Holographic Three-Point Functions of Giant Gravitons

Konstantinos Zoubos

Niels Bohr Institute

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Milos

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Based on arXiv:1103.4079 with A. Bissi, C. Kristjansen and D. Young

Motivation: Semiclassical States in AdS/CFT

- Context: AdS/CFT correspondence between $\mathcal{N} = 4$ SYM and string theory on $\mathrm{AdS}_5 \times \mathrm{S}^5$
- Interested in $\mathcal{N} = 4$ SYM operators with large quantum numbers, e.g. $\mathrm{Tr}(X^{J_1} Y^{J_2})$
- E.g. large R-charge $\Leftrightarrow$ large angular momentum along S^5
- Semiclassical states on $\mathrm{AdS}_5 \times \mathrm{S}^5$ play a significant role
- Their role is particularly crucial in the context of AdS/CFT integrability [Gubser, Klebanov, Polyakov '02]
- They are dual to long operators with a large number of impurities
- Examples: Folded and circular spinning strings, giant magnons, (cusped) Wilson loops...

Correlation functions in AdS/CFT

- Consider the set of all gauge invariant operators $\mathcal{O}_I$ in $\mathcal{N} = 4$ SYM ($\mathcal{O}_I \in \{\mathrm{Tr}(XYXZX\cdots), \mathrm{Tr}(D_\mu X\psi\bar{\psi}\cdots), \cdots\}$)
- Their two-point functions take the form:

$$\langle \mathcal{O}_I^{\Delta_I}(x)\overline{\mathcal{O}}_J^{\Delta_J}(y)\rangle = \frac{\delta_{IJ}}{|x-y|^{2\Delta_I}}$$

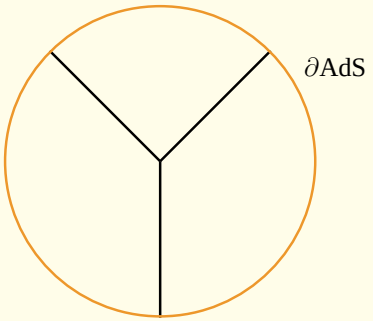
- Spectral problem: Determine Δ_I for all $\mathcal{O}_I$
- This is now believed to be solved for practically all operators in (planar) $\mathcal{N} = 4$ SYM
- Integrability techniques (Bethe ansatz, Y-system...)
- Next step: Three-point functions!

$$\langle \mathcal{O}_1^{\Delta_1}(x)\mathcal{O}_2^{\Delta_2}(y)\mathcal{O}_3^{\Delta_3}(z)\rangle = \frac{\mathcal{C}_{123}}{|x-y|^{\Delta_1+\Delta_2-\Delta_3}|x-z|^{\Delta_1+\Delta_3-\Delta_2}|y-z|^{\Delta_2+\Delta_3-\Delta_1}}$$

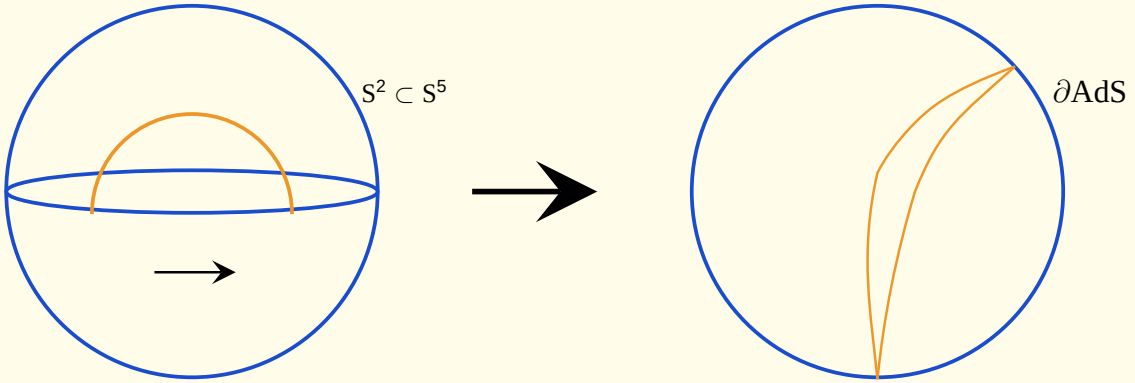
- Knowing all the $\mathcal{C}_{IJK}$ (as well as the Δ_I) would amount to solving the theory (in principle)

