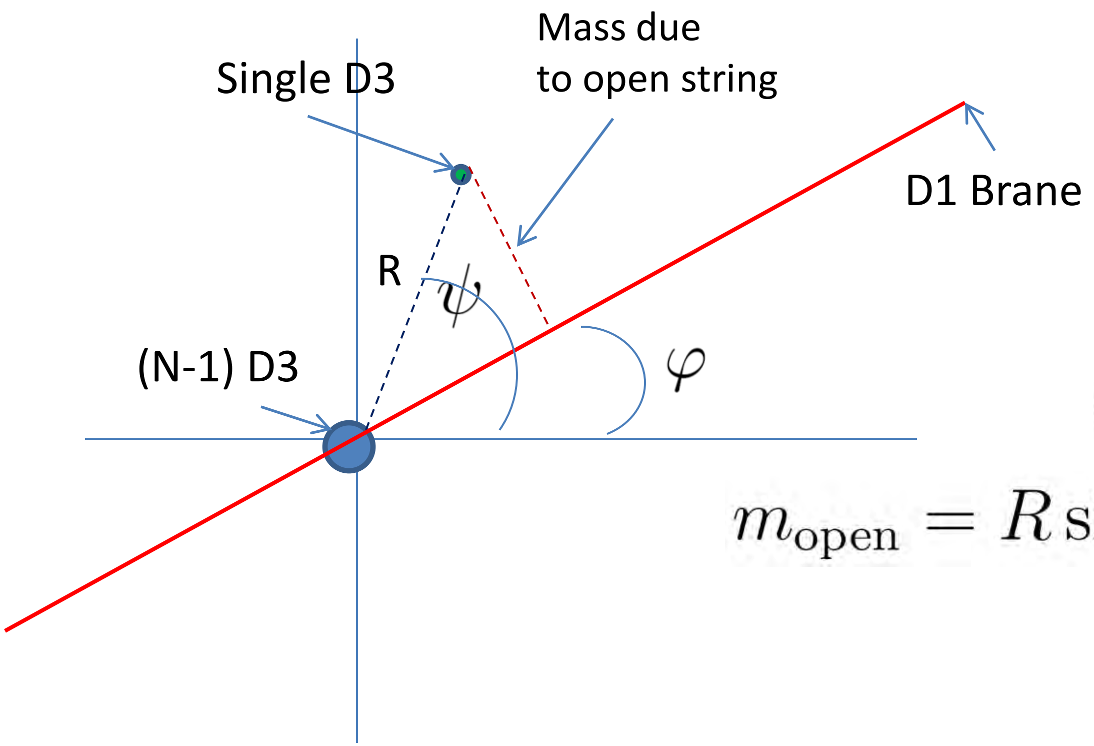
Description in the CFT

- The 0+1 dimensional defect CFT has 2 complex scalars in the fundamental of $SU(N)$, $Q_i \quad (i = 1, 2)$
- The 3 complex adjoint scalars of the N=4 theory (Φ_1, Φ_2, Φ_3) correspond to cartesian coordinates in the transverse C^3. Let the scalar corresponding to the (x^4, x^5) plane be Φ_3.
- Then the solution $\varphi(v)$ corresponds to a source term

$$\sum_i \bar{Q}_i [\mathrm{Im}(\Phi_3 \, e^{-i\varphi(t)})] Q_i$$

- *(Diaconescu)*

- We can check this by going to the Coulomb branch - i.e. separating one D3 brane from the others. Then this leads to a mass for the hypermultiplet fields

