Can Gravity be Localized ?

based on : CB, J. Estes, *arXiv:1103.2800 [hep-th]*

B. Assel, CB, J. Estes, J. Gomis, *1106.xxxx*

also: O. Aharony, L. Berdichevsky, M; Berkooz, I. Shamir, *arXiv:1106.1870 [hep-th]*

C. Bachas, ETH 06/11

Fields can be localized on (extended) **solitons** in **QFT**:

there are many spin-0 and spin-1/2 examples

Goldstone modes

e.g. superconducting strings

Can also localize abelian "spin-1" fields

Josephson junctions

Renormalizability requires $D \leq 4 \implies d \leq 3$

which severely restrict the possibilities.

In **string theory** D=10 and ∃ UV completion

so more room

can do *some* calculations without infinities

non-abelian spin-1 gauge fields localized on **D-branes**

but what about spin 2 ?

Einstein's theory is much harder to "tinker" with ......

This is closely related to the questions:

Can the graviton have mass?

Can it be a resonance?

Are sectors “hidden” from gravity possible ?

Other IR modifications of Einstein equations ?

The subject has a long history, to which I will not try to do justice here .....

see also Slava Mukhanov’s talk

In Minkowski spacetime, the answer seems to be NO

An important obstruction is the vDVZ discontinuity

van Dam, Veltman, Zakharov '70

Notice that for the photon the answer is YES

Indeed, the particle data group quotes the experimental bound:

$$m_\gamma < 10^{-18} eV$$

$\text{range} > 10^9\, km \sim 1\, \text{light hour}$ but could be finite!

To understand the difference, consider the linearized Lagrangian for a massive spin-1 particle:

$$\mathcal{L} = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{m^2}{2} A_\mu A^\mu + A_\mu j^\mu$$

Introducing a spurious field $A_\mu = A'_\mu + \frac{1}{m}\partial_\mu \phi$ and taking $m \to 0$ gives:

$$\mathcal{L} = -\frac{1}{4}(\partial_\mu A'_\nu - \partial_\nu A'_\mu)^2 + A'_\mu j^\mu - \frac{1}{2}\partial_\mu \phi \partial^\mu \phi + \frac{1}{m}\partial_\mu \phi j^\mu$$
$$= \mathcal{L}_{\mathrm{Maxwell}} - \frac{1}{2}\partial_\mu \phi \partial^\mu \phi$$

The dangerous last term drops out, provided the *e-m* current is conserved, so that the extra scalar mode decouples.

Now repeat the exercise for a massive spin-2 field.

The (ghost-free) massive Pauli - Fierz Lagrangian is:

$$\mathcal{L}_{\rm PF} = \mathcal{L}_{\rm EH} - \frac{m^2}{2}\left(h^{\nu\lambda}\,h_{\nu\lambda} - (h^{\rho}{}_{\rho})^2\right)$$

where

$$\mathcal{L}_{\rm EH} = -\frac{1}{2}\partial_\mu h^{\nu\lambda}\,\partial^\mu h_{\nu\lambda} + \partial^\mu h^{\nu\lambda}\,\partial_\nu h_{\mu\lambda} - \partial_\mu h^{\mu\nu}\,\partial_\nu h^\lambda{}_\lambda + \frac{1}{2}\partial_\nu h^\lambda{}_\lambda \partial^\nu h^\rho{}_\rho + h_{\mu\nu}T^{\mu\nu}$$

with $$\partial_\mu T^{\mu\nu} = 0$$

Introduce again compensators to restore gauge invariance:

$$h_{\mu\nu} = h'_{\mu\nu} + \frac{1}{m}(\partial_\mu A_\nu + \partial_\nu A_\mu) + \frac{2}{m^2}\partial_\mu\partial_\nu\phi$$

invariant under

$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$
$$\delta A_\mu = -m\xi_\mu + \partial_\mu \Lambda$$
$$\delta\phi = -m\Lambda$$

Inserting in $\mathcal{L}_{\mathrm{PF}}$ gives a free massless spin-1 field, and a two-derivative Lagrangian mixing ϕ and $h'_{\mu\nu}$.

PF was precisely devised for this !

Redefining fields to remove the mixing ($h'_{\mu\nu} = h''_{\mu\nu} + \eta_{\mu\nu}\phi$) finally gives:

$$\mathcal{L}_{\rm PF} = \mathcal{L}_{\rm EH} - \frac{1}{2} F_{\mu\nu} F^{\mu\nu} - 3\,\partial_\mu\phi\,\partial^\mu\phi + \boxed{\phi\, T^\rho{}_\rho}$$

The residual coupling is different for light, than for massive matter;

thus the Pauli-Fierz theory does not give Einstein's theory when $m \longrightarrow 0$

If we set Newton's law to its measured form,

light bending = 3/4 of measured effect

.... so however tiny the mass, it is ruled out !

The story looks more promising in AdS:

- The vDVZ discontinuity is absent if $m_{\rm gr} < 1/L_{\rm AdS}$

  Kogan - Mouslopoulos - Papazoglou;
  Porrati

- $\exists$ a simple "model", possibly embed-*able* in string theory

  Karch-Randall

- Supersymmetry can protect the required hierarchy

Of course, we don't seem to live in AdS spacetime !

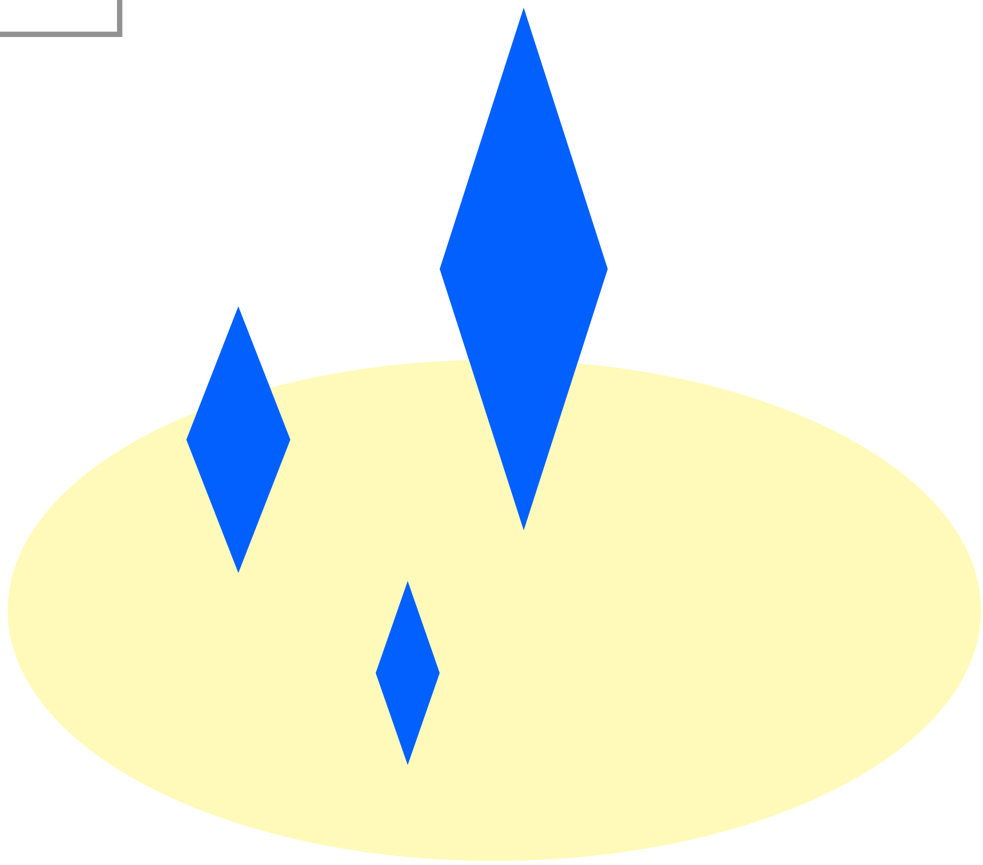
OK, take attitude that anything one can learn about IR gravity is interesting, and proceed.