Two-point functions of semiclassical states

- In AdS/CFT, correlation functions of single-trace operators are calculated using Witten diagrams

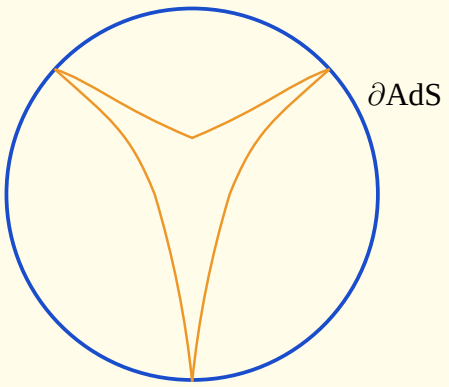


- We would like a similar prescription for semiclassical states
- For two-point functions, this was discussed in [Tsuji '06, Janik, Surowka, Wereszczynski '10]
- Appropriate Wick rotations take a spinning string solution to a configuration starting and ending at the boundary



Three-point functions of semiclassical states

- We would like to do something similar for three-point functions
- However, that would seem to involve knowing the geometric solution for a semiclassical string ending on the boundary at three points



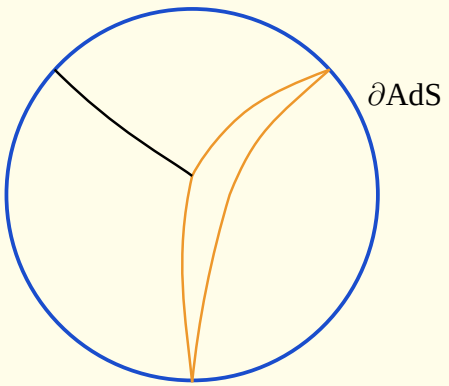
- Some recent progress, but the general problem is still open

  [Vicedo '11, Klose, McLoughlin '11]
- Expectation is that integrability will eventually give the answer for $\mathcal{C}_{IJK}$ while bypassing the precise solution

State of the art: Two heavy, one light

[Zarembo '10, Costa, Monteiro, Santos, Zoakos '10, Buchbinder/Tseytlin/Roiban/Russo '10]

- Take two of the states to be heavy (semiclassical) and one to be light (dual to a supergravity mode)



- Ignore backreaction of the light state on the heavy one
- The semiclassical trajectory is unchanged
- Integrate over the position of the insertion of the light state on the heavy state worldvolume

Three-point function prescription

- $\mathcal{C}_{IJK}$ is given by the following prescription: [Zarembo '10]

$$\frac{\langle \mathcal{W}\mathcal{O}_I(y)\rangle}{\langle \mathcal{W}\rangle} = \lim_{\epsilon\to 0}\frac{\pi}{\epsilon^{\Delta_I}}\sqrt{\frac{2}{\Delta_I - 1}}\left\langle \phi_I(y,\epsilon)\frac{1}{Z_{\text{heavy}}}\int DXe^{-S_{\text{heavy}}[X]}\right\rangle_{\text{bulk}}$$

- $\phi(y,\epsilon)$ is the supergravity mode dual to the single-trace chiral primary $\mathcal{O}_I$
- For a string, the action is:

$$S_{\text{heavy}} = \frac{\sqrt{\lambda}}{4\pi}\int d^2\sigma\sqrt{h}G_{MN}\partial^a X^M\partial_a X^N$$

  and couples to ϕ through $G_{MN} = g_{MN} + \gamma_{MN}$, $\gamma_{MN} = V^I_{MN}\phi_I$
- Several cases have been considered recently
- Weak coupling side less developed [Escobedo,Gromov,Sever,Vieira '10]

Back to the BPS sector

- All this was for semiclassical obects which are far from BPS
- But can these techniques also provide new input in the $\frac{1}{2}$-BPS sector?
- Much better control on the gauge theory side (often exact results exist)
- Could hope to find exact matching between the two sides of the duality
- In this talk, we will look at correlation functions involving operators in representations of order N
- We will identify and attempt to compute the same correlation functions holographically
- First review some facts about the $\frac{1}{2}$-BPS sector

