On the phase diagram of gauge theories with a large number of adjoint scalars

Gautam Mandal

6th Regional Meeting in String Theory, Milos, June 21, 2011

with T. Morita (arXiv:1103.1558)
with M. Mahato and T. Morita (arXiv:0910.4526)
with T. Morita (arXiv:1107.xxxx)
with R. Narayanan; P. Basu, T. Morita, S.R. Wadia; N. Iizuka; H. Isono (in progress)

Other works

T. Morita (2010)
K. Hashimoto and T. Morita (2011)
O. Aharony, J. Marsano, S. Minwalla, K. Papadodimas, M. Van Raamsdonk and T. Wiseman (2005)
R. Narayanan and H. Neuberger (2003-2007)
M. Unsal and L. G. Yaffe (2010)

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Gauge theory

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  YM on $S^1 \times S^2$: Papadodimas, Shieh, van Raamsdonk (2006)

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- Generation of a mass gap in pure YM theory calls for a non-perturbative treatment.
- In four-fermi theories, e.g. Gross-Neveu model, can prove

$$(\bar{\psi}_i \psi_i)^2 \to \bar{\psi}_i \psi_i \langle \bar{\psi}_i \psi_i \rangle$$

The condensate (at large N_f) satisfies a gap equation, like in BCS theory, and characterizes a nonperturtative vacuum with dynamical mass generation, symmetry breaking and asymptotic freedom.

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- Euclidean YM on S^1 of length β corresponds to thermal YM (at temperature $1/\beta$). Low temperature: temporal Wilson loop $W_0 = 0$ (unbroken Z_N symmetry $\to$ confinement); high temperature $W_0 \neq 0$ (brone Z_N symmetry $\to$ deconfinement). Deconfinement: $Z_N \to 1$.

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- YM on T^D have been studied holographically (Aharony, Marsano, Minwalla, Papadodimas, van Raamsdonk, Wiseman (2005)) and in the lattice (Narayanan, Neuberger et al, 2003-2011). Exotic phase structure: e.g. for YM_4 on T^4, with $L_0 = \beta > L_1 > L_2 > L_3$, the phase structure is found to be

Which radii small	Symmetry	Non-zero Wilson loops	Name of phase
None	Z_N^4	None	0_c
L_3	Z_N^3	W_3	1_c
L_2, L_3	Z_N^2	$W_2 W_3$	2_c
L_1, L_2, L_3	Z_N	W_1, W_2, W_3	3_c
L_0, L_1, L_2, L_3	1	W_0, W_1, W_2, W_3	4_c

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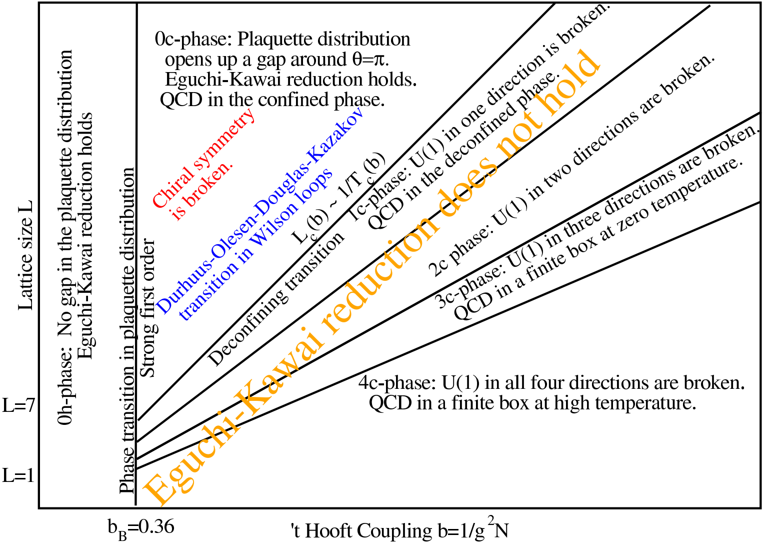
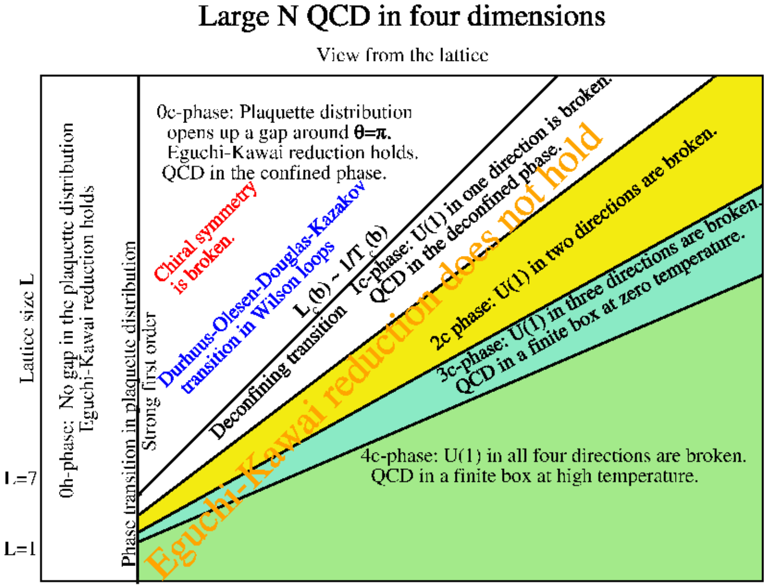
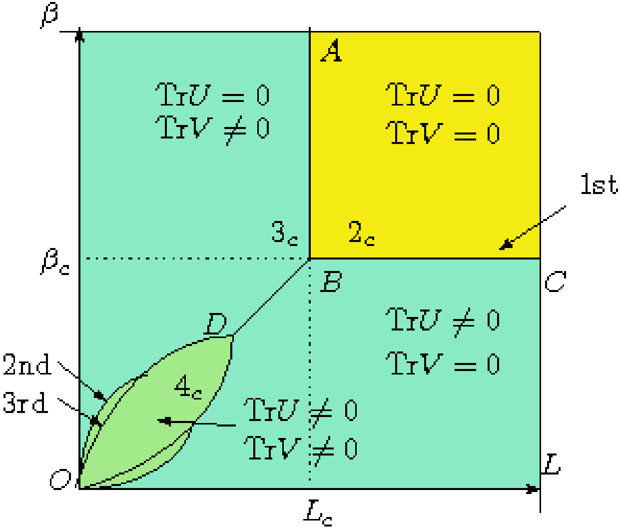
- 'Cascade': there is no phase boundary across which two Wilson lines acquire non-zero values simultaneously.
- We will be able to compute a number of these phase boundaries analytically and verify the above properties.