KK reduction for spin 2

Interested in *warped*-(A)dS geometries,

$$\widehat{ds^2} = e^{2A(y)}\bar{g}_{\mu\nu}(x)\,dx^\mu dx^\nu + \hat{g}_{ab}(y)\,dy^a dy^b$$

$$\bar{\mathcal{M}}_4 = \mathrm{AdS}_4,\ \mathbb{M}_4,\ \mathrm{dS}_4$$
$$k = -1, 0, 1$$



Consider (consistent reduction to) metric perturbations

$$ds^2 = e^{2A}\,(\bar{g}_{\mu\nu} + h_{\mu\nu})\,dx^\mu dx^\nu + \hat{g}_{ab}\,dy^a dy^b \ ,$$

with $$h_{\mu\nu}(x,y) \ = \ h^{[\mathrm{tt}]}_{\mu\nu}(x)\,\psi(y)$$

where $$(\bar{\Box}^{(2)}_x - \lambda)\,h^{[\mathrm{tt}]}_{\mu\nu} = 0 \qquad \text{and} \qquad \bar{\nabla}^\mu h^{[\mathrm{tt}]}_{\mu\nu} = \bar{g}^{\mu\nu} h^{[\mathrm{tt}]}_{\mu\nu} = 0 \ .$$

Pauli-Fierz $(\lambda = m^2 + 2k)$

Linearize the Einstein equations $$R_{MN} - \frac{1}{2} g_{MN} R = T_{MN}$$

to find the Schrodinger problem :

$$-\frac{e^{-2A}}{\sqrt{[\hat{g}]}} \left(\partial_a \sqrt{[\hat{g}]}\, \hat{g}^{ab} e^{4A} \partial_b\right) \psi = m^2 \psi$$

This is equivalent to a *scalar-Laplace* equation in d dimensions :

$$\frac{1}{\sqrt{\hat{g}}} \left(\partial_M \sqrt{\hat{g}}\, \hat{g}^{MN} \partial_N\right) h_{\mu\nu}(x, y) \;=\; 0 \;.$$

Important: the linearized equation depends only on the geometry, not on the detailed matter-fields that created it.

Csaki, Erlich, Hollowood, Shirman

CB, JE

Localization of spin-2 can only come from geometry

The wavefunction norm is

$$||\psi||^2 \equiv \int d^{d-4}y \sqrt{[\hat{g}]}\, e^{2A}\, |\psi|^2$$

The would-be massless graviton has $\psi(y) = \text{constant}$

It is normalizable iff the transverse volume is finite

Why can't the warp factor "help"?

When it does, *infinity* is an apparent horizon, so

-- geometry should be made geodesically complete

-- or should supplement *quantum* theory with boundary conditions at horizon ("IR brane")

In the cases

$$\mathcal{M}_4 = \mathbb{M}_4 \quad \text{or} \quad \mathrm{dS}_4$$

the energy conditions show (at least in codim = 1) that the warp factor A is monotonic, so it cannot turn around to form an effective “graviton trap”

But for $\mathcal{M}_4 = \mathrm{AdS}_4$ localization, and a tiny AdS graviton mass cannot be *a priori* ruled out.

Karch-Randall model

Starting point is 5D Einstein action plus a thin 3-brane

$$I_{\mathrm{KR}} \;=\; -\frac{1}{2\kappa_5^2}\int d^4x\,dy\,\sqrt{g}\left(R+\frac{12}{L^2}\right)+\lambda\int d^4x\,\sqrt{[g]_4}\;,$$

The solution is:

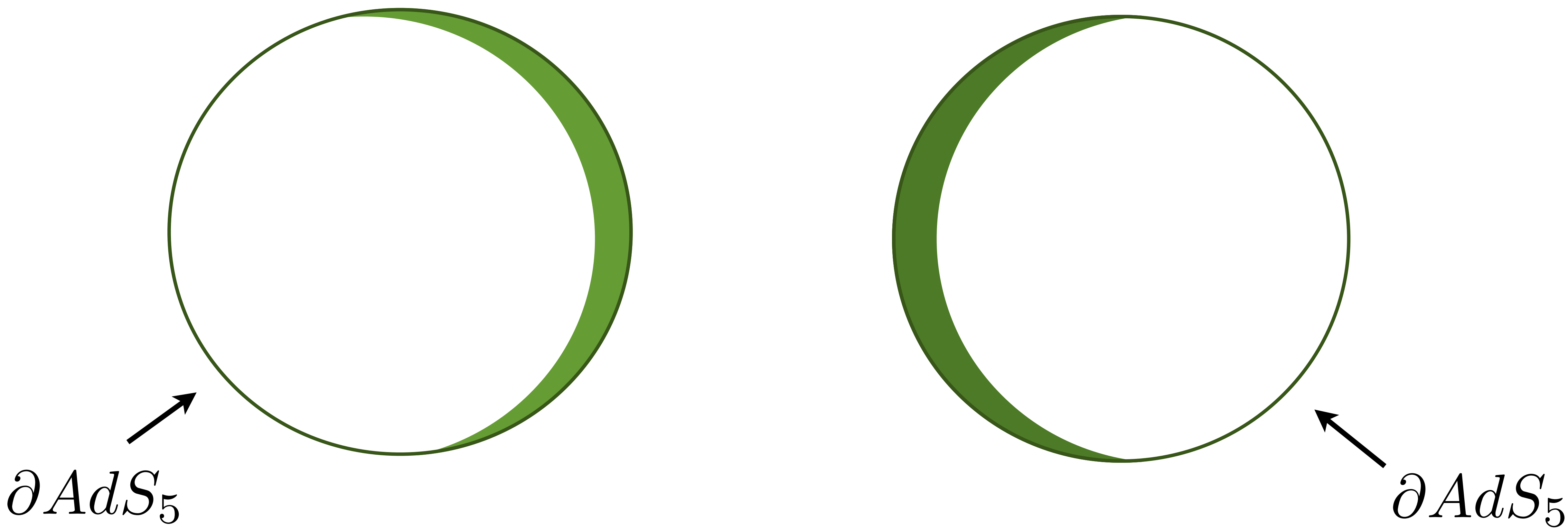
$$ds^2 = L^2\cosh^2\left(\frac{y_0-|y|}{L}\right)\bar{g}_{\mu\nu}dx^\mu dx^\nu + dy^2,$$ *where* $$y_0 = L\,\mathrm{arctanh}\left(\frac{{\kappa_5}^2\lambda L}{6}\right)$$

