AdS/CFT and the cosmological constant problem

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- In QFT vacuum energy not observable.
- In Gravity vacuum energy does backreact
- Quantum fluctuations $\Rightarrow \rho_{1-loop} \sim M_{cut}^4$
- If we take $M_{cut} \sim M_P$ then

$$\frac{\rho_{observed}}{\rho_{1-loop}} \sim 10^{-120}$$

Cosmological constant fine-tuning

- $\rho_{obs} = \rho_{bare} + \rho_{1-loop}$

- Two terms must be fine-tuned, one part in 10^{120}.

- (will not discuss what fixes ρ_{obs} or cosmic coincidence problem etc.)

Emergence of spacetime

- Why don't quantum fluctuations gravitate?
- Question independent of $\Lambda > 0$ or $\Lambda < 0$
- c.c. fine tuning also in AdS
- AdS/CFT: spacetime and gravity are emergent from CFT
- Should be able to **see** (via the dual CFT) whether quantum fluctuations gravitate

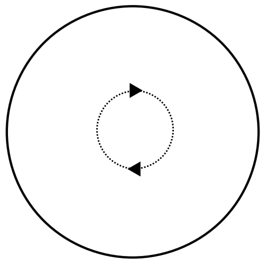
MAIN QUESTION

- How do we translate the bulk cosmological constant fine-tuning into a CFT question?

- Any CFT question can (in principle) be answered without the need for new physics.

- We should be able to understand the c.c. fine-tuning.

How do we proceed?



- Need to understand meaning of "1-loop diagram" and "counterterm' ($\sim$ bare c.c.) in CFT language.

- Difficult to map the bulk vaccum-to-vacuum amplitudes in CFT.

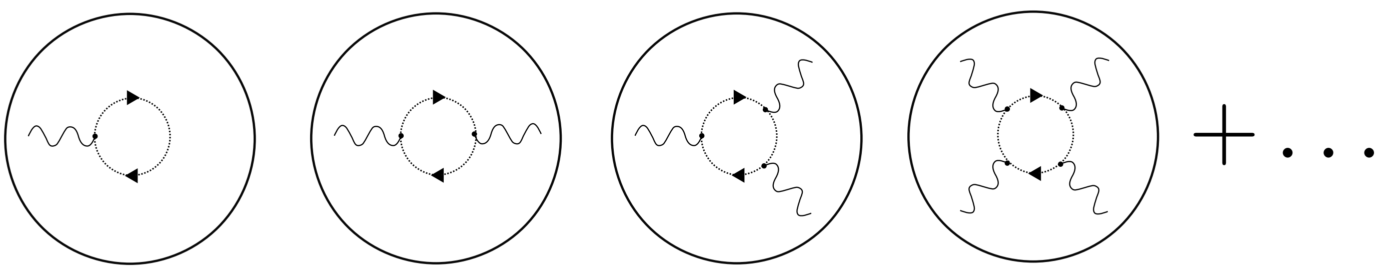
- More familiar with computation of Witten diagrams (diagrams with external legs).

- c.c. counterterm has the form

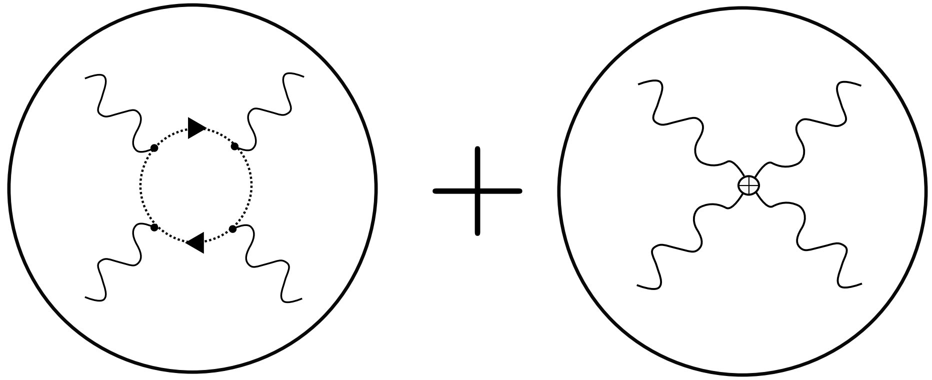
$$\int d^4x \sqrt{g}\, \Lambda_{bare}$$

- Expanding $\sqrt{g} \rightarrow$ generates graviton vertices with arbitrary number of external legs.