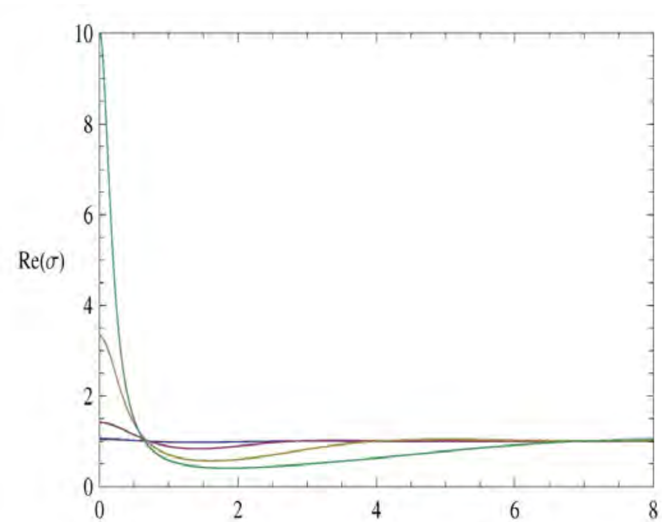


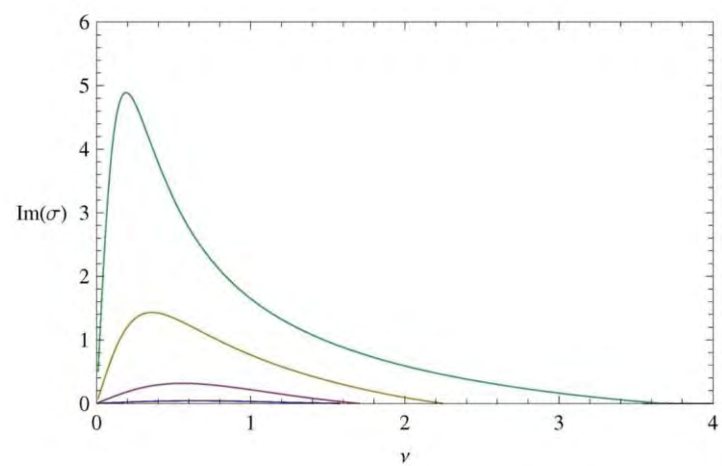
$$\Phi_3 = x_4 + i x_5$$

$$m_{\rm open} = R \sin(\psi - \varphi) = {\rm Im}\Phi_3 e^{-i\varphi}$$

Gauge Fields on D3 and Conductivity

- Another imanifestation is provided by D3 branes in the $AdS_5 \times S^5$ geometry, dual to a 2+1 dimensional field theory.
- There is a uniformly rotating solution together with a worldvolume gauge field – this has a worldvolume event horizon. This leads to an electrical conductivity for charge carriers in the boundary field theory





Gauge Fields on D3 and Conductivity

- There is a uniformly rotating solution together with a worldvolume gauge field

$$A_a = (A_t(r) = \Phi(r), 0, 0, 0)$$
$$\Phi(r) = \int_\omega^r ds \frac{s^2}{\sqrt{(1 + Cs^4)(s^4 + s^2\omega^2 + \omega^4)}}$$

- The constant C is related to the charge density

$$\rho = \lim_{r \to \infty} \Phi'(r) r^2 = C^{-1/2}$$

- Solving the linear equation for A_x perturbations obtain the electrical conductivity $\sigma(\nu)$

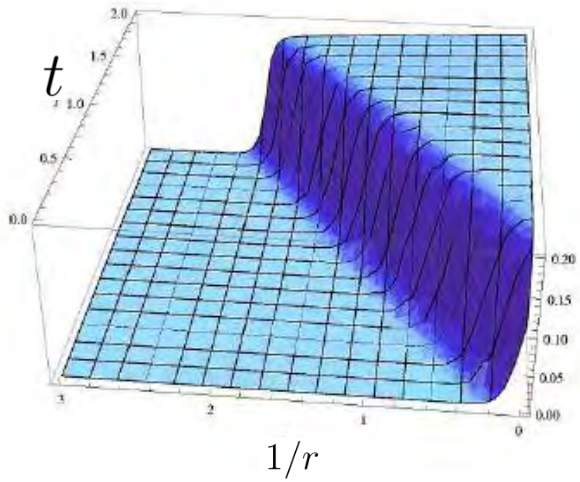
Other Examples

- 2+1 dimensional field theory dual to D5 branes in $AdS_5 \times S^5$
- Uniformly rotating D5 obtained earlier by *Evans and Threlfall*
- In this case the situation is best described in the following coordinates in $AdS_5 \times S^5$