It describes two (large) slices of AdS_5 glued along a AdS_4 brane

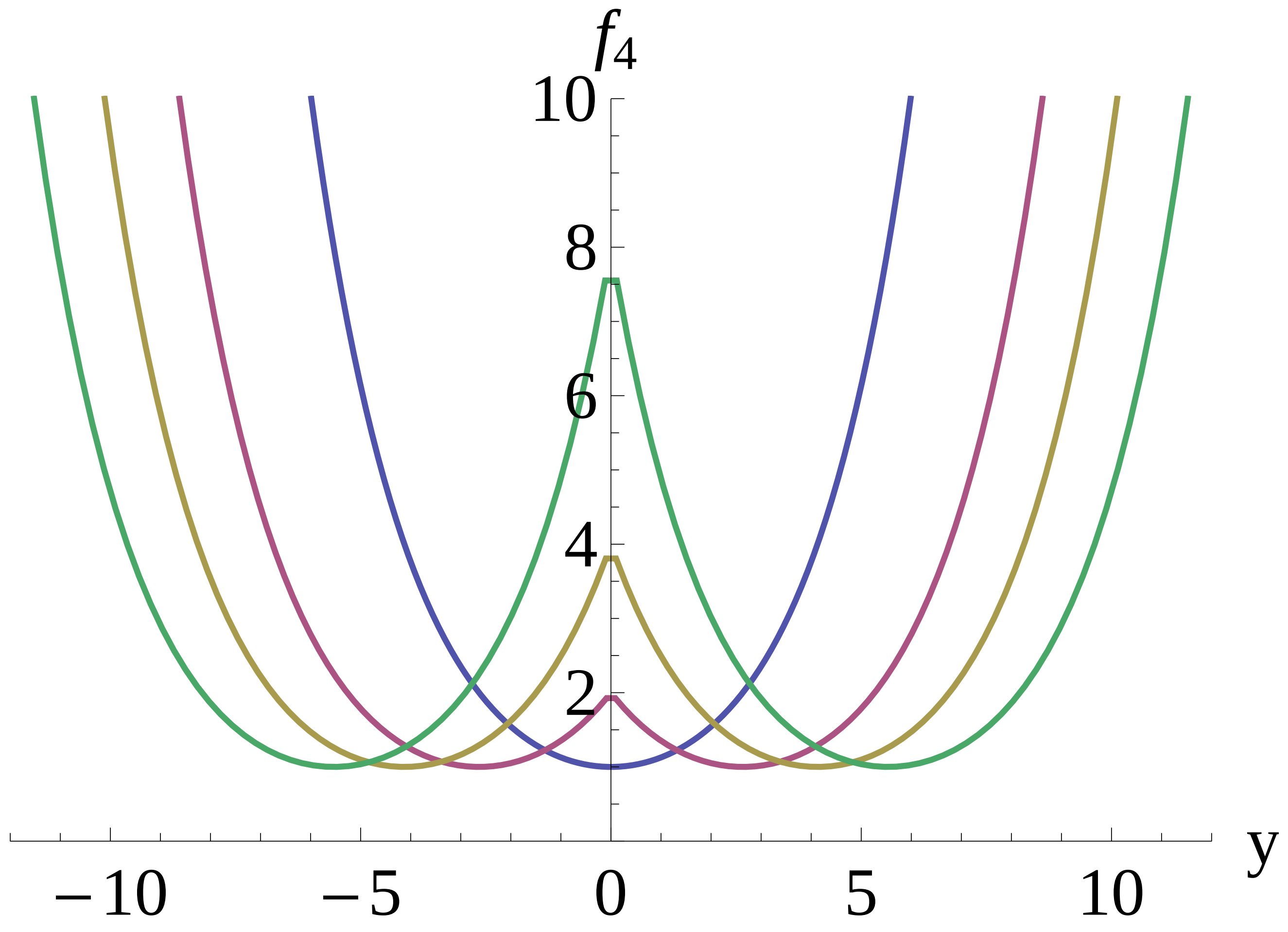
with radius

$$\ell^2 \;=\; e^{2A(0)} \;=\; L^2\cosh^2\left(\frac{y_0}{L}\right)\;.$$

One can tune λL **to make** $\frac{\ell}{L} \gg 1$



Cut away green slices, then glue the white ones in a symmetric fashion. Gives two 4D boundaries glued across two 3D defects (domain walls).



Warp factor $$e^{2A} \equiv {f_4}^2 = L^2 \cosh^2\left(\frac{y_0 - |y|}{L}\right)$$

as ℓ/L is gradually tuned up

4D parameters: $$8\pi G_N \simeq \kappa_5^2/L$$ *as in usual KK*

$$V_{\text{Newton}} + \Delta V \simeq -\frac{G_N m_1 m_2}{r}\left(1 + \gamma \frac{L^2}{r^2} + \cdots\right)$$

unlike standard KK

so $$\frac{\ell}{L} \sim 10^{31} - 10^{62}$$

Spectrum : - a nearly-constant, nearly massless mode $m_0^2 \simeq \dfrac{3L^2}{2\ell^2}$

- two towers of AdS5 modes

$$m^2 \simeq (2n+1)(2n+4) \qquad n = 0, 1, \cdots$$

These masses are in units of the AdS4 radius

so states with $m^2 \simeq o(1)$ mediate long-range interactions.

What "saves the day" is that the AdS5 states live at the bottom of the warp-factor well . Their wavefunctions are exponentially suppressed at the brane position

Furthermore, $$\int \psi_0 \psi^\dagger \psi \neq \text{universal}$$

so the nearly-massless graviton has non-universal couplings to the other fields !

The exact (super)gravity solutions

Karch and Randall proposed to embed their model in IIB string theory, by inserting 5-branes in the $AdS_5 \times S^5$ geometry of D3-branes.

The exact geometry of these configurations was discovered recently by *D'Hoker, Estes and Gutperle*

Try to understand whether graviton in these geometries is localized.

The solutions are $AdS_4 \times S^2 \times S^2$ fibrations over a surface Σ .

They depend on two harmonic functions h_1 , h_2 subject to certain global consistency conditions.

metric :

$$ds^2 = f_4^2 ds^2_{\mathrm{AdS}_4} + f_1^2 ds^2_{\mathrm{S}_1^2} + f_2^2 ds^2_{\mathrm{S}_2^2} + 4\rho^2 dz d\bar{z} \ ,$$

$$f_4^8 = 16 \frac{N_1 N_2}{W^2} \ , \quad f_1^8 = 16\, h_1^8 \frac{N_2 W^2}{N_1^3} \ , \quad f_2^8 = 16\, h_2^8 \frac{N_1 W^2}{N_2^3}$$

dilaton :

$$e^{4\phi} = \frac{N_2}{N_1}$$

where :

$$W = \partial h_1 \bar{\partial} h_2 + \bar{\partial} h_1 \partial h_2 = \partial\bar{\partial}(h_1 h_2) \ ,$$

$$N_1 = 2 h_1 h_2 |\partial h_1|^2 - h_1^2 W \ , \qquad N_2 = 2 h_1 h_2 |\partial h_2|^2 - h_2^2 W \ .$$

There are also *3-form* and *5-form* backgrounds, and 1/4 unbroken supersymmetry.

The solutions of interest have $\sum$ = infinite strip
with h_1, h_2 obeying N or D conditions, possibly
with isolated singularities on the boundary, e.g.

The harmonic functions for this choice are:

$$
\begin{aligned}
h_1 &= \left[-i\alpha \sinh(z-\beta) - \gamma \ln\left(\tanh\left(\frac{i\pi}{4} - \frac{z-\delta}{2}\right)\right)\right] + \text{c.c.} \\
h_2 &= \left[\hat{\alpha} \cosh(z-\hat{\beta}) - \hat{\gamma} \ln\left(\tanh\left(\frac{z-\hat{\delta}}{2}\right)\right)\right] + \text{c.c.}
\end{aligned}
$$

The simplest **Janus** solution $\gamma_i = 0$ is a *dilaton domain wall*

$$L^4 = 16\,|\alpha_1\alpha_2|\,\cosh\Delta\phi \qquad\qquad e^{2\phi_\pm} = \left|\frac{\alpha_2}{\alpha_1}\right| e^{\pm\Delta\phi}$$

