# Shock waves in strongly coupled plasmas

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#### Introduction

Examples AdS/CFT and hydrodynamics

#### Shock waves

Shock waves in ideal hydro Weak shocks in gravity Strong shocks in gravity

#### Other directions

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Examples AdS/CFT and hydrodynamics

## Shock waves

- When objects move in fluids supersonically they generically create shock waves.
- Shock waves are perturbations that propagate supersonically and are observed as discontinuities in hydrodynamical quantities.
- Many examples from everyday life.

Examples AdS/CFT and hydrodynamics

#### Examples: 1



# Supersonic bullet in water. Photograph (Shadowgraph) taken by E. Mach in 1887.

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Shock waves in strongly coupled plasmas

Examples AdS/CFT and hydrodynamics



- Shock waves in quark gluon plasmas.
- There is evidence that there are shock waves created by a hard parton in the Quark Gluon Plasma created in RHIC.
- The evidence comes from two and three jet correlators.

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Examples AdS/CFT and hydrodynamics

# Initial Thermalization of the plasma

- Shock waves may also be very important in the initial stage of the collision.
- In ideal hydrodynamics they are the only mechanism with which one can create entropy.
- Can be used to explain initial thermalization of the plasma in the Landau model.

Examples AdS/CFT and hydrodynamics

## Description of shock waves in ideal hydro

- Impose continuity of energy and momentum across the shock.
- Also charge, particle number etc. if they are present.
- There are two solutions: One that transforms kinetic energy to thermal energy,
- and one that transforms thermal energy to kinetic (violating the second law).

Examples AdS/CFT and hydrodynamics

#### Sketch of planar shock wave



Examples AdS/CFT and hydrodynamics

# Hydrodynamics in the $\mathcal{N}=4$ superconformal plasma

- ► There are no characteristic lengths in the N = 4 plasma except for inverse temperature <sup>1</sup>/<sub>T</sub>.
- ► Hydrodynamics is a long wavelength (compared to <sup>1</sup>/<sub>T</sub>) expansion of the theory.
- The discontinuity in the shock means that the hydrodynamics expansion breaks down at the core of the shock.
- ► The fields change across distances much smaller than the hydrodynamic expansion parameter <sup>1</sup>/<sub>T</sub>.

Examples AdS/CFT and hydrodynamics

# Using AdS/CFT

- Using AdS/CFT we can relate the dynamics of the fluid to dynamics of a black hole.
- Hydrodynamics might break when derivatives of fields are large but gravity does not.
- ▶ We can use gravity to resolve the discontinuity!
- Shock waves should propagate on the horizon of black holes!

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Examples AdS/CFT and hydrodynamics

# Hydrodynamics of the $\mathcal{N}=4$ SYM plasma

- Low energy excitations of a plasma are given by temperature variations and displacements, characterized by  $T, u_{\mu}$ . 4 variables .
- ► But  $T_{\mu\nu}$  has 9 independent components. We define T,  $u_{\mu}$  through:

• 
$$T^{\mu\nu}u_{\nu} = -3(\pi T)^4 u^{\mu}$$
.

- There are three more eigenvectors (3 variables) and 3 eigenvalues (2 variables).
- Hydrodynamics determines these 5 variables in terms of *T*, *u*<sub>μ</sub>.
   Then *T*, *u*<sub>μ</sub> are determined by the equations of motion:

$$\blacktriangleright \ \partial_{\mu}T^{\mu\nu}=0 \ .$$

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Examples AdS/CFT and hydrodynamics

## Hydrodynamics from gravity

(Battacharya, Hubeny, Minwalla, Rangamani)

- For any conserved T<sub>μν</sub> we can find a dual asymptotically AdS metric that is generically singular.
- ► For long wavelengths we can systematically find the  $T_{\mu\nu}$  that gives rise to a non-singular metric:

$$T^{\mu\nu} = (\pi T)^4 (\eta^{\mu\nu} + 4u^{\mu}u^{\nu}) - 2(\pi T)^3 \sigma^{\mu\nu} + (\pi T)^2 \left( (\ln 2) T_{2a}^{\mu\nu} + 2T_{2b}^{\mu\nu} + (2 - \ln 2) \left( \frac{1}{3} T_{2c}^{\mu\nu} + T_{2d}^{\mu\nu} + T_{2e}^{\mu\nu} \right) \right) ,$$
  

$$\sigma^{\mu\nu} = P^{\mu\alpha} P^{\nu\beta} \partial_{(\alpha} u_{\beta)} - \frac{1}{3} P^{\mu\nu} \partial_{\alpha} u^{\alpha} , \quad \mathcal{D} = u^{\alpha} \partial_{\alpha},$$

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Examples AdS/CFT and hydrodynamics