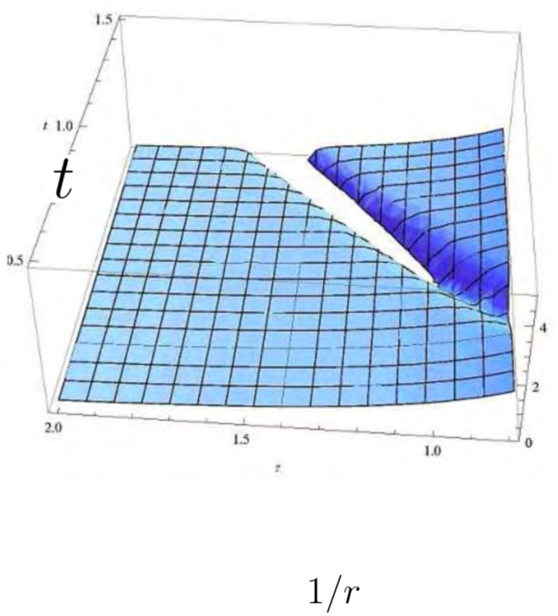
$$ds^2 = (r^2 + y^2)[-dt^2 + d\vec{x}^2] + \frac{1}{r^2 + y^2}[dr^2 + r^2 d\Omega_2^2 + dy^2 + y^2 d\bar{\Omega}_2^2]$$

- The D5 brane is wrapped on r, x_1, x_2, Ω_2
- The coordinate y is the distance between the D3 and D5 branes.
- Thus $y(t,r)$ is dual to a time dependent mass $m(t)$ of hypers.
- Construct the bulk solution for $y(t,r)$ with a mass $m(t)$

$$m(t) = m_0(1 + tanh(kt))$$



Plot of $y(t,r)$



Plot of $-g_{tt}(r,t)$. The zero of this quantity is the location of the apparent horizon.

Hairpin zero near t = 1/r

Quench of chemical potential

- One interesting application of this mechanism of formation of apparent horizons on brane worldvolumes is the understanding of thermalization of the meson sector in N=2 gauge theories – *Iizuka, Hashimoto and Oka*
- By introducing quarks via a source for the worldvolume gauge field one gets a time dependent electric field – this corresponds to a time dependent chemical potential
- Fluctuations of the brane in this background perceive an induced metric which has an apparent horizon.

Quench across critical points

- So far our examples did not involve a critical point.
- In the rest of the talk we will concentrate on quench across critical points.
- *(P. Basu and S.R.D – arXiv:1107.xxxx)*

Condensation of a neutral scalar

- A rather simple "*phenomenological*" model of a holographic phase transition is provided by a neutral scalar field in AdS_4 black brane background – *(Iqbal, Liu, Mezei and Si)*
- The background metric is

$$ds^2 = r^2[-f(r)dt^2 + d\vec{x}^2] + \frac{dr^2}{r^2 f(r)} \qquad f(r) = 1 + \frac{3\eta}{r^4} - \frac{1+3\eta}{r^3}$$

$$T = 3(1-\eta)$$

- The lagrangian density for the scalar field is

$$L = \frac{1}{2\kappa^2\lambda}\sqrt{-g}[-\frac{1}{2}(\partial\phi)^2 - \frac{1}{4}(\phi^2+m^2)^2 - \frac{m^4}{4}]$$

- When the coupling λ is large, the field can be treated in a probe approximation.

- The equation of motion is

$$\frac{1}{r^2}[-\frac{1}{f(r)}\partial_t^2 + \partial_r(r^4 f(r)\partial_r)]\phi - m^2\phi - \phi^3 = 0$$

- The asymptotic form of the solution is

$$\phi(r, x^\mu) \sim r^{3-\Delta}[\phi_0(x^\mu) + O(1/r^2)] + r^\Delta[\psi_0(x^\mu) + O(1/r^2)]$$

- Want to look for solutions with vanishing ϕ_0. Then ψ_0 determines the one point function.
- There is always the solution $\phi(r) = 0$ which has $<\hat{\mathcal{O}}>= 0$
- When the mass lies in the range,

$$-\frac{9}{4} < m^2 < -\frac{3}{2}$$

there is a critical temperature below which this solution is unstable.