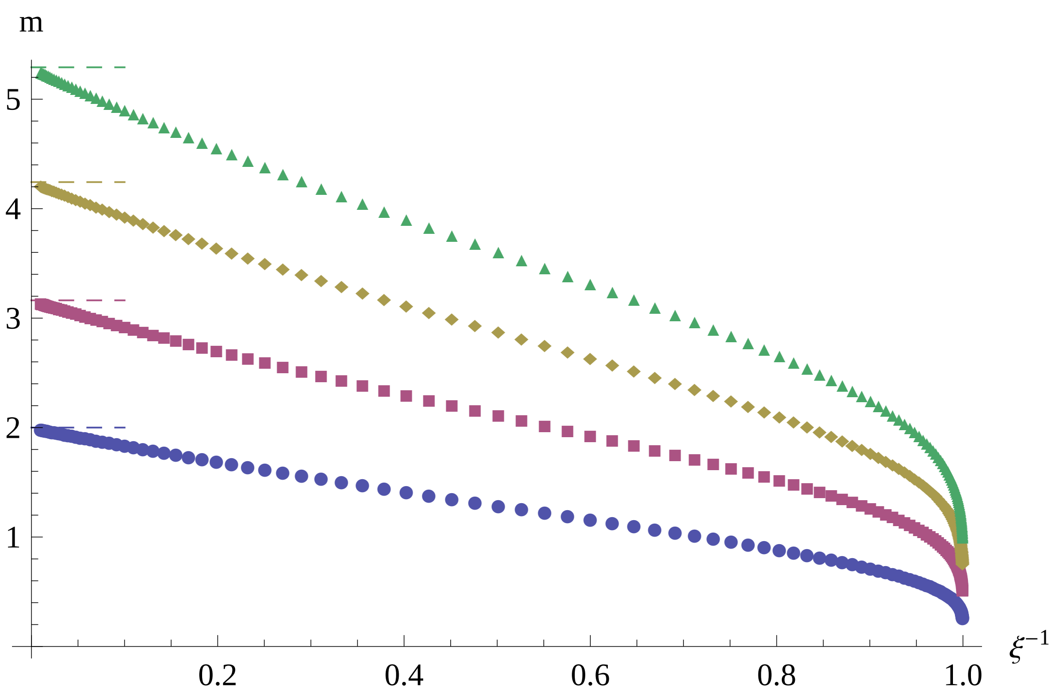
radius of asymptotic $AdS_5 \times S^5$

dilaton jump

The spectral equation reduces to a ODE with 4 regular singular points (Heun's equation) which can be solved with fast numerics

The results are not particularly exciting:

There is one free parameter, $(\tanh\frac{\Delta\phi}{2})^2 \;\equiv\; \xi^{-1} \in [0,1)$



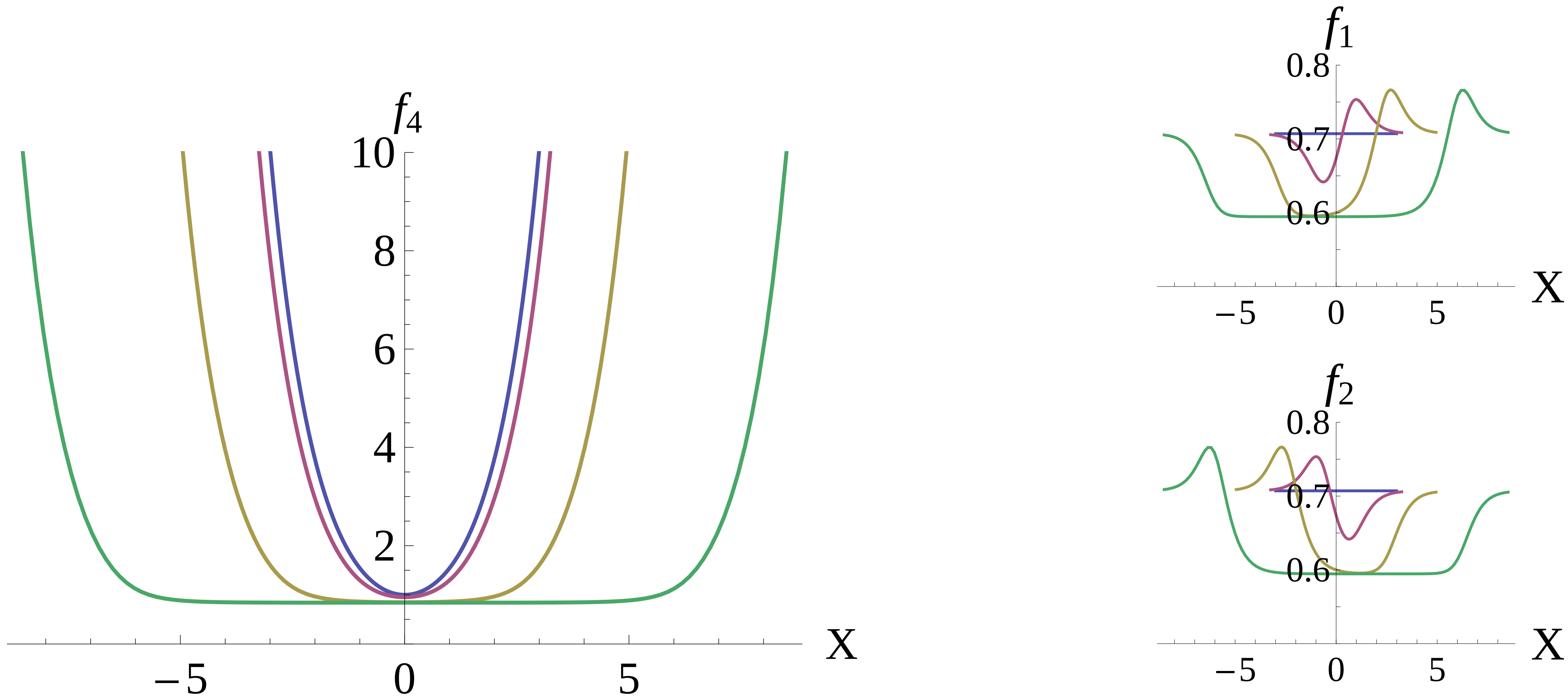
$AdS_5 \times S^5$

$\Delta\phi = \infty$

The only interesting limit is one in which a linear-dilaton dimension decompactifies, and the geometry becomes

$$AdS_4 \times \mathbb{R}_\phi \times_w \tilde{S}^5$$

$$\Delta\phi = 0, 1, 4, 10$$

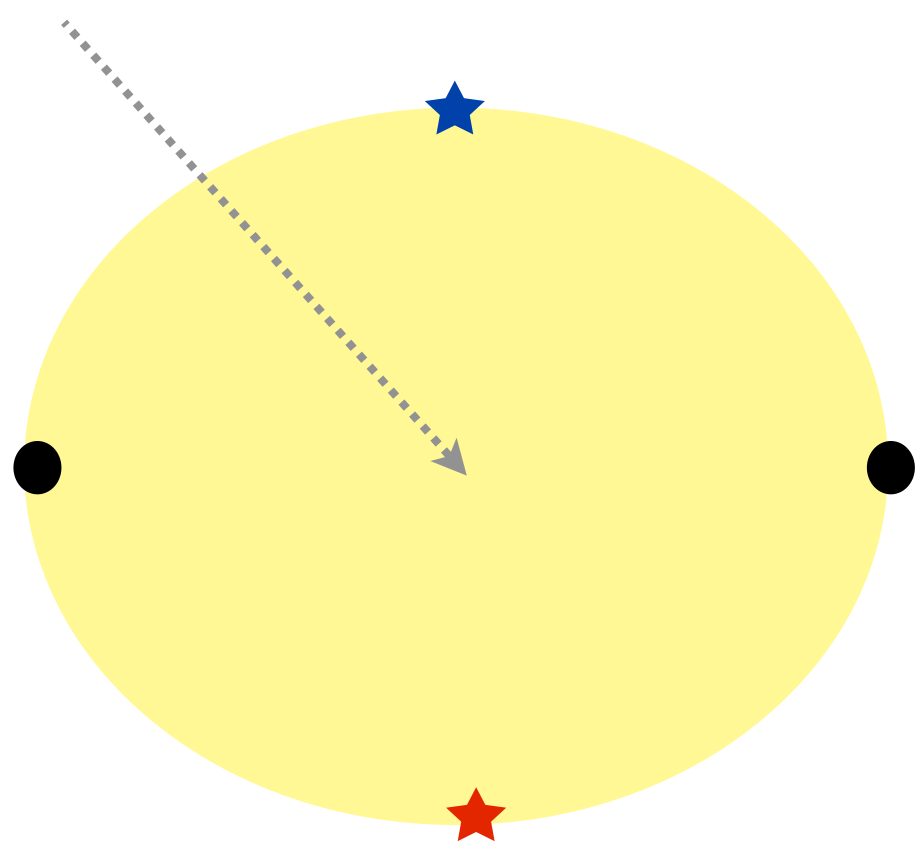


The “problem” is that the dilaton has no (super)potential, so its domain wall spreads to infinite thickness.

