

Holographic Models of the Quantum Hall Effect

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References

D3-D7'

- O. Bergman, G. Lifschytz, N. Jokela, MSL
Quantum Hall effect in a holographic model
JHEP **1010** (2010) 063; arXiv:1003.4965 [hep-th]
- G. Lifschytz, N. Jokela, MSL
Magneto-roton excitation in a holographic quantum Hall fluid
JHEP **1102** (2011) 104; arXiv 1012.1230 [hep-th]
- O. Bergman, G. Lifschytz, N. Jokela, MSL
Striped Instability of a holographic Fermi-like liquid
arXiv 1106.3883 [hep-th]

D2-D8'

- N. Jokela, M. Järvinen, MSL
A holographic quantum hall model at integer filling
JHEP **1105** (2011) 101; arXiv: 1101.3329 [hep-th]
- N. Jokela, M. Järvinen, MSL
Fluctuations of a holographic integer quantum Hall fluid
in preparation

Outline

Part I - This talk

1. Introduction
 - Motivation
 - QHE review
2. D2-D8' Model Construction
 - Set-up
 - Stabilization
3. Embeddings and Conductivity

Part II - Next Talk

1. Thermodynamics and Phase Structure
2. Fluctuations
3. Open Questions

Holographic fermions

Many phenomena involve strongly-coupled fermions:

- Chiral Symmetry Breaking
- Quantum Critical Behavior
- Fractional quantum Hall effect (FQHE)
- ...

But interesting = difficult

Top-Down Holographic Approach:

- Study concrete string models
- Known field theory duals
- Gives new effective theories
- Look for universal features

Quantum Hall Effect (QHE)

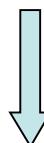
Experimental Setup:

e^- in 2+1 dimensions
high magnetic field H
low temperature T

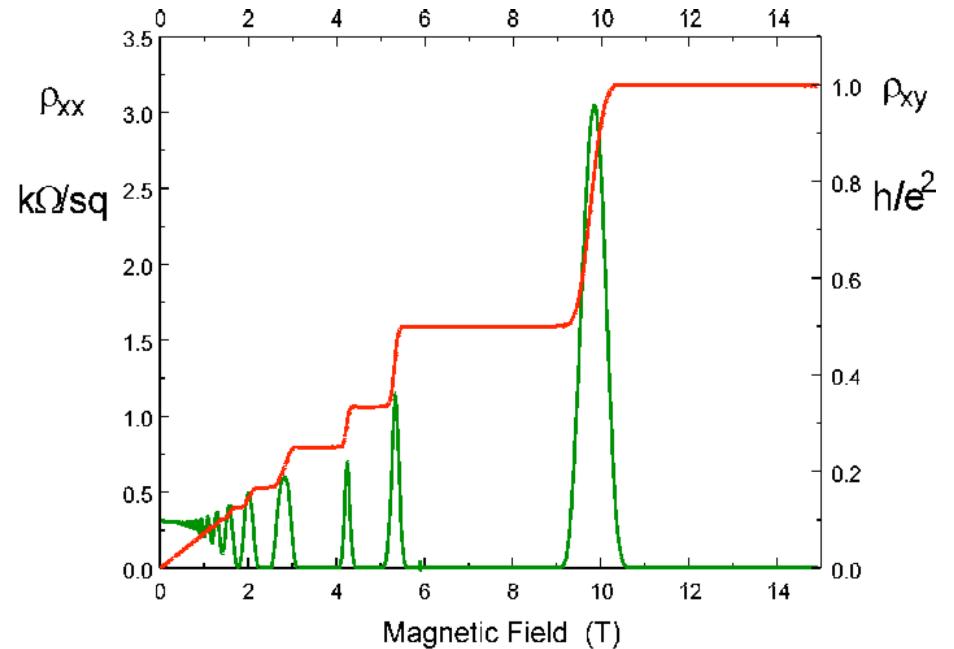
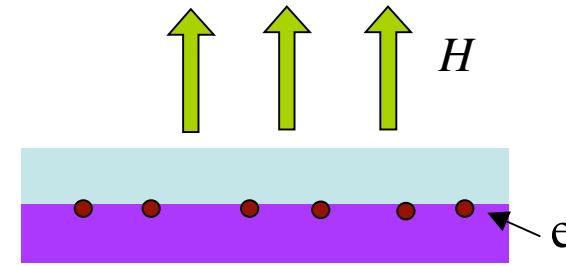
Conductivity

Longitudinal: $\sigma_{xx} = \frac{j_x}{E_x} = 0$

Hall: $\sigma_{xy} = \frac{j_y}{E_x} = \frac{e^2}{2\pi\hbar}\nu$



Filling fraction



Filling Fraction

$$\nu \equiv \frac{2\pi\hbar D}{eH} \sim \frac{\# \text{ electrons}}{\# \text{ flux quanta}}$$

QHE states for certain ν

$\nu \in \mathbf{Z}$  Integer QHE
Landau levels

$\nu \notin \mathbf{Z}$  Fractional QHE
strongly-coupled, Laughlin wave function

Open questions:

- microscopic description
- allowed ν 's
- transitions

D p -D q Models

Brane intersections with #ND=6

- fundamental fermions at intersection
- D q probe in D p background
- SUSY  stability?