Large N QCD in four dimensions

View from the lattice




Large N QCD in four dimensions
View from the lattice
Lattice size L
L=7
L=1
0h-phase: No gap in the plaquette distribution
Eguchi-Kawai reduction holds
Phase transition in plaquette distribution
Strong first order
0c-phase: Plaquette distribution opens up a gap around θ=π. Eguchi-Kawai reduction holds. QCD in the confined phase.
Chiral symmetry is broken.
Durhuus-Olesen-Douglas-Kazakov transition in Wilson loops
L_c(b) ~ 1/T_c(b)
Deconfining transition
1c-phase: U(1) in one direction is broken. QCD in the deconfined phase.
Eguchi-Kawai reduction does not hold
2c phase: U(1) in two directions are broken.
3c-phase: U(1) in three directions are broken. QCD in a finite box at zero temperature.
4c-phase: U(1) in all four directions are broken. QCD in a finite box at high temperature.


β
A
$\mathrm{Tr}U = 0$
$\mathrm{Tr}V \neq 0$
$\mathrm{Tr}U = 0$
$\mathrm{Tr}V = 0$
1st
3_c
2_c
β_c
B
C
D
$\mathrm{Tr}U \neq 0$
$\mathrm{Tr}V = 0$
2nd
4_c
3rd
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O
L
L_c

Gravity motivations

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- Studying the holographic duals of gauge theories on T^D leads to new proposals for strong-coupling continuations of the deconfiment transition. [cf. Takeshi's talk yesterday on Deconfinement in 4D YM].

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- Phase transitions in gauge theory correspond to interesting phase transitions in gravity (Hawking-Page, Gregory-Laflamme, Scherk-Schwarz,...).
- Studying the holographic duals of gauge theories on T^D leads to new proposals for strong-coupling continuations of the deconfiment transition. [cf. Takeshi's talk yesterday on Deconfinement in 4D YM].
- The end-point of a dynamical Gregory-Laflamme transition is interesting to study (especially from the viewpoint of the appearance of a naked singularity). We will study the dynamical transition in the gauge theory. Basu-Mandal-Morita-Wadia (in progress).

Plan

- The Technique: $(d+D)$ dimensional YM on T^D (large D)
- d=0 (Bosonic IKKT)
- d=1 ($\leftarrow$ SS reduction of D1)
- d=2 ($\leftarrow$ SS reduction of D2)
- Dynamical Gregory Laflamme in gauge theory
- Conclusions and open problems

The large D technique

- Consider a $d+D$ dimensional bosonic YM theory on a small T^D

$$S = \frac{1}{4}\int d^d x Tr \left(F_{\mu\nu}^2 + \frac{1}{2}\sum_{I=1}^{D} D_\mu Y^I D^\mu Y^I - g^2 \sum_{I,J} \frac{1}{4}[Y^I, Y^J]^2 \right)$$

Can we treat the Y^4 term in a fashion similar to 4-fermi terms as in Gross-Neveu or NJL models?

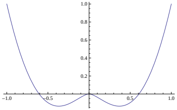
Hotta-Nishimura-Tsuchiya 1999, Mahato-Mandal-Morita 2009

Large N_f

Recall Gross-Neveu model:

$$S = \int d^2x \left(\bar{\psi}_i \partial_\mu \gamma^\mu \psi_i - g (\bar{\psi}_i \psi_i)^2 \right)$$

The technique to solve Gross-Neveu model is to introduce an auxiliary dynamical field ϕ, $g(\bar{\psi}_i\psi_i)^2 = \phi\bar{\psi}_i\psi_i - \phi^2/(4g)$ and integrate out the fermions to get



$$S_{eff}[\phi] = N_f \, \mathrm{logDet}(\gamma^\mu \partial_\mu + 2\phi) + \phi^2/(4g)$$

In the large N_f limit, $N_f g = \lambda$ fixed, the 1-loop term competes with the tree level term. Hence, a non-trivial value of the flavour-singlet condensate

$$< \phi >= \frac{2\lambda}{N_f} < \bar{\psi}_i \psi_i > \neq 0 = \Lambda \exp[-\alpha/(gN_f)]$$

appears at the new saddle point. [BCS, χSB, ...]