- Exactly at m_c^2 there is a time independent solution of the *linearlized* equation of motion

$$\frac{1}{r^2}\partial_r(r^4 f(r)\partial_r)\phi - m^2\phi = 0$$

- This is the zero mode which will play a key role soon.
- For a smaller value of m^2 there is a new solution of the full *nonlinear* equations of motion – the asymptotic behavior of this new solution determines the expectation value of the order parameter.
- At any non-zero temperature the transition is mean-field

$$< \hat{O} >\sim (m_c^2 - m^2)^{1/2} \quad (\phi_0 = 0)$$

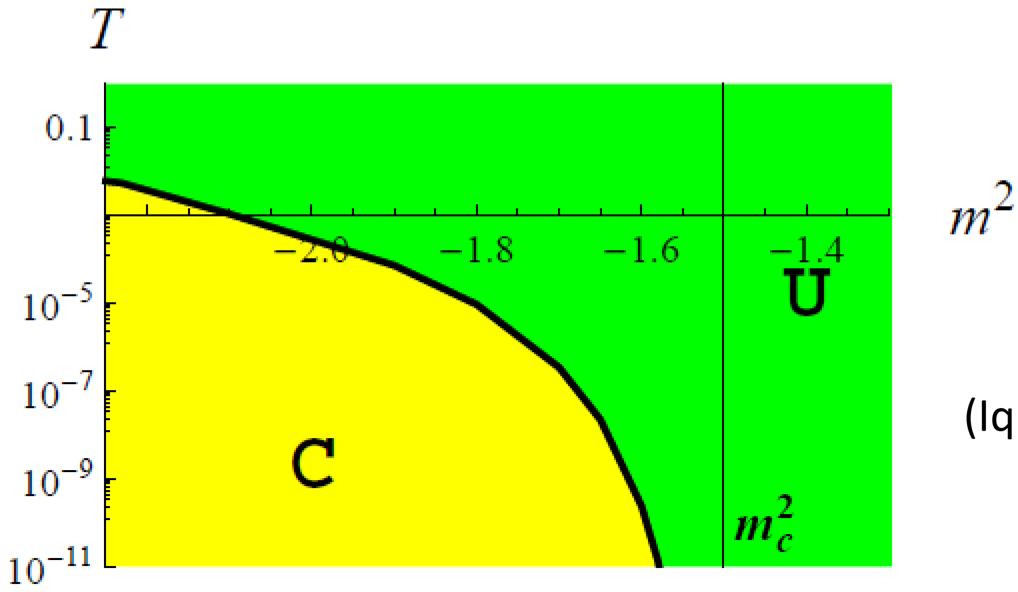
$$< \hat{O} >\sim \phi_0^{1/3} \quad (m^2 = m_c^2)$$

- Exactly at zero temperature the transition is Berezinski-Kosterlitz-Thouless type

$$T_c \sim \exp[-\pi/\sqrt{-3/2 - m^2}]$$

$$< \hat{O} > \sim \exp[-\pi\sqrt{6}/2\sqrt{-3/2 - m^2}]$$

- This case is rather subtle – we will not consider this for the moment.



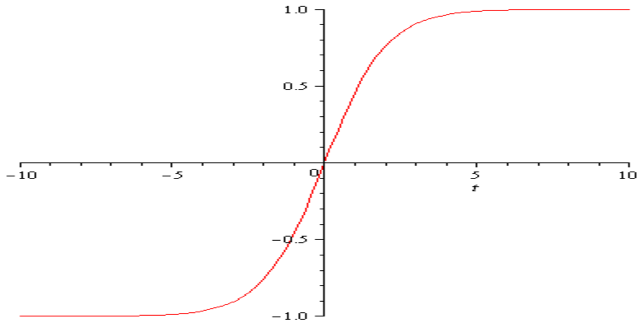
(Iqbal et.al.)