Example:

Sakai Sugimoto model: D4-D8- $\overline{\text{D}8}$

Rey

Kraus & Keski-Vakkuri

Myers & Wapler, etc

Hong & Yee

Takayanagi et al.

...

Two other examples:

- D3-D7' Model

2+1-dim fermions, 3+1-dim gauge

FQHE, $\nu = \text{irrational}$, set by internal flux

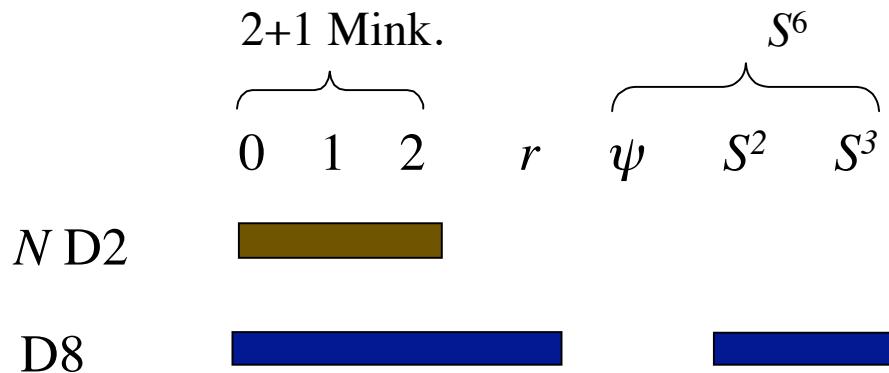
- D2-D8' Model

2+1-dim gauge + fermions

IQHE, $\nu = 1 \ \forall \text{internal flux} \neq 0$

Focus of today's talks

D2-D8' system



$$d\Omega_6^2 = d\psi^2 + \cos^2 \psi \; d\Omega_2^2 + \sin^2 \psi \; d\Omega_3^2$$

D8 wraps $S^2 \times S^3 \subset S^6$

Embedding:

solve for $\psi(r)$
 $\psi(r \rightarrow \infty) \equiv \psi_\infty = \arctan \sqrt{2/3}$

Stabilization

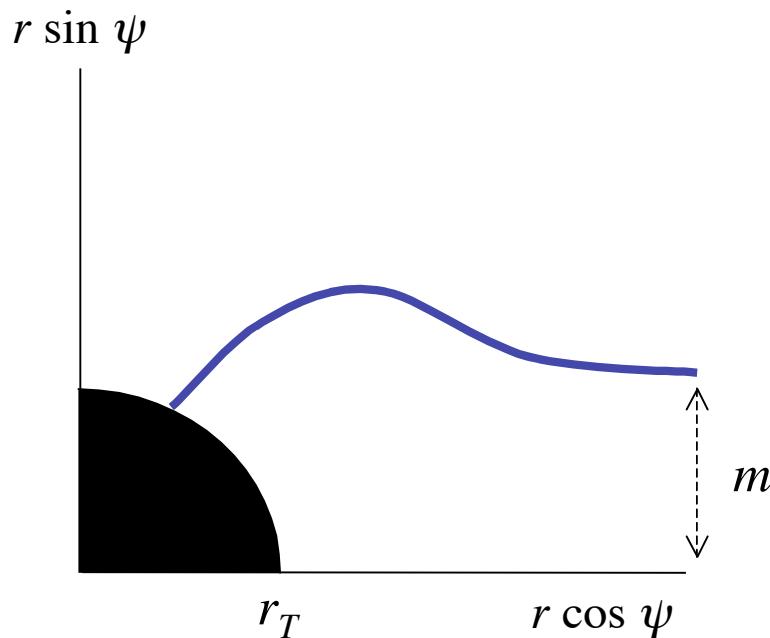
lowest mode for ψ tachyon (slipping mode)