Back to YM

- Can we write $Y^4 = -B^2/4 + BY^2$ etc. to get a non-trivial vacuum with $< Y^2 > \neq 0$?
  What could a 'singlet' Y^2 be? It can't be of the form $Tr[Y, Y]$ which trivially vanishes. It can be $Tr(Y^I Y^J)$, but we can't write $Tr([Y_I, Y_J]^2) = B_{IJ} Tr[Y^I Y^J] - B_{IJ}^2/4$ (single trace $\neq$ double trace).

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- Turns out that by considering gauge-non-invariant, but SO(D)-invariant auxiliary fields, we CAN write

$$Tr[Y_I, Y_J]^2 \equiv -Y_a^I Y_b^J M_{ab,cd} Y_c^J Y_d^J = B_{ab} M^{-1}_{ab;cd} B_{cd} - 2i B_{ab} Y_a^I Y_b^I$$

where we have written $Y^I = Y_a^I \lambda_a$, and

$$M_{ab,cd} = -\frac{1}{4}\Big\{Tr[\lambda_a, \lambda_c][\lambda_b, \lambda_d] + (a \leftrightarrow b) + (c \leftrightarrow d) + (a \leftrightarrow b, c \leftrightarrow d)\Big\}$$

Now Y is only quadratic; integrating over Y, we get

Large D saddle point

$$Z = \int DA_\mu DB_{ab} \exp[-S_{eff}[A,B], \; S_{eff}[A,B] =$$

$$= \int d^d x [\frac{1}{4g^2} \left(F^2_{\mu\nu} + B_{ab} M^{-1}_{ab;cd} B_{cd} \right)] + (D/2)\mathrm{logDet}(-D^2_\mu \delta_{ab} + iB_{ab})$$

The idea now is to take a 'tHooft-like limit $D \to \infty, g^2 \to 0$ with $g^2 D = (\hat{g})^2$ held fixed. The determinant term will now compete with the tree level term, leading to a new large D saddle point for $< B_{ab} >= iM_{ab,cd} < Y^I_c Y^I_c >$ Note complex contour.

- In the examples we consider below, we will obtain saddle point values of the form $< B_{ab} >= i\Delta^2 \delta_{ab}$, which will imply dynamical generation of a condensate of the form

$$(1/D) < Y^I_a Y^I_b >= \Delta^2 \delta_{ab}$$

or, equivalently a mass gap $M_Y = \Delta$ (cf. the BY^2 term). In the large D saddle point, the field B_{ab} can be treated as classical, leading to a large D evaluation of $S_{eff}[A]$.

d=0

Yang-Mills integrals (cf. Bosonic IKKT model)

$$Z = \int dY^I \exp[-\frac{1}{4g^2} Tr \sum_{I,J} [Y^I, Y^J]^2]$$
$$= \int DY_a^I DB_{ab} \exp[\frac{1}{4g^2} B_{ab} M^{-1}_{ab,cd} B_{cd} - \frac{i}{2g^2} B_{ab} Y_a^I Y_b^I]$$
$$= \int DB_{ab} e^{-\mathcal{S}}, \mathcal{S} = \frac{1}{4g^2} B_{ab} M^{-1}_{ab,cd} B_{cd} + D/2 \mathrm{logdet}[B_{ab}] \qquad (1)$$

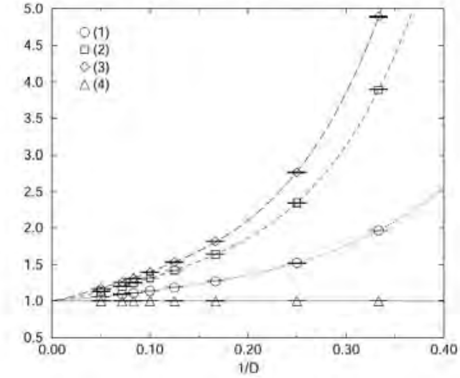
This can be computed at finite N, in a large D expansion! The leading term comes from the trace part $B_{ab} = B_0 \delta_{ab}$:

$$\mathcal{S} = \frac{NB_0^2}{8\hat{g}^2} + \frac{(N^2-1)}{4} \log\left(-\frac{B_0^2}{\hat{g}^2 N}\right)$$

where $(\hat{g})^2 = g^2 D$. At large N,

$$F = -\frac{\log Z}{DN^2} = -\frac{1}{4} + \frac{\log 2}{4} + \frac{1}{D}\left(-\frac{5}{8} + \frac{1}{2}\log\frac{3}{2}\right) + O\left(\frac{1}{D^2}\right).$$

d=0: comparison with numerics



The circles represent numerical values of 1/(DN) $< trY^I Y^I > /(\hat{g}/\sqrt{2})$ (extrapolated to $N = \infty$), while the dotted line represents the 1/D result discussed above. [The analytic result was also independently obtained by Hotta-Nishimura-Tsuchiya].

d=1

This is the first non-trivial dimension involving a gauge field A_μ. Consider the size of the Euclidean dimension to be finite, β.

$$Z = \int DA_0 DY^I e^{-S},$$

$$S = \int_0^\beta dt\, Tr\left(\sum_{I=1}^{D} \frac{1}{2}\left(D_0 Y^I\right)^2 - \sum_{I,J} \frac{g^2}{4}[Y^I, Y^J][Y^I, Y^J]\right). \qquad (2)$$

Step 1: Wilson loop:
For finite β, can't gauge away A_0; fix gauge $\partial_t A_0 = 0$ [Aharony et al]

$$\Delta_{FP} = \exp[-S_{FP}], S_{FP} = N^2 \sum_{n=1}^{\infty} |u_n|^2/n$$

where $u_n = (1/N)TrU^n, U = P\exp[i\oint dtA_0]$. Thus, A_0 reduces only to the Wilson loop (Polyakov loop).