- This phase transition is similar to that in holographic superconductors, which involve a charged field in a charged black brane background.
- However a similar instability and condensation mechanism happens for neutral scalars in a charged black brane background
- This has been suggested as a toy model for antiferromagnetic transitions.

- We will have no comment on this interpretation, but will simply use this as a convenient model to study quntum quench across the critical point.

- One way to explore the dynamics of the critical point is to consider a time dependent mass $m^2(t)$ which passes m_c^2 at some time, chosen to be $t = 0$, e.g.

$$\alpha(t) \equiv m_c^2 - m^2(t) = m_0^2 \tanh(vt)$$

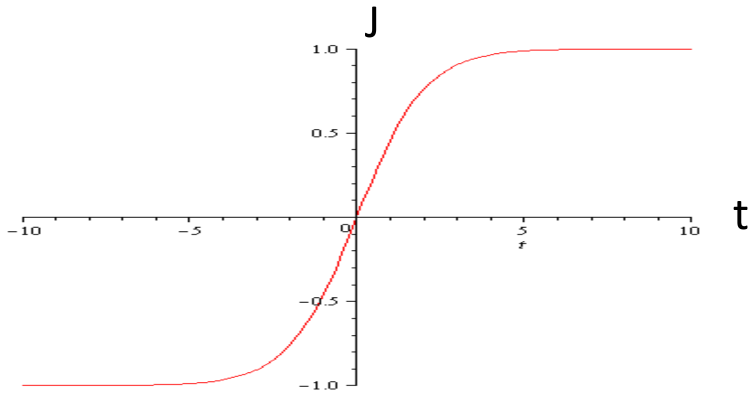


- Suppose we start at early times in the ordered phase, so that the scalar field has a static solution .
- The idea is to use this as the initial condition for time evolution of the field.
- The boundary condition is the same as before – a vanishing ϕ_0 , since we have not turned on a source for $\hat{\mathcal{O}}$ itself.

- Another way to study the critical point is to work exactly at m_c^2 but introduce a source for the order parameter of the form

$$\mathcal{H} \to \mathcal{H} + J(t)\hat{\mathcal{O}}(t)$$

- Such that



- Then at $t=0$ we can cross the critical point.
- In this case, we have to change the boundary condition on the dual bulk field and impose

$$\phi_0(t) = J(t)$$

- Suppose at the initial time the rate of change of $\alpha(t)$ is slow enough (*how slow – we will quantify this soon*). We would then expect that the solution can be obtained in an adiabatic expansion.
- For a given static solution $\phi_0(r; m)$ this is given by

$$\phi(r, t; m(t)) = \phi_0(r; m(t)) + \epsilon^2 \phi_1 + \cdots$$

- Where ϵ is a slowness parameter introduced to keep track of the adiabatic expansion by rescaling time.
- The lowest order ϕ_1 satisfies the ODE

$$\mathcal{D}\phi_1 = \ddot{\alpha}\partial_\alpha \phi_0 + \dot{\alpha}^2 \partial_\alpha^2 \phi_0$$

- where

$$\mathcal{D} = -f(r)\partial_r (r^2 f(r) \partial_r) + r^2 f(r)(m^2 + 3\phi_0^2)$$

- When the mass-square equals m_c^2 adiabaticity fails because of two related reasons
- (1) The operator $\mathcal{D}$ has a zero mode – so the Green's function does not exist.
- (2) The α derivatives of ϕ_0 themselves blow up, since as we saw

$$< \hat{\mathcal{O}} > \sim (m_c^2 - m^2)^{1/2}$$

- The condition for adiabaticity is in fact

$$\frac{\ddot{\alpha}}{\alpha^2}, \frac{(\dot{\alpha})^2}{\alpha^3} \ll 1$$

- In the critical region, and for the rate v small at early times, the dynamics is in fact dominated by the zero mode. This has to be now treated in an exact fashion – not in an adiabatic approximation.