wrap magnetic flux on internal S^2

$$\psi(r \rightarrow \infty) \equiv \psi_\infty = 0$$

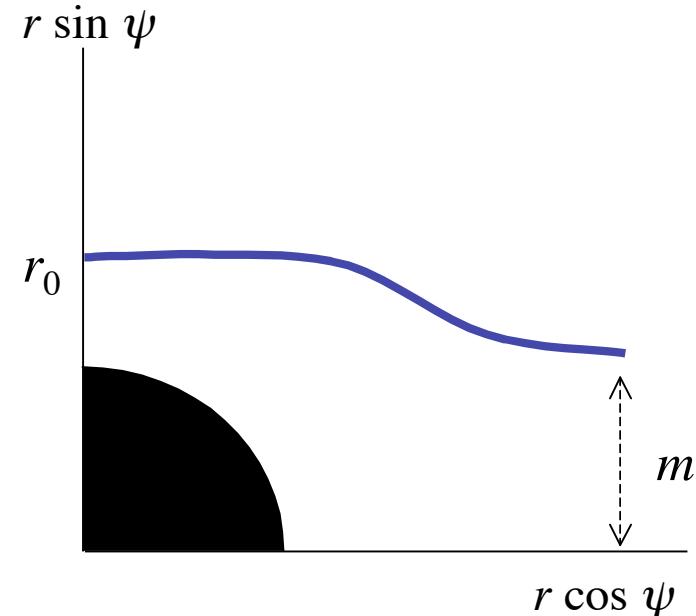
Embeddings

Black Hole (BH)



D8 enters horizon

Minkowski (MN)



D8 ends where S^3 shrinks

Add charges and magnetic field

Charge density

$$2\pi\alpha' F_{r0} = a'_0(r)$$

Magnetic field

$$2\pi\alpha' F_{xy} = H$$

Chern-Simons

$$\begin{aligned} S_{CS} &\sim \int C_5 \wedge F \wedge F \\ &\sim H \int dr \ c(r) a'_0(r) \quad \text{where} \quad c(r) \sim \int_{S^2 \times S^3} C_5 \end{aligned}$$

→ C_5 flux and H induce charge

Where's the charge?

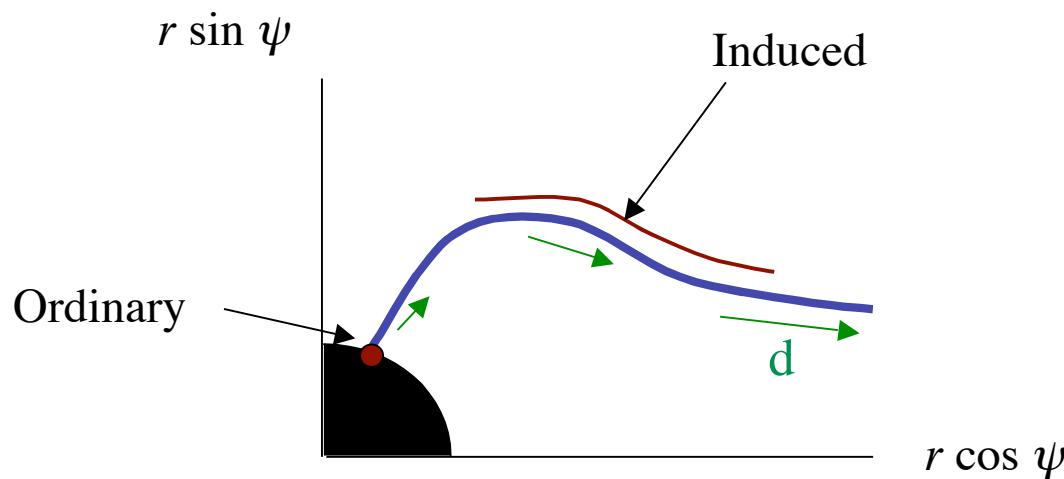
Charge density \longleftrightarrow radial displacement flux

$$d(r) \equiv \frac{\delta S_{D8}}{\delta a'_0} = d_\infty - Hc(r)$$

Total charge density: $D = d_\infty$

Split between two types:

1. Induced charge: $Hc(r_{\min})$
2. Ordinary charge: $D - Hc(r_{\min})$



BH Embeddings

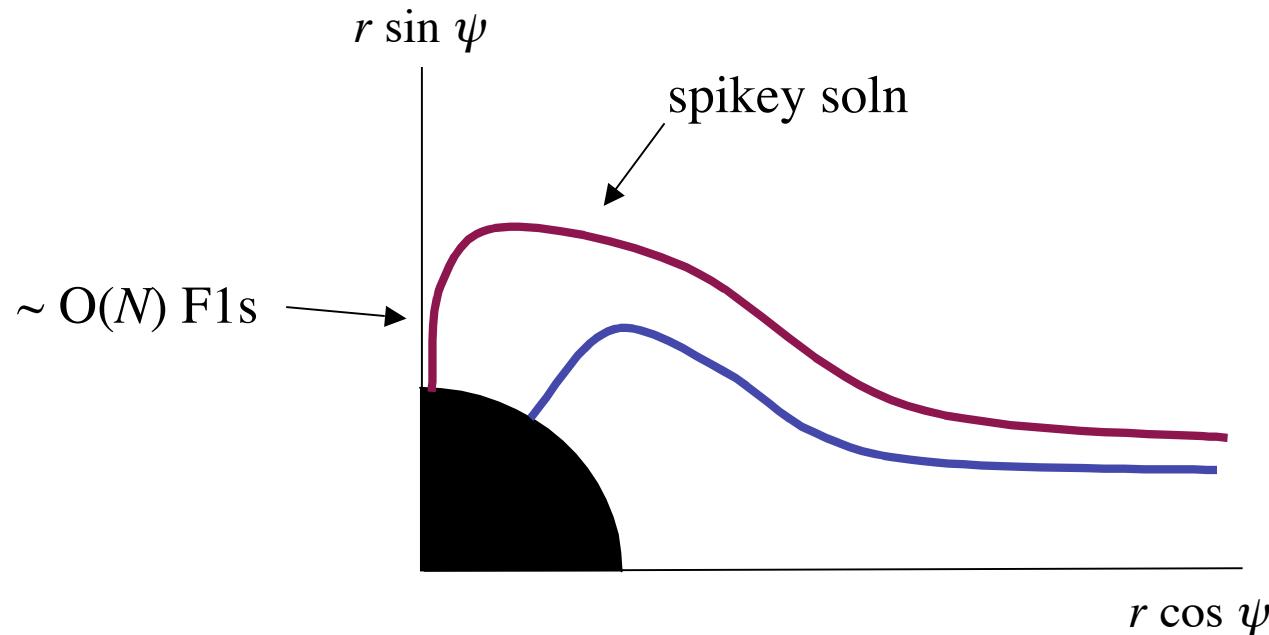
Metallic state

- gapless charged excitations
- longitudinal current (via Karch-O'Bannon)

Solutions become spiky as $\nu \equiv \frac{2\pi D}{N H} \rightarrow 1$



filling fraction per fermion



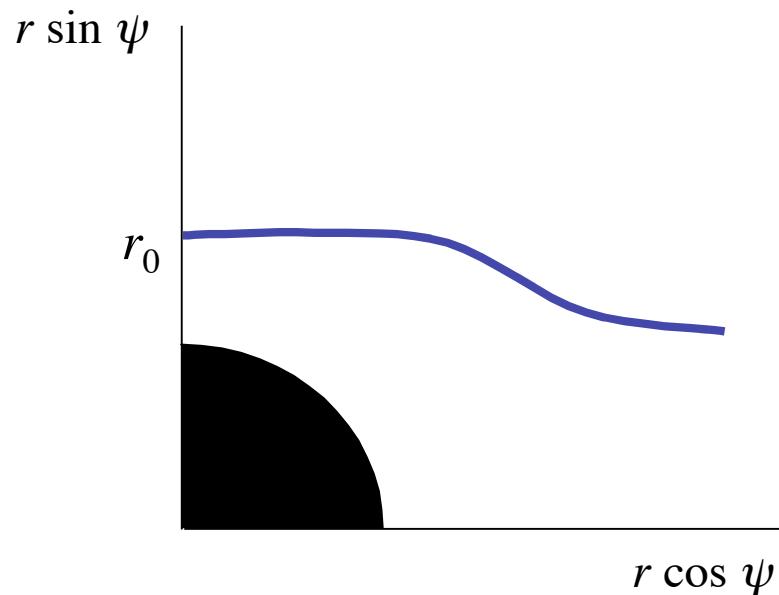
MN Embeddings

QHE state

- no sources at tip, all charge induced

$$\longrightarrow D = \frac{N}{2\pi} H$$

- $\nu = 1$ independent of internal flux
- gap for charged excitations $m_g \sim r_0$



MN Conductivity

Modified Karch-O'Bannon

1. Turn on electric field and radial fields dual to j_x and j_y
2. But require regularity of gauge field at tip (no pseudohorizon)
3. Conductivity

$$\sigma_{xx} = 0$$

$$\sigma_{xy} = v/2\pi$$

Summary of Part I

Two top-down models

Charged fermions in 2+1 dim

MN embedding = QH fluid

Features:

- Quantized ν
- Mass gap
- Conductivities

Bugs:

- Only one QH state per model
- Limited choice of ν