$u_1 = 0$: centre symmetry unbroken (“confined” phase); $u_1 \neq 0$: centre symmetry broken (“deconfined” phase).

d=1: Integrate Y's

Step 2: Integrate out Y^I:
We show results only for the dominant mode $B_{ab}(t) = i\Delta^2\delta_{ab}$

$$\frac{D}{2}\log\left(\det\left(-D_0^2 + \triangle^2\right)\right) = \frac{DN^2\beta\triangle}{2} - D\sum_{n=1}^{\infty}\frac{x^n}{n}|u_n|^2.$$

Combining with the classical B^2 term, and Δ_{FP} we get

$$\frac{\mathcal{S}(\triangle,\{u_n\})}{DN^2} = -\frac{\beta\triangle^4}{8\tilde{\lambda}} + \frac{\beta\triangle}{2} + \sum_{n=1}^{\infty}\left(\frac{1/D - x^n}{n}\right)|u_n|^2.$$

where $\tilde{\lambda} = \lambda D = g^2 ND$ is the large D 'tHooft coupling.

d=1: Large D saddle point

Step 3: Evaluate Δ at the saddle point

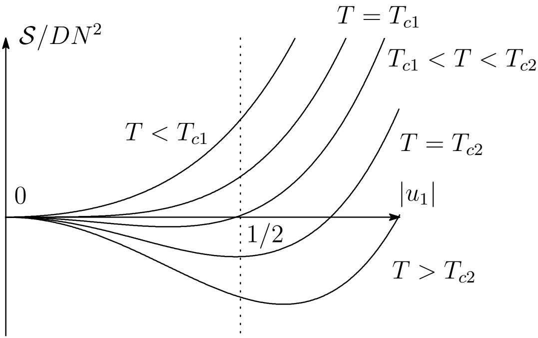
$$\triangle_0(\{u_n\}) = \tilde{\lambda}^{1/3}\left(1 + \frac{2}{3}\sum_{n=1}^{\infty}\bar{x}^n|u_n|^2\right) + \cdots ,$$

where $\bar{x} = \exp[-\beta\tilde{\lambda}^{1/3}]$.
Step 4: Put this back in $\mathcal{S}[\Delta, \{u_n\}]$:

$$\begin{aligned}\frac{\mathcal{S}(\{u_n\})}{DN^2} &= \frac{3}{8}\beta\tilde{\lambda}^{1/3} + a_1|u_1|^2 + b_1|u_1|^4 + \sum_{n=2}^{\infty} a_n|u_n|^2 + \cdots , \\ a_n &= \frac{1}{n}\left(1/D - \bar{x}^n\right), \\ b_1 &= \frac{1}{3}\beta\tilde{\lambda}^{1/3}\bar{x}^2, \end{aligned} \tag{3}$$

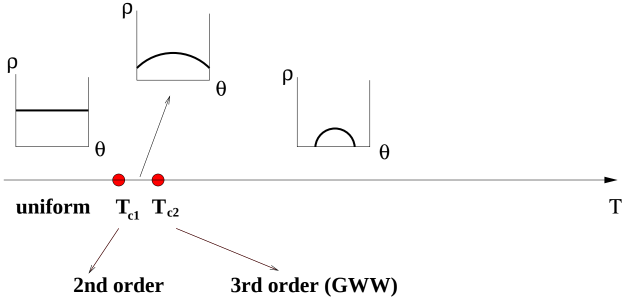
where the $\cdots$ involve other u_n^4 terms for $n > 1$, which are down at large D. $u_1 = (1/N)\mathrm{Tr}U$.

d=1: Landau-Ginzburg



$u_1 = \mathrm{Tr}U/N$. As T crosses T_{c1}, u_1 becomes tachyonic and there is a second order phase transition which signals an onset of non-uniformity in the eigenvalue distribution $\rho(\alpha)$. At $T = T_{c2}$, characterized by a potential minimum at $|u_1| = 1/2$, a gap develops in the eigenvalue distribution, signalling a GWW transition.

d=1: phase diagram

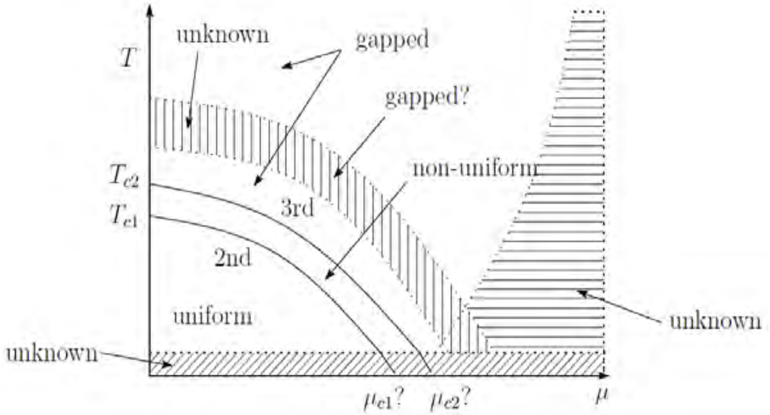


	T_{c1}	T_{c2}	R^2	F_0
Numerical result	0.8761	0.905	2.291	6.695
Leading large-D result	0.947	0.964	2.16	7.02
Large-D including $1/D$ effect	0.895	0.917	2.28	6.72