# Hydrodynamics from gravity

$$\begin{split} & T_{2a}^{\mu\nu} = \epsilon^{\alpha\beta\gamma(\mu}\sigma_{\gamma}^{\nu)}u_{\alpha}l_{\beta} , T_{2c}^{\mu\nu} = \partial_{\alpha}u^{\alpha}\sigma^{\mu\nu} , \\ & T_{2b}^{\mu\nu} = \sigma^{\mu\alpha}\sigma_{\alpha}^{\nu} - \frac{1}{3}P^{\mu\nu}\sigma^{\alpha\beta}\sigma_{\alpha\beta} , \\ & T_{2d}^{\mu\nu} = \mathcal{D}^{\mu}\mathcal{D}u^{\nu} - \frac{1}{3}P^{\mu\nu}\mathcal{D}u^{\alpha}\mathcal{D}u_{\alpha} , \\ & T_{2e}^{\mu\nu} = P^{\mu\alpha}P^{\nu\beta}\mathcal{D}\left(\partial_{(\alpha}u_{\beta)}\right) - \frac{1}{3}P^{\mu\nu}P^{\alpha\beta}\mathcal{D}\left(\partial_{\alpha}u_{\beta}\right) , \\ & \bullet \ l_{\mu} = \epsilon_{\alpha\beta\gamma\mu}u^{\alpha}\partial^{\beta}u^{\gamma} . \end{split}$$

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Examples AdS/CFT and hydrodynamics

# Hydrodynamics from gravity

- This construction is important because for every solution to the hydrodynamical equations we can find a gravity dual.
- and finding a gravity solution will gives us a solution of hydrodynamics.

Shock waves in ideal hydro Weak shocks in gravity Strong shocks in gravity

#### Shock waves in ideal hydro and asymptotics

• 
$$T^{\mu\nu} = (\pi T)^4 (\eta^{\mu\nu} + 4u^{\mu}u^{\nu}) , \quad v_s = \frac{1}{\sqrt{3}}$$

• Matching  $T^{tx}$ ,  $T^{xx}$  gives

• 
$$v_1 v_2 = \frac{1}{3}, \left(\frac{T_2}{T_1}\right)^4 = \frac{9v_1^2 - 1}{3(1 - v_1^2)}$$

 There is entropy generation. More entropy comes out than comes in.

$$\Delta s^{x} = s^{x}_{subsonic} - s^{x}_{supersonic} = \frac{4\pi^{2}T_{1}^{3}}{\sqrt{1-v_{1}^{2}}} \left( 3^{-3/4} \left( \frac{9v_{1}^{2}-1}{1-v_{1}^{2}} \right)^{1/4} - v_{1} \right)$$

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## Weak shocks in gravity

- We use the BHMR construction and expand in the amplitude of the shock.
- The higher order hydrodynamics terms break time reversal. The second law violating solution is excluded.
- The solution is given by:  $\xi = \frac{4\pi}{3}u_{\infty}T_0x$

• 
$$u(\xi) = \frac{1}{\sqrt{2}} + u_{\infty} \delta u_{(1)}(\xi) + u_{\infty}^2 \delta u_{(2)}(\xi) + \dots$$

$$\bullet \ \delta u_{(1)} = \tanh(-\xi) \ ,$$

$$\delta u_2(\xi) = \frac{1}{6} \left[ 4\sqrt{2}(1 - \ln 2) \frac{\ln \cosh \xi}{\cosh^2 \xi} + 5\sqrt{2} \left( \tanh^2 \xi + \tanh \xi + \frac{\xi}{\cosh^2 \xi} \right) \right]$$

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#### Weak shocks in gravity

The metric is given by

$$ds^{2} = -2u_{\mu}dx^{\mu}dr + \frac{1}{r^{2}}((\pi T)^{4} + k)u_{\mu}u_{\nu}dx^{\mu}dx^{\nu} + r^{2}\eta_{\mu\nu}dx^{\mu}dx^{\nu} + \frac{1}{r^{2}}j\tilde{u}_{\mu}u_{\nu}dx^{\mu}dx^{\nu} + \alpha r^{2}(\tilde{u}_{\mu}\tilde{u}_{\nu}dx^{\mu}dx^{\nu} - \frac{1}{2}(dy^{2} + dz^{2})),$$

where  $u^{\mu} = (\sqrt{1+u^2}, u, 0, 0)$ ,  $\tilde{u}^{\mu} = (u, \sqrt{1+u^2}, 0, 0)$  and u should be expanded as  $u = u_{(0)} + u_{(1)} + u_{(2)}$ . The temperature is also expanded as

$$T = T_{(0)} - \frac{\sqrt{2}}{3} T_{(0)} u_{(1)} - \frac{\sqrt{2}}{3} T_{(0)} u_{(2)} + \frac{1}{3\pi} u'_{(1)} ,$$

whereas the other functions entering the metric are given by

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#### Weak shocks in gravity

$$k = \frac{2}{3}r^{3}(u'_{(1)} + u'_{(2)}) - \frac{\sqrt{2}}{3}r^{2}u''_{(1)},$$
  