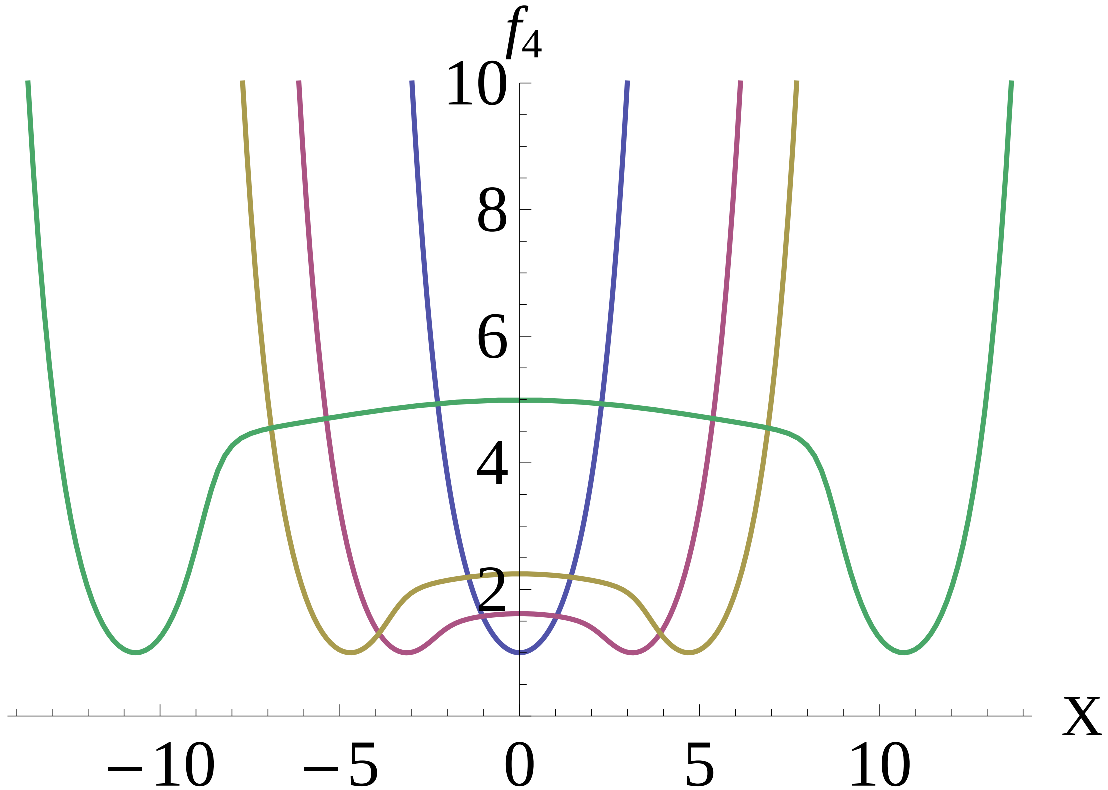
Adding one type of 5-branes does not help: the dilaton adjusts to (∞ ly) small or large value, so as to minimize 5-brane tension.

The only interesting limit is one with both NS5 and D5 charges, and with $\dfrac{Q_5}{Q_3} \gg 1$.

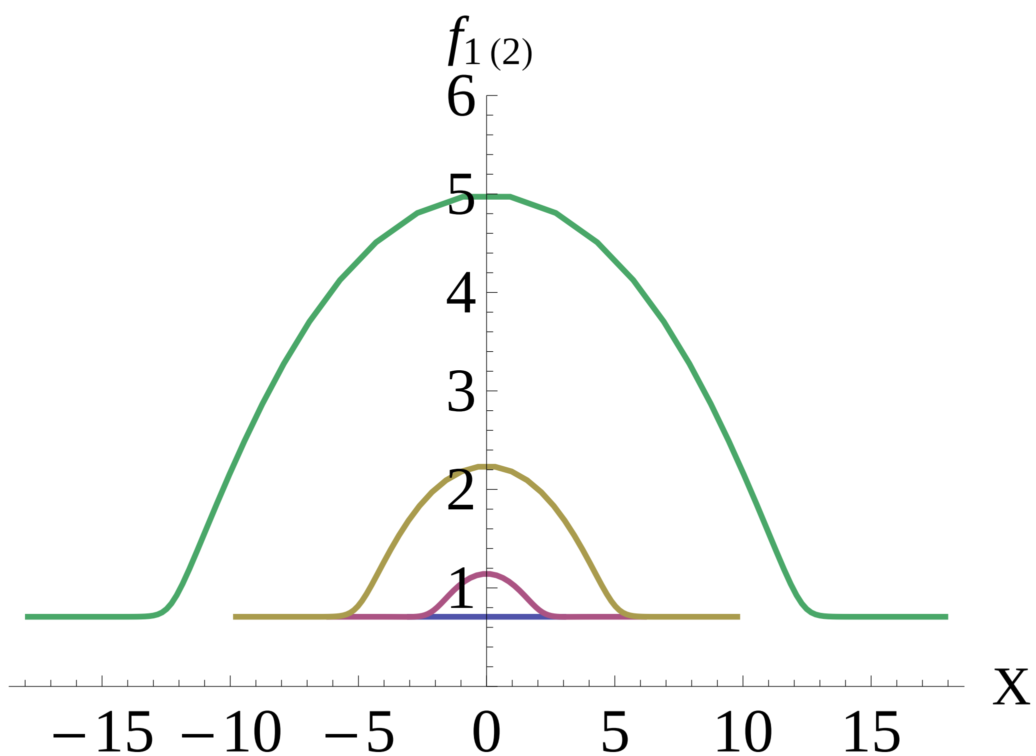
Inspection of the geometry shows that this creates a bubble of almost factorized $AdS_4 \ltimes K$ geometry in the central region.



The $AdS_5 \ltimes S^5$ regions are much more curved



warp factor



sphere radii

Actually the limit $Q_3 \longrightarrow 0$ is smooth, transverse space compactifies:

the asymptotic regions $AdS_5 \times S^5$ go over to smooth $AdS_4 \times \mathcal{D}_6$ caps

These $AdS_4 \ltimes K$ solutions must be gravity duals to

3-dimensional (super)conformal field theories

Which ones ?