2nd and 3rd rows are our results, with $D = 9$ (10-dimensional YM theory compactified to d=1). Numerical results are from Nishimura and Kawahara. The agreement between the 3rd row and the 1st row are within 1% (which is $1/D^2$).

Works even for $D = 2$!

d=1: chemical potential

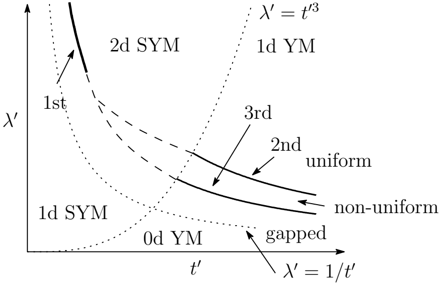


Morita 2010

Gravity correspondence: D1 branes

2d Euclidean SYM on $S^1_{\beta_2} \times S^1_L$ with (AP, P) spin structure for fermions $\leftrightarrow$ black string wrapped along $S^1_{\beta_2}$ at temperature β (D1 at large L, smeared D0 at small L).

As the box size increases beyond horizon size, D0 branes clump, leading to a Gregory-Laflamme transition. [figure] The weak coupling version are the clumping of eigenvalues of U.



$\lambda' = \lambda_2 L^2, t' = L/\beta$. $\lambda' < t'^3$ described by 1D YM since temporal KK modes (and fermions) are massive. Phase transitions: $\lambda' t' = 1/T_{c1}^3, 1/T_{c2}^3$.

d=2

Consider $d = 2$ Euclidean YM theory with D ajoint scalars, compactified on a 2-torus T^2.

$$S = \int_0^\beta dt \int_0^L dx \, Tr \left(\frac{1}{2g^2} F_{01}^2 + \sum_{I=1}^{D} \frac{1}{2} \left(D_\mu Y^I \right)^2 - \sum_{I,J} \frac{g^2}{4} [Y^I, Y^J][Y^I, Y^J] \right)$$

We now have two Wilson lines $U = P \exp[i \oint^\beta A]$ and $V = P \exp[i \oint^L A]$ along the two cycles. There are now possibly 4 or more phases, corresponding to whether TrU, TrV are zero or non-zero and whether a non-zero Wilson line can exist in 2 distinct phases (non-uniform vs gapped eigenvalue distribution).

d=2: small L

- For small enough L, the problem reduces to $d = 1$, with A_1 turning into an extra Y, which we have solved above.
- Large N volume independence vs KK reduction. In the centre symmetric phase (Tr V=0: uniform eigenvalues), KK reduction does not work in the usual fashion since new soft modes, with mass $\sim 1/(NL)$, appear. However, for small enough L, eigenvalues of A_1 are clumped near 0 (this is consistent with eigenvalues of A_0 getting more and more clumped at low enough β) hence centre symmetry along L is broken (Tr $V \neq 0$). Hence KK reduction works along L and the problem simplifies to the $d = 1$ model.

d=2: large L

Need to evaluate the 1-loop effective action

$$S^{(1)}(A_\mu, \Delta) = \frac{D}{2} \log\det\left(-D_\mu^2 + \triangle^2\right)$$

where $B_{ab}(x,t) = i\Delta^2\delta_{ab}$ is, as usual, the dominant mode at large D. Under the assumptions $L\Delta \gg 1, \Delta \gg \sqrt{\tilde{\lambda}}$, it turns out that the Wilson line V decouples from the dynamics, yielding (Semenoff-Tirkonnen-Zarembo 1996, Basu-Ezhutachan-Wadia 2005)

$$S/DN^2 = \int_{-\infty}^{\infty} dx \left[\frac{1}{2N} Tr\left(|\partial_x U|^2\right) - \frac{\xi}{N^2}|TrU|^2\right].$$

where $\xi = \sqrt{\frac{\triangle_0}{2\pi\tilde{\lambda}^2\beta^3}}e^{-\triangle_0\beta}$ and Δ_0 is an analog of Λ_{QCD} (Asymptotic freedom, dynamical mass generation)

$$\triangle_0 = \sqrt{\frac{\tilde{\lambda}}{2\pi}\log\left(\frac{2\pi\Lambda^2}{\tilde{\lambda}}\right)} + \cdots, \quad \tilde{\lambda} = (2\pi\Delta_0^2)/\log(\Lambda^2/\Delta_0^2)$$

Full formula involves Lambert's W-function.