- Necessary to (partly) cancel diagrams of the form



Example: graviton 4-point function



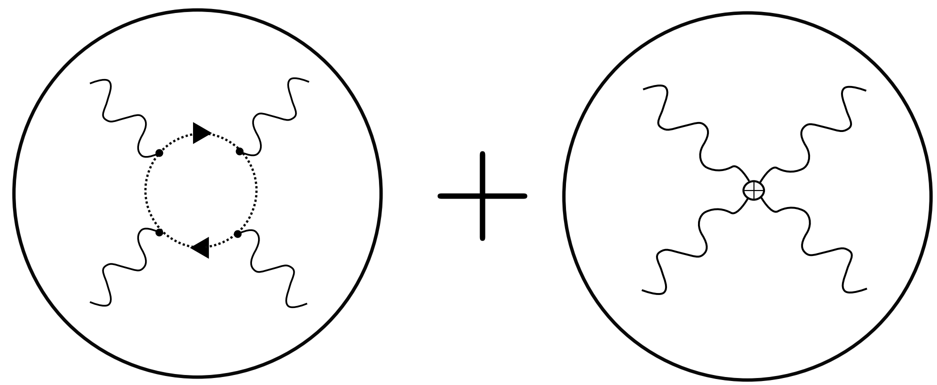
- If observed c.c. is small, then connected 4-point function should be small i.e. of the order

$$(M_P R)^{-2}$$

- If we take AdS as big as the universe, 4-point function should be of order

$$10^{-120}$$

Example: graviton 4-point function



- Estimate 1-loop diagram:

$$G_N^2 \int (d^4p)\ p^4 \ \left(\frac{1}{p}\right)^4 \sim M_P^{-4} M_{cut}^4 \sim \mathcal{O}(1)!$$

- Two diagrams above are both $\mathcal{O}(1)$ and exactly cancel up to something of order $\mathcal{O}(10^{-120})$.

Summary

- c.c. fine tuning can be understood in terms of bulk correlation functions. (Witten diagrams)

- via AdS/CFT $\Rightarrow$ c.c. fine tuning should be visible in CFT correlators

Fine-tuning in CFT

- CFTs with holographic duals $\rightarrow$ large N expansion.

- Small (observered) c.c. $\leftrightarrow$

$$\langle \widetilde{T}(x_1)...\widetilde{T}(x_n)\rangle_{con} \sim N^{2-n}$$

- **IS THE LARGE N EXPANSION NATURAL?**

- For example 4-point function should be of order

$$\frac{1}{N^2}$$

is the $1/N$ suppression achieved in a **natural** way, or via cancellations between parametrically larger terms?

- Any CFT correlator $\Rightarrow$ expanded in "conformal blocks"

$$\langle\widetilde{T}(x_1)\widetilde{T}(x_2)\widetilde{T}(x_3)\widetilde{T}(x_4)\rangle_{con} = \sum_{\mathcal{A}} |C^{\mathcal{A}}_{TT}|^2 \mathbf{G}_{\mathcal{A}}(x_1, x_2, x_3, x_4)$$

OUR CONJECTURE:

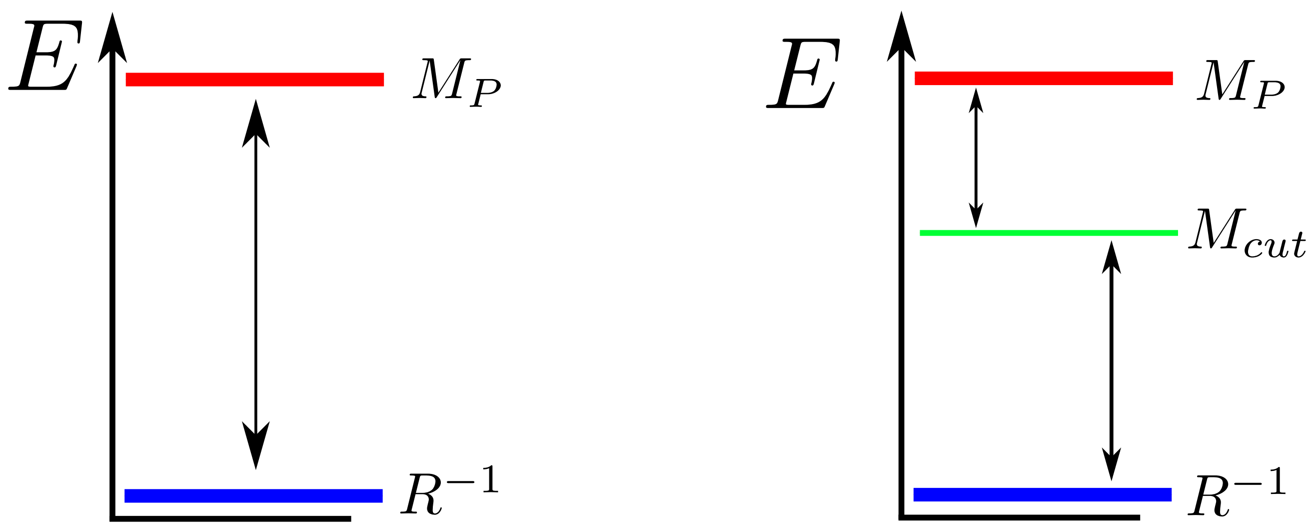
- Bulk c.c. fine-tuning problem $\Rightarrow$ "un-naturalness" of large N expansion in CFT

- Final $1/N$ suppression of correlators is achieved via delicate cancellations between (parametrically larger) conformal blocks.

- Notice: $1/N$ expansion and conformal block expansion are not the same thing!

Comments

- Hierarchy vs fine-tuning



- "Fine tuned" if

$$M_{cut}^4 M_P^{-2} R^2 \gg 1$$

- Need high (bulk) cutoff.

Large N gauge theories

- Large N gauge theories in 't Hooft limit $\Rightarrow$ weakly coupled string theories (Hagedorn growth)

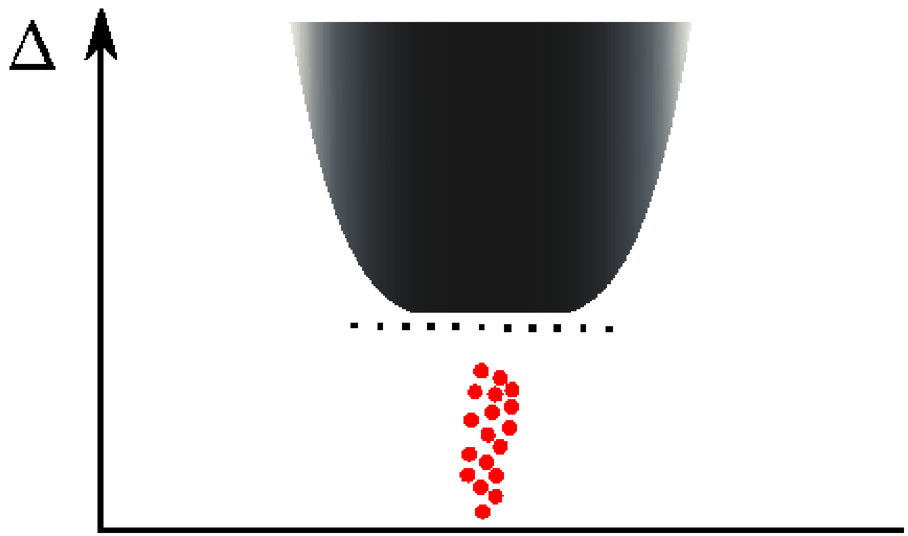
- String scale $M_s \sim f(\lambda)/R$ (does not scale with N), which implies

$$M_{cut} \ll M_P$$

- $\Rightarrow$ Cannot pose a "sharp" c.c. fine-tuning problem

- Same for higher spin gravity (Vasiliev etc.)