By studying the flat-space configurations, *Gaiotto and Witten* have proposed the existence of a class of interacting SCFTs in three dimensions that they called

$$T^{\hat{\rho}}_{\rho}(SU(N))$$

They are in 1-to-1 corrspondence with solutions of Nahm's equations:

$$\frac{dX^a}{dt} = i\epsilon_{abc}[X^b, X^c]$$

on the interval, with boundary conditions that are simple poles,

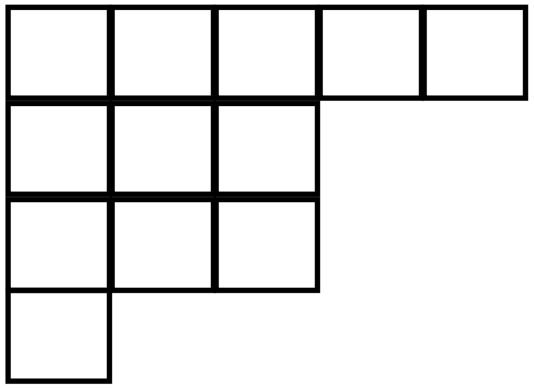
$$X^a \sim \frac{J^a}{t}$$

N-dimensional generators of SU(2)

This problem has been solved by *Kronheimer and Nakajima*

One can associate a partition of N with each choice of the J^a

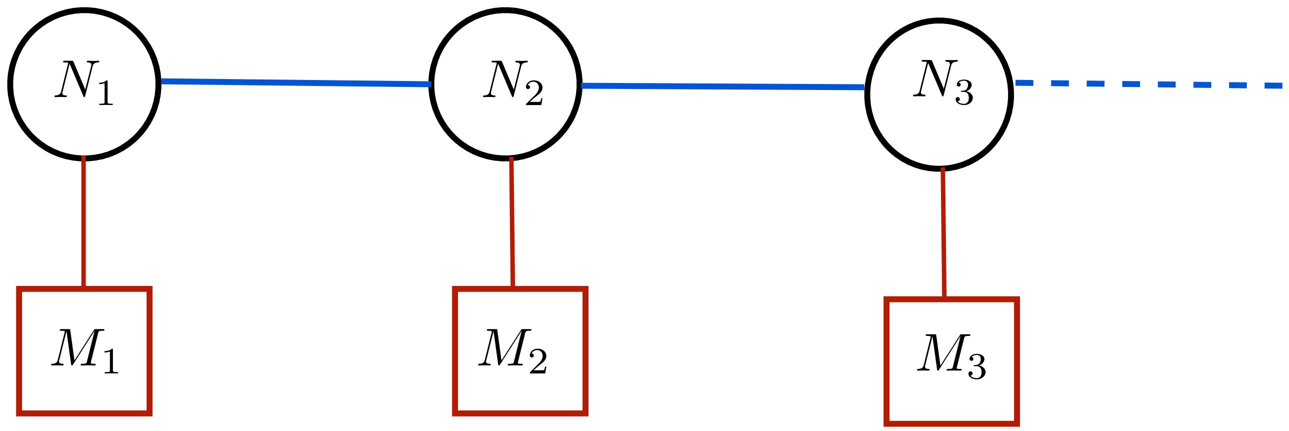
e.g. ρ : 12 = 5 + 3 + 3 + 1



K & N have shown that solutions exist iff $\rho^T > \hat{\rho}$

where these are the two partitions at the interval ends.

The underlying gauge theories are described by linear quivers



$$U(N_1) \times U(N_2) \times U(N3) \times \cdots$$

The quiver data can be read from the partitions:

$$\rho : \qquad N \quad = \quad l_1 + \ldots + l_k$$

linking numbers of D5-branes

$$= \underbrace{1 + \ldots + 1}_{M_1} + \underbrace{2 + \ldots + 2}_{M_2} + \ldots + \ldots \ .$$

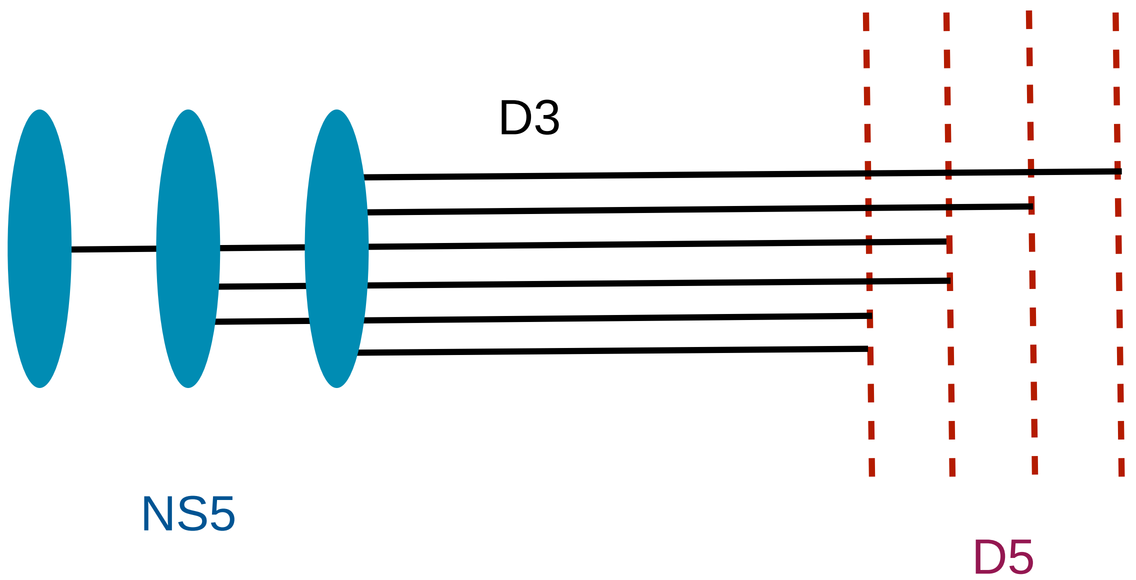
$$m_{l+1} = m_l - M_l$$

: transposed Young tableau

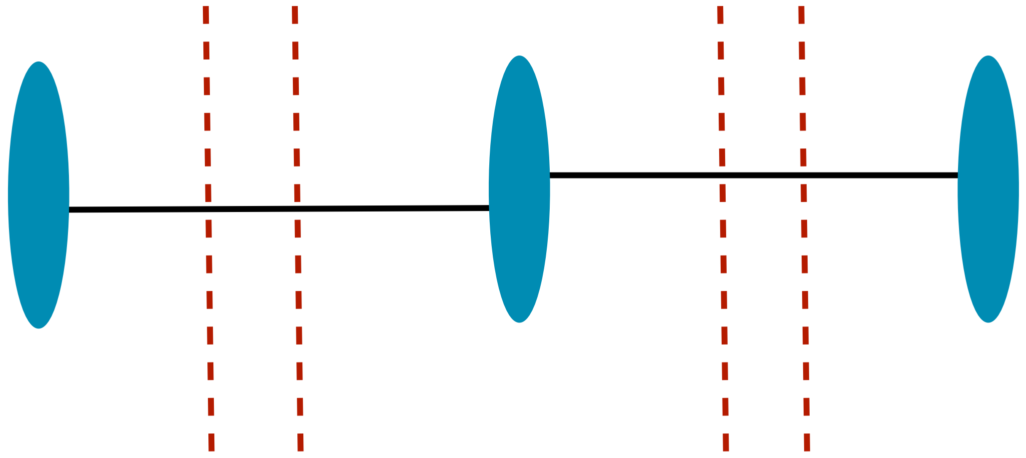
$$N_j = N_{j-1} + m_j - \hat{l}_j \qquad \text{for} \qquad j = 2, \cdots \hat{k} - 1 \ .$$