d=2: large L phase transition

The double trace action was analyzed in [Semenoff-Zarembo, Basu-Ezhuthachan-Wadia], using the eigenvalue density

$$\rho(\theta, x) = \frac{1}{N} \sum_{i=1}^{N} \delta(\theta - \theta_i(x))$$

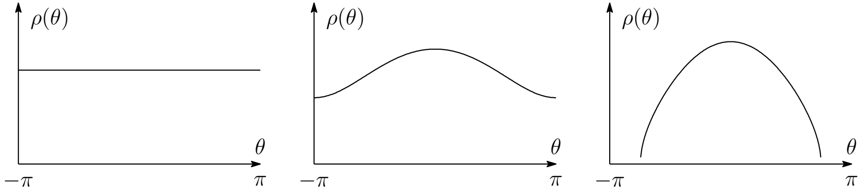
The hamiltonian becomes (at large N)

$$H = \int d\theta \left(\frac{1}{2} \rho v^2 + \frac{\pi^2}{6} \rho^3 - \xi \left| u_1 \right|^2 \right) .$$

where $v = \partial_\theta \Pi$. The hamiltonian admits x-independent solutions

$$\rho(\theta) = \frac{\sqrt{2}}{\pi} \left(\sqrt{E + 2\xi\rho_1 \cos\theta} \right)$$

The eigenvalue density can be uniform, non-uniform or gapped, for various ξ-values.



d=2: Landau-Ginzburg potential

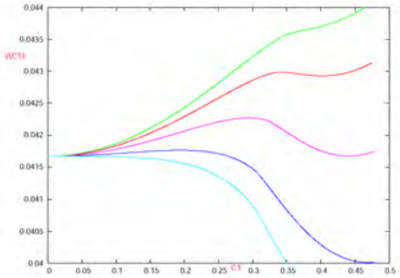
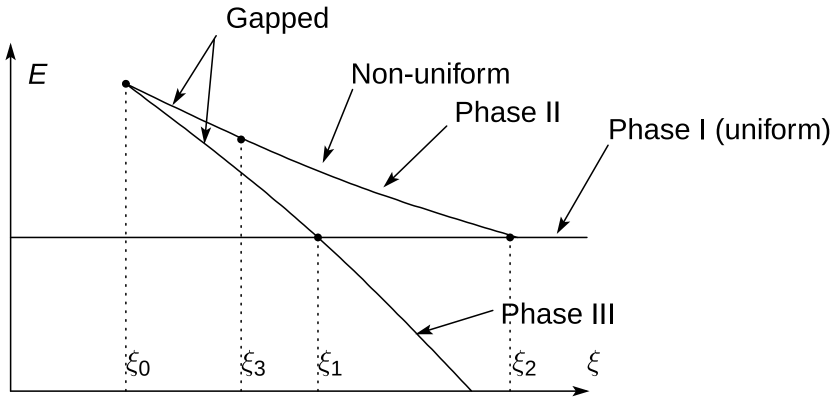


Figure 1: Plot of $V(C_1)$ with C_1 with $\xi = 0.22$, $\xi = 0.23$, $\xi = 0.237$, $\xi = 0.245$ and $\xi = 0.25$, with value of ξ increasing from the top curve to the bottom.

Here C_1 is roughly $< TrU >$ (in a static phase), and $V(C_1)$ can be regarded as an on-shell evaluation of the action S in the previous slide. There is a clear first order phase transition.

d=2: Stability and order of transition



Energy vs ξ for three types of eigenvalue distribution of the Wilson line U. ξ is a monotonically increasing function of T. Note the 1st order transition at ξ_1.

d=2: phase diagram

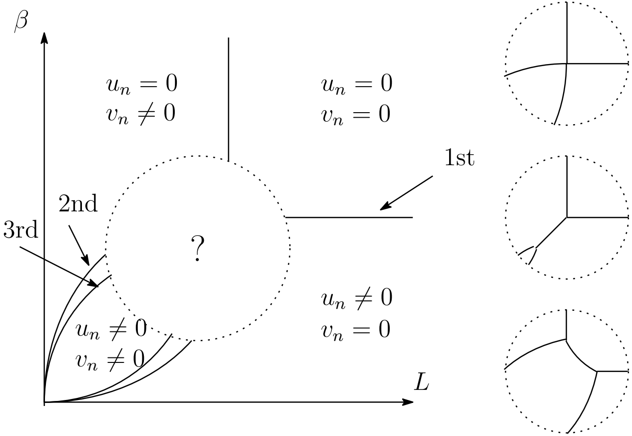


Figure: Phases at small and large L. The second joining pattern is picked out by gravity calculations. This supports the 'cascade' found in lattice calculations.

Gravity correspondence: D2 branes

- To get a gravity dual of $d = 2$ bosonic YM, start with D2 branes= 3d SYM on T^3 with radii β, L_1, L_2.
- Consider AP b.c. for fermions along L_2. For small enough L_2 the corresponding KK modes and all fermions decouple $\Rightarrow$ $d = 2$ YM.
- However, for very small L_2, the gravity analysis is not reliable; hence L_2 cannot be taken too small, $\Rightarrow$ fermions persist.
- Phase diagram depends on fermion boundary conditions along β, L_1: (P,P), (AP, P), (P, AP), (AP,AP).
- Gravity solutions (phases) include D0, D1 and D2 branes (smeared/ localized) and AdS solitons which are double Wick rotations of these.