CFTs with holographic duals



1. **Large central charge c**
2. **Few low-lying operators (c-independent)**
3. **Factorization of correlators**

   For semi-classical gravity
4. **Gap between spin 2 and higher.**

Candidate CFTs with c.c. problem

Conditions:

1. CFT must have holographic dual (large c, few operators etc.)

2. CFT should not be supersymmetric

3. Conformal dimension where new (higher spin) states appear must satisfy

$$\Delta_{cut} \gg c^{1/d-1}$$

- 1 and 2 but not 3: large N gauge theories, $O(N)$/WZW/coset models, non-susy orbifolds of $\mathcal{N} = 4$.
- 1 and 3 but not 2: ABJM, $k = 1,\ N \to \infty$
- 1 and 2 and 3 ?

Conformal block expansion

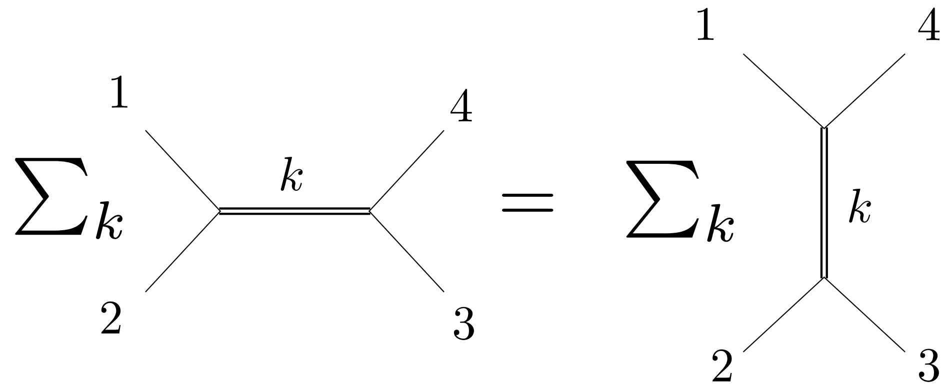
- Operator Product Expansion

$$\mathcal{O}_i(x)\mathcal{O}_j(y) = C_{ij}^k \mathcal{O}_j(y) + \dots$$

- Any n-point function can be computed by OPEs.

$$\langle \mathcal{O}_i(x_1)\mathcal{O}_j(x_2)\mathcal{O}_m(x_3)\mathcal{O}_n(x_4)\rangle = \sum_k C_{ij}^k C_{mn}^k \mathbf{G}_k(x_1,x_2,x_3,x_4)$$

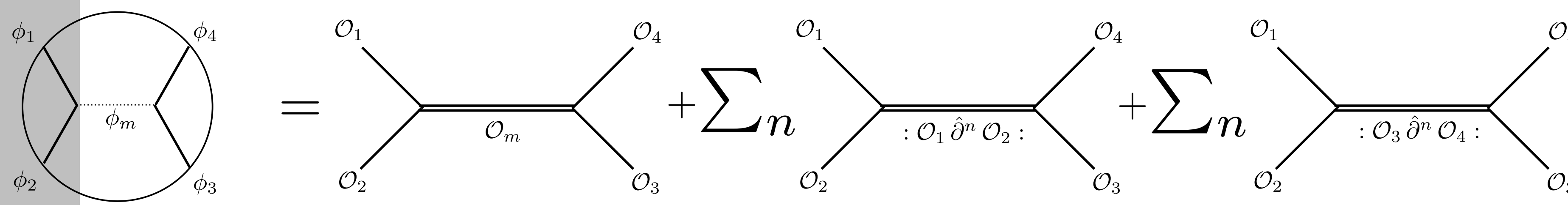
- Consistency condition: "conformal bootstrap"



Witten diagrams and conformal blocks

- Interactions in bulk $\Rightarrow$ Witten diagrams.

- Witten diagram expansion $\sim$ conformal block expansion.