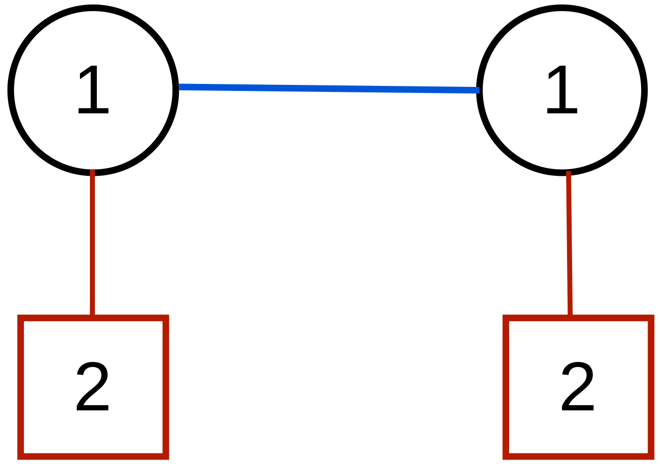
ranks of gauge groups

for example:



$$N = 6 \ ; \ \rho = (2,2,1,1) \ ; \ \hat{\rho} = (3,2,1)$$

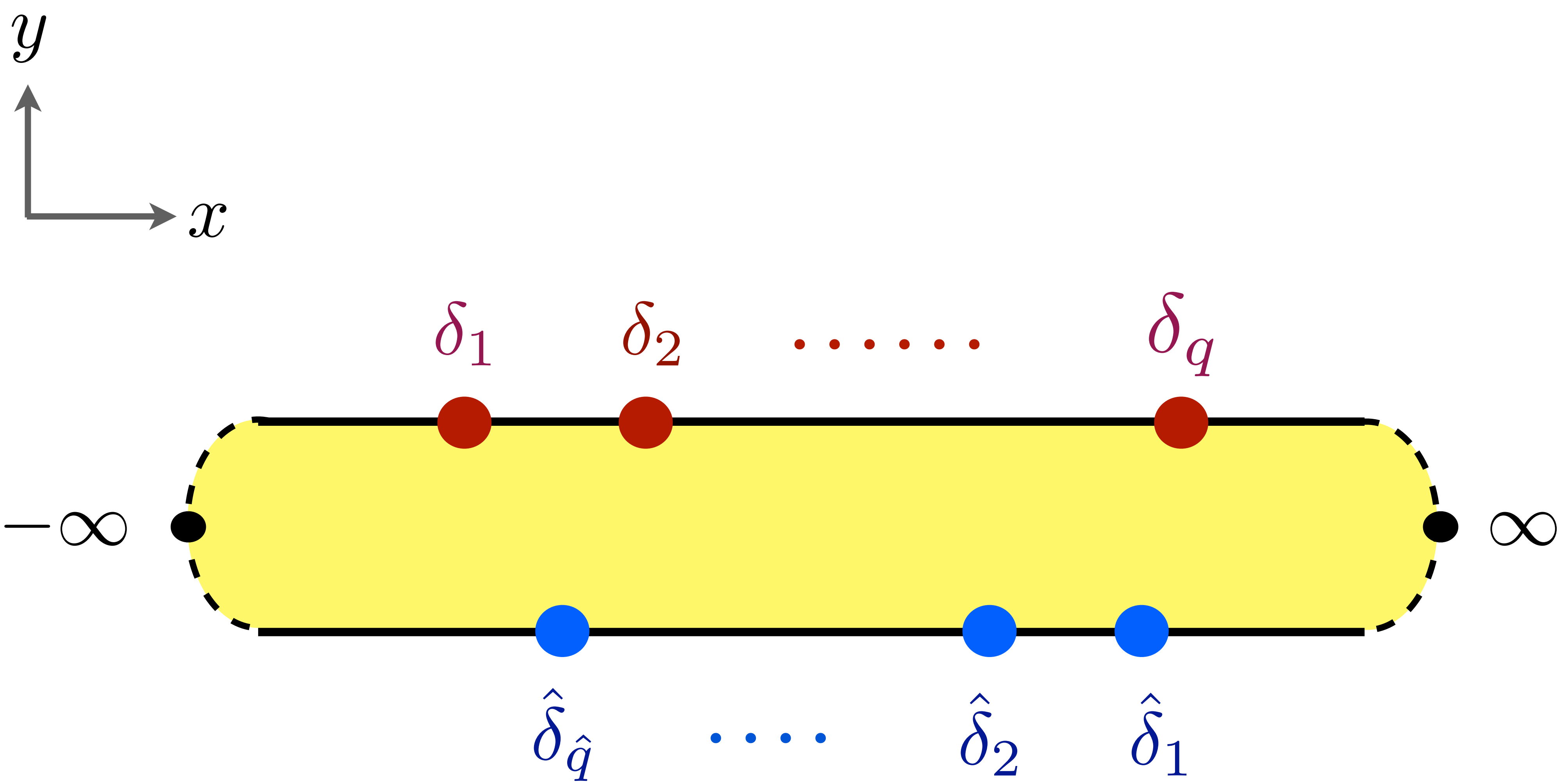




General result (by moving branes):

$$\text{supersymmetry} \iff \hat{\rho}^T \geq \rho \ , \quad \text{and} \quad \text{irreducibility} \iff \hat{\rho}^T > \rho \ .$$

When the inequality is saturated, the quiver breaks down to disjoint pieces.



$$h_1 = \left[-i\alpha\sinh(z-\beta) - \sum_{a=1}^{q}\gamma_a \ln\left(\tanh\left(\frac{i\pi}{4} - \frac{z-\delta_a}{2}\right)\right)\right] + c.c.$$

$$h_2 = \left[\hat{\alpha}\cosh(z-\hat{\beta}) - \sum_{b=1}^{\hat{q}}\hat{\gamma}_b \ln\left(\tanh\left(\frac{z-\hat{\delta}_b}{2}\right)\right)\right] + c.c.$$

D3-brane Page charges in fivebrane stacks:

$$Q_{D3}^{\mathrm{inv}(a)} = \int_{\mathcal{C}_a} F_5 - B_2 \wedge F_3 + \int_{\mathcal{C}_a} F_3 \wedge B_2 \Big|_{z=\infty}$$

$$= 2^8 \pi^3 \left(\hat{\alpha}\, \gamma_a \sinh(\delta_a - \hat{\beta}) - 2\, \gamma_a \sum_{b=1}^{\hat{q}} \hat{\gamma}_b \arctan(e^{\hat{\delta}_b - \delta_a}) \right)$$

$$\hat{Q}_{D3}^{\mathrm{inv}(b)} = \int_{\hat{\mathcal{C}}_b} F_5 + C_2 \wedge H_3 - \int_{\hat{\mathcal{C}}_b} H_3 \wedge C_2 \Big|_{z=-\infty}$$

$$= 2^8 \pi^3 \left(\alpha\, \hat{\gamma}_b \sinh(\hat{\delta}_b - \beta) + 2\, \hat{\gamma}_b \sum_{a=1}^{q} \gamma_a \arctan(e^{\hat{\delta}_b - \delta_a}) \right) .$$

$$N_{D3}^{(a)} \quad = -N_{D5}^{(a)} \sum_{b=1}^{\hat{q}} \hat{N}_{NS5}^{(b)} \frac{2}{\pi} \arctan(e^{\hat{\delta}_b - \delta_a}) \ ,$$

$$\hat{N}_{D3}^{(b)} \quad = \hat{N}_{NS5}^{(b)} \sum_{a=1}^{q} N_{D5}^{(a)} \frac{2}{\pi} \arctan(e^{\hat{\delta}_b - \delta_a})$$

Compute the linking numbers:

$$l^{(a)} \equiv -\frac{N_{D3}^{(a)}}{N_{D5}^{(a)}} \qquad \text{and} \qquad \hat{l}^{(b)} \equiv \frac{\hat{N}_{D3}^{(b)}}{\hat{N}_{NS5}^{(b)}} \ .$$