Brane free energies

$Dp_{L_0,(L_1,L_2,...,L_p)}$ denotes a Dp brane wrapped on a contractible cycle of length L_0 (cf. cigar), and non-contractible $L_1, ..., L_p$.

$$\Rightarrow \langle TrU_0\rangle \neq 0, \langle TrU_1\rangle = \langle TrU_2\rangle = ... = 0$$

$$ds^2 = \alpha' \left[F(u)\left(f(u)dt^2 + \sum_{i=1}^{p} dx_i dx_i\right) + \frac{du^2}{F(u)f(u)} + G(u)d\Omega_{8-p}^2\right]$$
$$F(u) = \frac{u^{(7-p)/2}}{\sqrt{d_p\lambda_{p+1}}}, \quad G(u) = \sqrt{d_p\lambda_{p+1}}u^{(3-p)/2}, \quad f(u) = 1 - \left(\frac{u_0}{u}\right)^{7-p}$$
$$\lambda_{p+1} = g_{p+1}^2 N \tag{4}$$

$$S/N^2 = C_p\lambda_{p+1}^{\frac{p-3}{5-p}}L_1\cdots L_p\beta\left(-\beta^{-\frac{2(7-p)}{5-p}} + H(U_{\mathrm{reg}})\right),$$
$$H(U_{\mathrm{reg}}) = \left(\frac{2a_p}{\sqrt{\lambda_{p+1}}}\right)^{2(7-p)/(5-p)} U_{\mathrm{reg}}^{7-p} \tag{5}$$

D2: (P, P, AP)

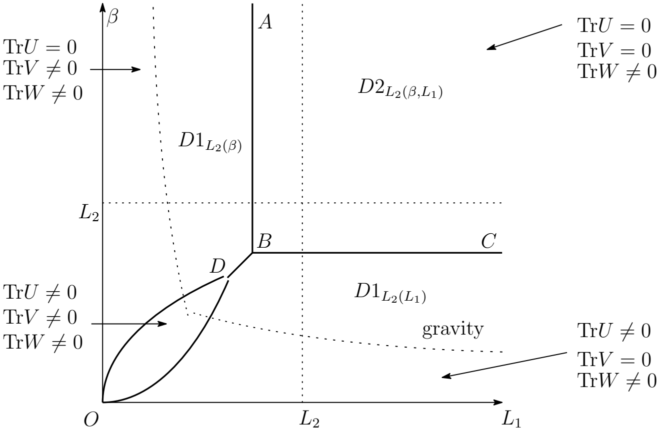
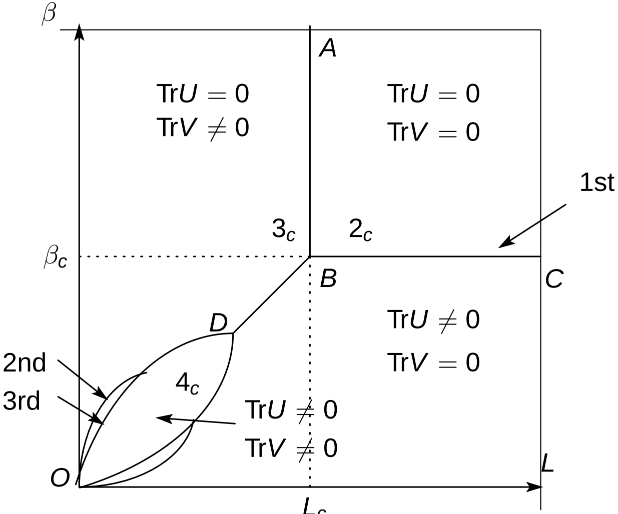


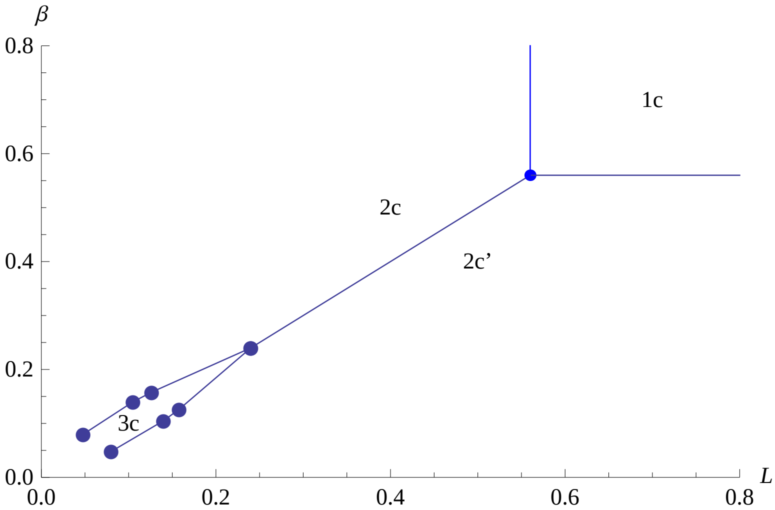
Figure: $D2$ brane on T^3 with (P,P,AP) boundary condition. Gravity description reliable above dotted lines.

d=2: Combining gauge theory & gravity-extrapolated



[cf. Takeshi's talk yesterday on Deconfinement in 4D YM]

New results from lattice [in collaboration with R. Narayanan]



d=2: Dynamical transitions

- We derived the large L effective action above. By flipping $t \leftrightarrow x_1$, we get the following effective action at large β (low temperature)

$$S(A)/DN^2 = \int_0^\infty dt \left(\frac{1}{2N} \mathrm{Tr}\left(|\partial_t V|^2 \right) + \sqrt{\frac{\triangle_0}{2\pi\tilde{\lambda}^2 L^3}} e^{-\triangle_0 L} \left| \frac{1}{N} TrV \right|^2 \right)$$

where $V(t) = P \exp[i \int A_1(x,t) dx]$.