A Digression – Landau-Ginsburg

- To understand the dynamics of this zero mode let us forget about AdS/CFT for the moment and consider the problem of a spatially homogeneous Landau-Ginsburg field whose equations of motion is

$$\frac{d^2\chi}{dt^2} + \alpha(t)\chi + \chi^3 = 0$$

- and the time dependent mass term is exactly what we had

$$\alpha(t) = m_0^2 \tanh(vt)$$

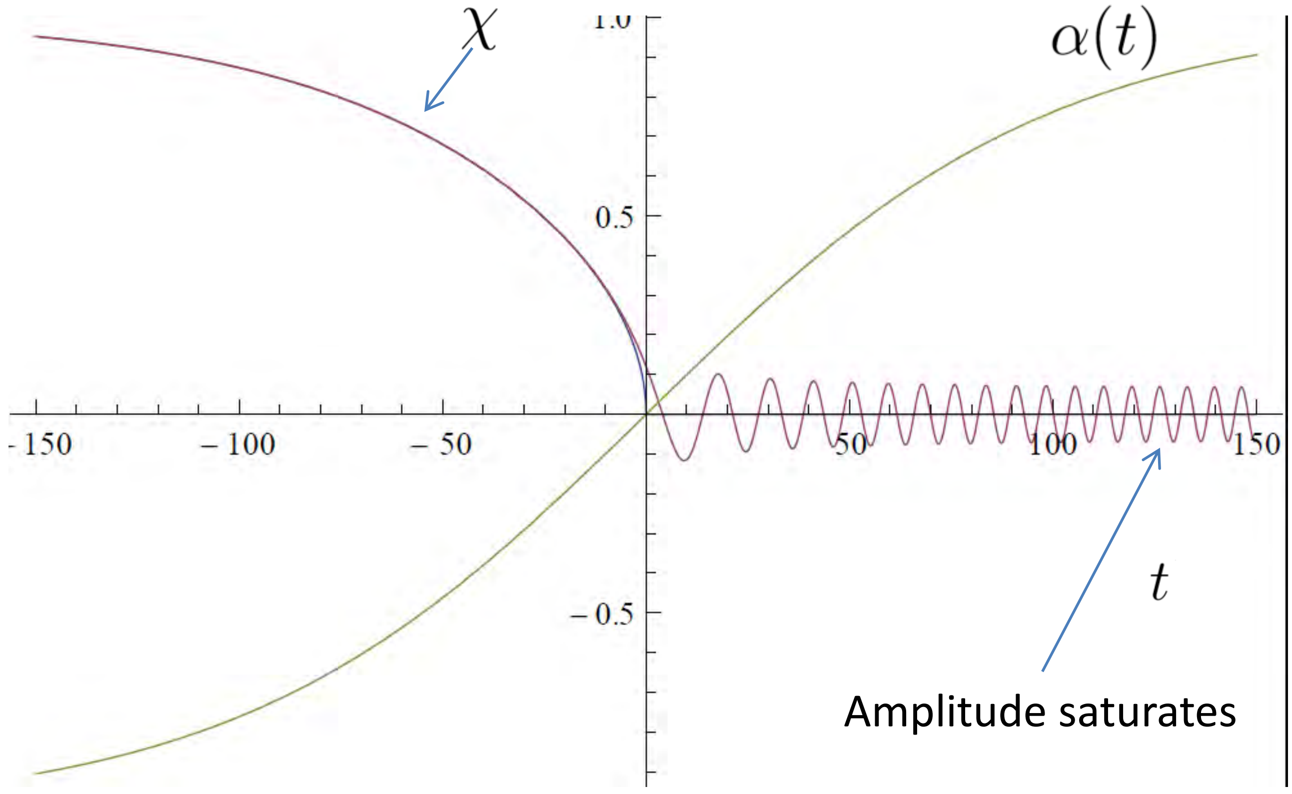
- In equilibrium, there would be a critical point when