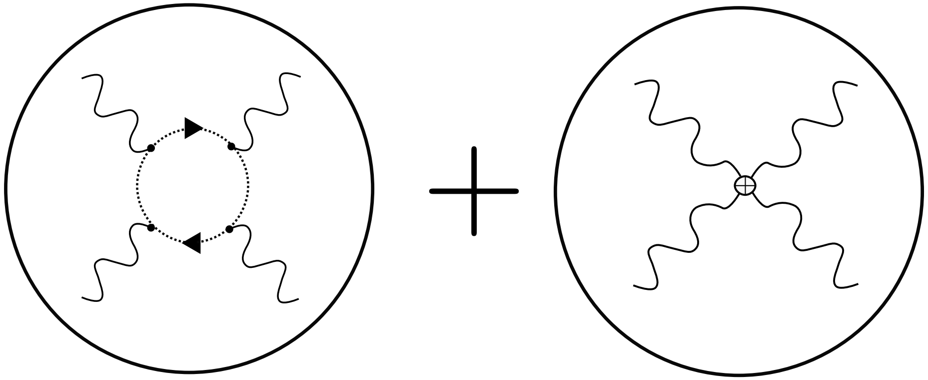
- Witten diagram basis: easy solutions of conformal bootstrap

4-point function of $T_{\mu\nu}$

- It has a large N expansion

$$\langle\widetilde{T}(x_1)\widetilde{T}(x_2)\widetilde{T}(x_3)\widetilde{T}(x_4)\rangle = \\ = G_0(x_1,...,x_4) + \frac{1}{N^2}G_1(x_1,...,x_4) + \frac{1}{N^4}G_2(x_1,...,x_4) + ... \tag{1}$$

- $G_2(x_1,...,x_4)$ gets contributions (at least from)



- The full $G_2(x_1, .., x_4)$ is $\mathcal{O}(1)$. However if we split it into Witten diagrams $\rightarrow$ each of them is $\mathcal{O}(N^{\#})$.

- Delicate cancellations between Witten diagrams.

$\Rightarrow$

- In CFT language: expand $G_2(x_1, .., x_4)$ in conformal blocks. While the G_2 is $\mathcal{O}(1)$, contributions from individual conformal blocks can be of order $\mathcal{O}(N^{\#})$.

1-loop and "counterterm" diagrams

- 1-loop Witten diagram corresponds to the exchange of

$$:\mathcal{O}(\partial^2)^n\partial_1...\partial_l\mathcal{O}:$$

- Sum over n, l gives very large contribution.

- Contact diagram: exchange of

$$:T\partial...\partial T:$$

  also comes with very large coefficient in order to cancel divergence of the previous sum.

CONCLUSION

- COSMOLOGICAL CONSTANT FINE TUNING IN ADS

  $\Rightarrow$

- UN-NATURALNESS OF LARGE N EXPANSION (from conformal block point of view)

Can we study this?

Is there a specific CFT where this fine-tuning takes place?

- No example known yet..
- If no such CFT exists $\Rightarrow$ AdS gravity without (low energy) supersymmetry inconsistent.

IS THERE A CFT RESOLUTION OF THE FINE TUNING ?

- Assume that large N expansion looks fine-tuned, as seen from conformal block expansion point of view.
- Could there be another way to calculate the correlation functions, in which the $1/N$ suppression becomes natural?
- For example in large N gauge theories, conformal block expansion: expansion in terms of "exchanged" gauge singlets (**glueballs, mesons** etc.) in intermediate channels
- **BUT**: fundamentally correlators are computed by "double-line" diagrams in terms of **quarks and gluons**. In this description $1/N$ suppression is manifest.

$$
\begin{tikzcd}
\boxed{\text{QFT}} \arrow[d] & \\
\boxed{\text{CFT}} \arrow[r] & \boxed{\text{Gravity}}
\end{tikzcd}
$$

QFT: description in terms of fundamental fields
CFT: description in terms of "gauge singlets"/CFT data
$\{\Delta_I, C_{ijk}\}$

While mathematically equivalent, large N expansion may look natural in first but fine-tuned in second.

- If this possibility is true $\Rightarrow$ Impossible to understand c.c. fine-tuning in terms of low-energy fields (gravitons etc.)

- We need to understand what they are “made out of” (in the dual QFT).

- The underlying stabilizing mechanism would guarantee cancellations between terms of different order in perturbation theory (unlike supersymmetry which acts at given loop order).

Conclusions

- To the extent that there is a consistent theory of QG with c.c. problem in AdS, we argued that in the dual CFT this would be manifested as an apparent fine-tuning of the large N expansion.

- We argued that the CFT may admit an underlying description where this fine-tuning becomes naturally resolved.

- WE NEED A TOY MODEL... (does not have to be a full-fledged QFT)

THANK YOU