- The static solutions, as mentioned before, are given by uniform, non-uniform and gapped eigenvalue distributions. The stability of these depends on the value of L.

- By using the above action, we can consider dynamical transitions between these phases, which would include gauge theory duals of dynamical Gregory-Laflamme transitions.

Gapless → gapped

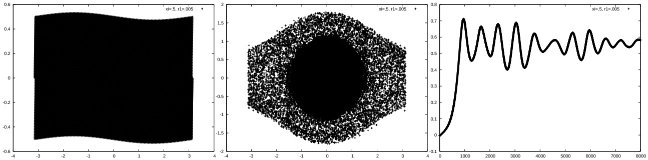


Figure: The figure on the left shows a slightly perturbed gapless distribution at $t = 0$. The figure in the middle shows a nearly gapped distribution (t=8000). The figure on the extreme right depicts $\rho_1(t)$ as it changes from 0 at $t = 0$ to 0.55 at $t = 8000$

Gapless $\rightarrow$ gapped: density plot

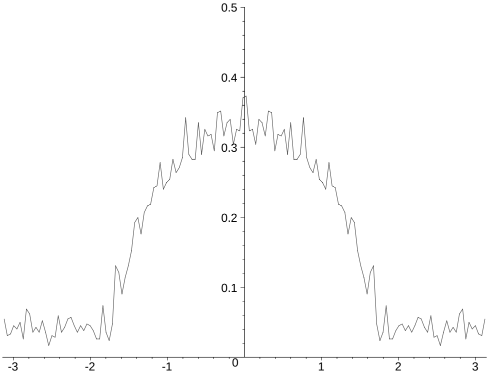


Figure: Coordinate space fermion distribution corresponding to the central figure of Fig 3. The 'waist' does not vanish at very large times. cf. Horowitz-Maeda conjecture: 'no naked singularity'.

Gapped $\rightarrow$ gapless

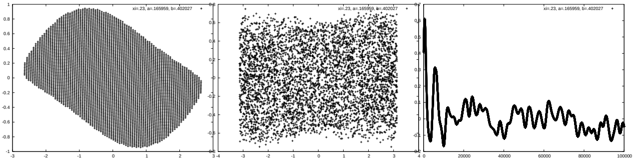


Figure: The figure on the left shows a slightly perturbed gapped distribution at $t = 0$. The value of ξ is 0.23. The figure in the middle shows a gapless distribution at $t = 10000$. The figure on the extreme right depicts $\rho_1(t)$ as it changes from 0.5 at $t = 0$ to 0 at $t = 8000$

Open problems and work in progress

- Fermions [work in progress with Hiroshi Isono]. Schematically,

$$\psi^2 Y + Y^4 = BY^2 + B^2 + \psi^2 Y$$
$$= B(Y + 1/(2B)\psi^2)^2 - \psi^4/(4B) = B(\tilde{Y})^2 - F^2/(4B) + \psi^2 F$$

  $\Rightarrow$ SSB of SO(D). Large D vs SUSY.
- Higher dimensions ($d \geq 3$). In addition to $\log(D_\mu^2 + B)$, the kinetic term $F_{\mu\nu}^2$ plays an important role. Makes analysis difficult.
- Dynamical transitions: end-point of GL, equilibration, time arrow [with Basu, Morita, Wadia; Iizuka, Morita]
- Large D as a new classical limit: $\langle TrY^I Y^I \rangle/(ND) \sim \Delta_0^2$. In fact, $\Psi(Y^2)$ turns out to be (under certain circumstances) $\sim \delta(Y^2 - Y_0^2)$. Appearance of size (horizon?). Need to compute Wilson line in the bulk to compute the location of horizon.
- Saddle point configuration corresponds to black objects, with entropy $O(N^2)$. How does this appear in the Y^I quantum mechanics? Splitting of the $O(N^2)$ level....

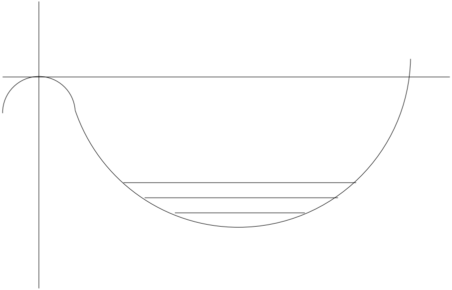
$\mathcal{M}$	type of phase transition
small $T^D \times$ small S^1	2nd+3rd
small $T^D \times$ large S^1	1st
small S^2	2nd+3rd
small S^3	1st

Table: Confinement/deconfinement type transitions in lower dimensional pure Yang-Mills theories on $S^1_\beta \times \mathcal{M}$. Here "small S^1" and "small T^D" refer to sizes small enough to ensure (a) that the Z_N symmetries in the S^1 and T^D directions, respectively, are broken, and (b) that all the KK modes can be integrated out. "Large S^1" ensures that the Z_N is not broken.

Glueball

Bhanot, Daley and Klebanov:
$d = 2, D = 1$

Tr $(Y(k_1)Y(k_2)...Y(k_r))$ Hagedorn spectrum of glueballs



S.H. Oscillation of the condensate