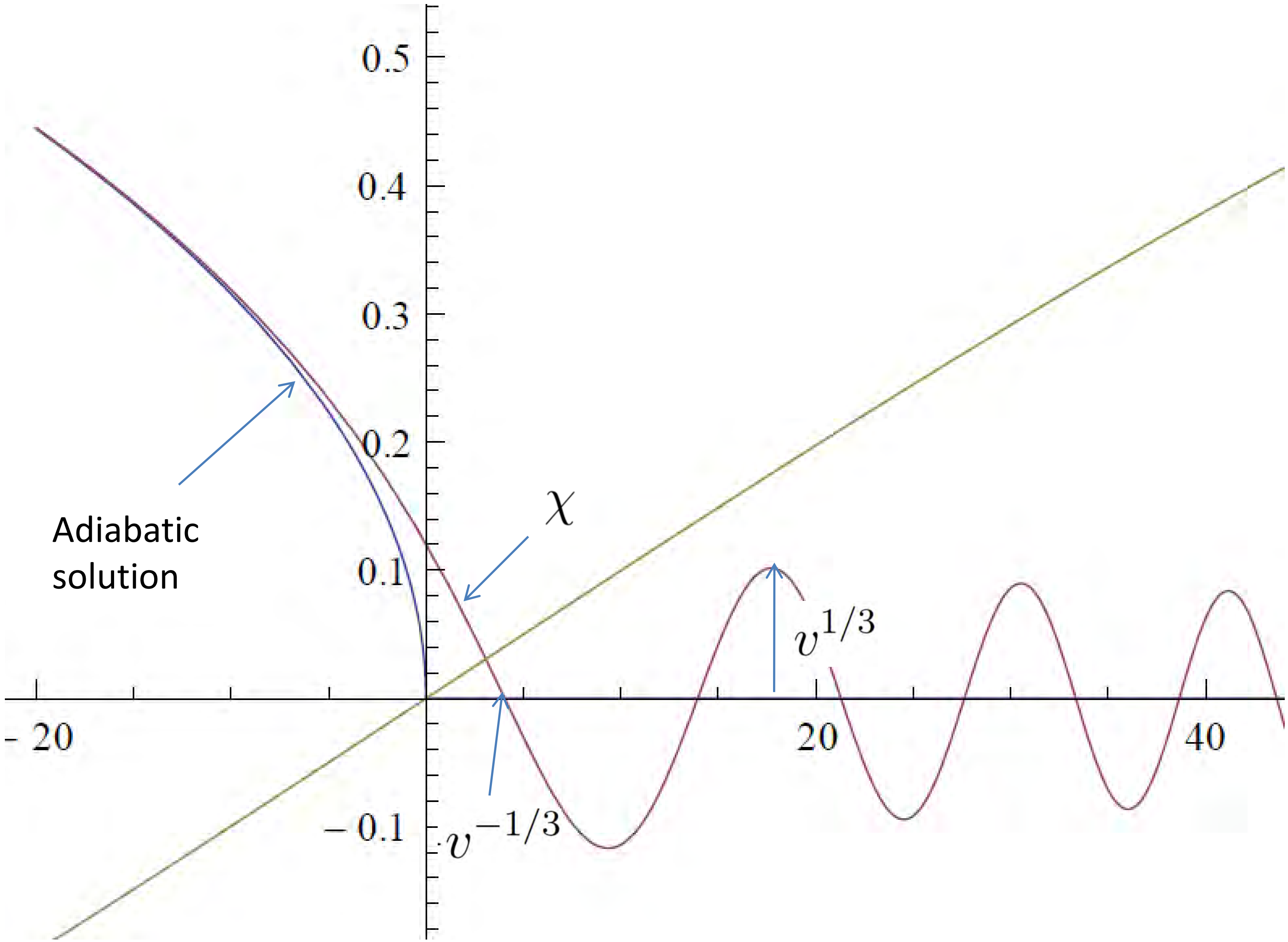
$$\alpha = 0$$

We will start in the ordered phase with adiabatic initial conditions

$$\chi(-T) = m_0\sqrt{\tanh(vT)} \qquad \dot{\chi}(-T) = -m_0 v \frac{\operatorname{sech}^2(vT)}{2\sqrt{\tanh(vT)}}$$