$$j = -\frac{2}{\sqrt{3}}r^{3}(u'_{(1)} + u'_{(2)}) + \frac{4\sqrt{2}}{3\sqrt{3}}u_{(1)}u'_{(1)}(\pi T_{(0)})^{3}F_{2}\left(\frac{r}{\pi T_{(0)}}\right),$$
  

$$\alpha = \frac{u'_{(1)} + u'_{(2)}}{3\pi T_{(0)}}F_{1}\left(\frac{r}{\pi T_{(0)}}\right) - \frac{4\sqrt{2}}{9\pi T_{(0)}}u_{(1)}u'_{(1)}F_{3}\left(\frac{r}{\pi T_{(0)}}\right),$$

with the  $F_{1,2,3}$  known functions.

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#### Entropy generation in weak shocks

The entropy current and the entropy generation are given by

$$s^{\mu} = 4\pi\eta u^{\mu} - rac{ au_{\pi}\eta}{4T}\sigma^{\kappa
u}\sigma_{\kappa
u}u^{\mu} \; ,$$



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# Surface gravity

- For stationary solutions the surface gravity should be constant. However a second global Killing vector is needed to define it. It can be shown that a second global Killing vector does not exist for our solution.
- Even under much milder assumptions for non expanding horizons, the surface gravity cannot be defined.

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## Strong shocks in gravity

- We can only study the asymptotics. Using gravity we study the non-hydrodynamic regime.
- We can use linearized gravity:
- $g_{\mu\nu} = g^0_{\mu\nu} + r^2 H_{\mu\nu} e^{-i\omega t + iqx}$ .  $g^0_{\mu\nu}$  is the boosted black hole solution.
- *H* has components  $H_{00}$ ,  $H_{01}$ ,  $H_{11}$  and  $H_{zz} = H_{yy} = H$ .
- ► The sound mode is described by  $Z(r) = H_{00} + \left(1 + \gamma^2 \frac{r_0^4}{r^4}\right) H(r), \ \gamma = \frac{1}{\sqrt{1-v^2}}.$

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# Strong shocks in gravity

• Using new coordinates  $u = \frac{r_0^2}{r^2}$  the sound mode satisfies

► 
$$Z''(u) + P(u)Z'(u) + Q(u)Z(u) = 0$$
,

► 
$$P(u) = \frac{3-5\gamma^2 u^2}{u(\gamma^2 u^2 - 3)}, \ Q(u) = -\frac{4\gamma^2 u^2}{\gamma^2 u^2 - 3} + q^2 \frac{\gamma^2 u^2 - 1}{4u}.$$

We impose boundary conditions on the horizon that in the unboosted frame correspond to infalling gravitons.

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# Strong shocks in gravity

Results

- Fluid at rest: v = 0, q = -i2.3361.
- The speed of sound  $v = \frac{1}{\sqrt{3}}$ , q = 0.
- The singular point  $v = \sqrt{\frac{2}{3}}$ ,  $q = -i\sqrt{2}$ .
- The ultrarelativistic limit  $v \to 1$ ,  $q \to i1.895(1-v^2)^{-1/4}$ .
- This can be found analytically since the singular point approaches the boundary. This scaling comes from the asymptotic AdS region and it is possible that any theory with an asymptotic AdS boundary will have this behavior.

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# Strong shocks in gravity

#### Numerical results



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# Strong shocks in gravity

Numerical Results: Comparison with hydrodynamics:



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#### Dispersion relations

- First and second order hydrodynamics and Israel-Stewart theory give wrong qualitative results.
- ► A simple curve that reproduces the numerical results (up to 4%) is given by:  $\frac{iq}{\pi T} = 4\left(\frac{3}{2}\right)^{\frac{1}{4}}\sqrt{\gamma}\left(v \frac{1}{\sqrt{3}}\right)$ .
- ► Going back to the unboosted brane we can get an effective dispersion relation for sound

$$q'(q'^2-\omega'^2)^{3/2}=16\left(rac{3}{2}
ight)^2\left(\omega'+rac{q'}{\sqrt{3}}
ight)\;.$$

 This might be useful for effective theories of hydrodynamics and/or numerical simulations.

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## Future and ongoing work

- ► The asymptotic behavior of q as v → 1 probes the UV of the theory. It should be interesting to study in other cases.
- In particular QCD has asymptotic freedom in this regime and a perturbative calculation could be done.
- In other dimensions for conformal plasmas  $q \sim i \gamma^{2/d}$ .

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## Future and ongoing work

- Perturbations of shock waves generate sound. Is there a process similar to superradiance in the bulk?
- Are the solutions stable under small perturbations?
- What is the gravity dual for a strong shock?

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# Conclusions

- We have performed a systematic study of shock waves.
- ▶ We have analyzed weak shocks and found their gravity dual.
- We have studied the asymptotic behavior of strong shocks.
- Strong shocks are probes of the microscopic theory and merit further study.

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#### Thank you!

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