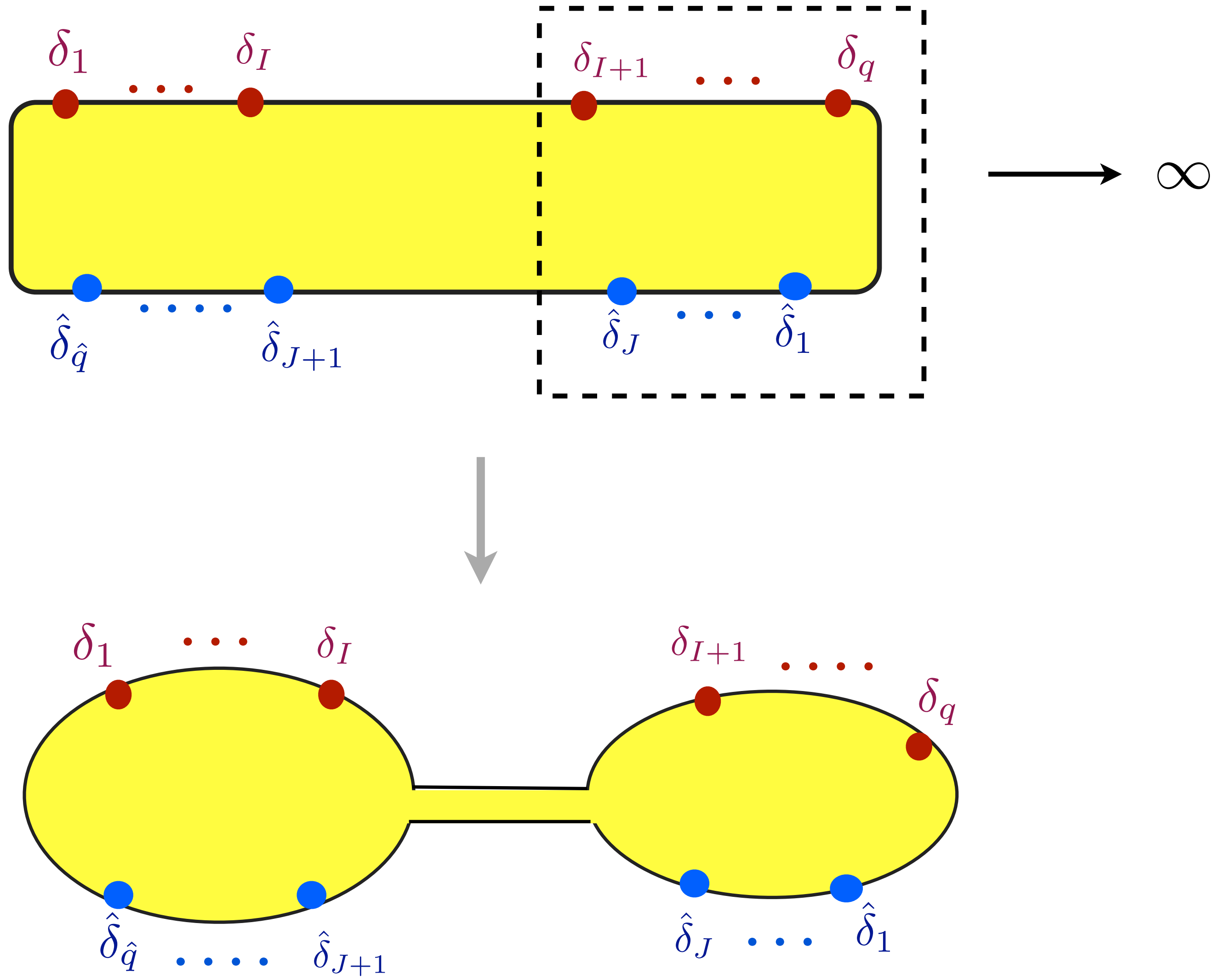
Prove $\rho^T > \hat{\rho}$ using the fact that $acrtan\theta \leq \pi/2$

The interesting limits $\hat{\rho} \simeq \rho^T$ correspond to **severing**

one (or more) **link**, by taking $N_i \longrightarrow 0$

This corresponds to factorizing the 5-brane singularities.

.... more on blackboard


δ_1
δ_I
δ_{I+1}
δ_q
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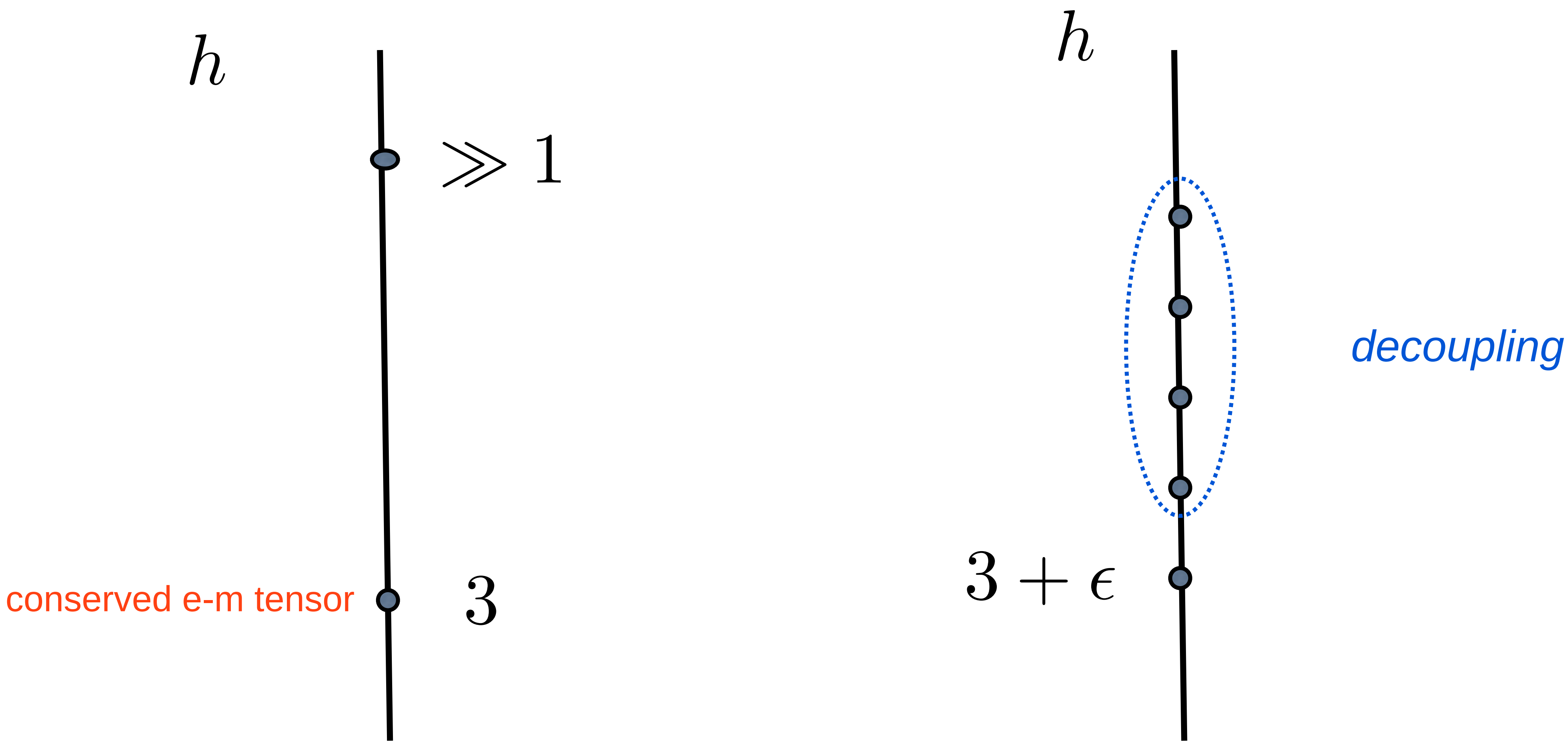
Holographic comment

on massive AdS gravity theories:



CFT spectrum

defect CFT spectrum

Two important observations:

Thank you