- And track the dynamics of the order parameter by solving this equation




0.5
0.4
0.3
0.2
0.1
- 0.1
- 20
20
40
Adiabatic
solution
χ
$v^{1/3}$
$v^{-1/3}$

- At early times the order parameter has small oscillations around the adiabatic solution.
- At time $-v^{-1/3}$ the order parameter starts deviating from the adiabatic solution.
- At time $v^{-1/3}$ the order parameter hits a zero. This is after the mass-squared has crossed zero.
- After that time, for a while the order parameter oscillates with amplitude $v^{1/3}$ and a period $v^{-1/3}$.
- The late time amplitude saturates to a constant and goes as $v^{1/2}$
- The temperature at which the average order parameter drops to zero is in fact higher than the equilibrium critical temperature. This may be related to similar effects known in the condensed matter literature for a long time, explored in holographic phase transitions by *Bao, Dong, Silverstein and Torraba*.

- Close to $t = 0$ the equation of motion becomes

$$\frac{d^2\chi}{dt^2} + vt\chi + \chi^3 = 0$$

- In terms of a new time η and a new order parameter ξ

$$\eta = tv^{1/3} \qquad \xi = v^{-1/3}\chi$$

this becomes

$$\frac{d^2\xi}{d\eta^2} + \eta\xi + \xi^3 = 0$$

- Therefore $$\chi(t;v) = v^{1/3}\chi(tv^{1/3};1)$$

- This explains the scaling properties described above

- This kind of analysis can be extended to situations where $\alpha(t)$ goes to zero as some higher power of t
- This behavior is universal – and follows from arguments similar to Kibble-Zurek scaling.
- The late time amplitude is however not universal – this depends on the details of how the mass saturates at late times. In this particular case this may be explained by solving the linearized equation (which is valid at late times) in a WKB approximation and matching the solution to the critical solution.

- A different passage across the critical point involves an external time dependent source for the order parameter (a time dependent external magnetic field) while the temperature is tuned to exactly the critical temperature.
- Now the equation is

$$\frac{d^2\chi}{dt^2} + \chi^3 + J(t) = 0$$

- If the source has the behavior

$$J(t) \sim vt \qquad t \sim 0$$

- The critical region solution scales as

$$\chi(t;v) = v^{1/4}\chi(v^{1/4}t;1)$$

Back to AdS/CFT

- In the holographic context, the role of the field χ is played by the zero mode of the operator

$$-f(r)\partial_r(r^4 f(r)\partial_r) + r^2 f(r)(m^2 - m_c^2)$$

- The equation of motion satisfied by this mode in the critical region is similar to the Landau-Ginsburg problem

$$\frac{d^2\xi}{d\eta^2} + \eta\xi + \xi^3 = 0$$

- The higher modes become stiff in this region. They satisfy equations of the form (in rescaled variables)

$$\frac{d^2\xi_n}{d\eta^2} + n^2 v^{-2/3}\xi_n + \eta\xi_n + O(\xi^3) = 0$$

- The second term is large – and basically kills all dynamics.
- Beyond the critical region, these modes start to play a role. In fact at late times, the field oscillates around zero and the mass becomes constant – thus the solutions are in fact the quasinormal modes
- The imaginary part of their frequencies then imply that the amplitude of late time oscillations decay – unlike our Landau Ginsburg example.

- We are solving the nonlinear equation numerically
- Our results are consistent with what I described, but so far too noisy to check these expectations accurately.

To Summarize

- We described a mechanism for thermalization in probe brane theories – viz. formation of apparent horizons in induced metric on worldvolume.
- We have explored quench dynamics near critical points in a class of theories with holographic phase transitions.

- So far the model(s) we studied had mean field exponents. The power laws in the critical region follow directly from LG – no need for holography.
- The decay of the late time amplitude is of course absent in standard LG theory.
- We regard our investigations as a viability study – since the same techniques can be used for transitions with holographic duals which have non-mean-field exponents.