The trace basis

- The simplest basis of $\frac{1}{2}$-BPS operators is made up products of traces of a single $\mathcal{N} = 4$ scalar

$$\mathrm{Tr}(Z^J)\ ,\ \mathrm{Tr}(Z^{J-1})\mathrm{Tr}(Z)\ ,\ \mathrm{Tr}(Z^{J-2})\mathrm{Tr}(Z^2)\ ,\ \ldots$$

- Focus on single-trace chiral primaries: $\mathcal{O}^J = \mathrm{Tr}Z^J$
- Dual to gravity modes in the dual theory
- Two- and three-point functions [Lee et al. '98, D'Hoker et al. '98, Kristjansen et al. '02, Constable et al. '02]

$$\langle \mathcal{O}^J \overline{\mathcal{O}}^J \rangle = JN^J \left(1 + O(1/N^2)\right)$$

$$\langle \mathcal{O}^J \mathcal{O}^K \overline{\mathcal{O}}^{J+K} \rangle = N^{J+K-1} JK(J+K) \left(1 + O(1/N^2)\right)$$

- Structure constants

$$\mathcal{C}_{J,K,K+J} = \frac{\langle \mathcal{O}^J \mathcal{O}^J \overline{\mathcal{O}}^{J+K} \rangle}{\sqrt{\langle \mathcal{O}^J \overline{\mathcal{O}}^J \rangle \langle \mathcal{O}^K \overline{\mathcal{O}}^K \rangle \langle \mathcal{O}^{J+K} \overline{\mathcal{O}}^{J+K} \rangle}}$$

$$= \frac{1}{N} \sqrt{JK(J+K)} \left[1 + O(1/N^2)\right]$$

Operators of very large dimension

- We are working in the planar limit $N \to \infty$
- What happens when we consider trace operators whose dimension $J \sim N$?
- Relations appear between single and multitrace states $\Rightarrow J$ bounded!
- The $\mathcal{O}^J$ cease to be orthogonal in this limit
- The usual $1/N^2$ counting for non-planar diagrams is upset by huge combinatoric factors
- Correlation functions of the $\mathcal{O}^J$ are not well-behaved

  [Balasubramanian et al. '01, Dhar, Mandal, Smedbäck '05]
- Does there exist a better $\frac{1}{2}$-BPS basis for $J \sim N$?

Schur polynomial operators

[Corley, Jevicki, Ramgoolam '01]

- Defined by a representation R_n of the symmetric group S_n

$$\chi_{R_n}(Z) = \frac{1}{n!} \sum_{\sigma \in S_n} \chi_{R_n}(\sigma) Z_{i_1}^{i_{\sigma(1)}} \cdots Z_{i_n}^{i_{\sigma(n)}}$$

- They can be expanded in a trace basis
- For the antisymmetric representation:

$$\begin{aligned}
O_2^A &= -\frac{1}{2}\mathrm{Tr}(Z^2) + \frac{1}{2}\mathrm{Tr}(Z)^2 \\
O_3^A &= \frac{1}{3}\mathrm{Tr}(Z^3) - \frac{1}{2}\mathrm{Tr}(Z^2)\mathrm{Tr}(Z) + \frac{1}{6}\mathrm{Tr}(Z)^3 \\
O_4^A &= -\frac{1}{4}\mathrm{Tr}(Z^4) + \frac{1}{3}\mathrm{Tr}(Z^3)\mathrm{Tr}(Z) + \frac{1}{8}\mathrm{Tr}(Z^2)^2 \\
&\quad - \frac{1}{4}\mathrm{Tr}(Z^2)\mathrm{Tr}(Z)^2 + \frac{1}{24}\mathrm{Tr}(Z)^4 \\
&\ldots
\end{aligned}$$

Schurs vs. Multi-traces

- The Schurs are a better basis when $\Delta \sim N$

  [Corley, Jevicki, Ramgoolam '01, Dhar, Mandal, Smedbäck '05]
- Orthogonal for any value of N

$$\langle \chi_R(Z)\chi_S(\overline{Z})\rangle = \delta_{R,S} \prod_{i,j\in R} (N - i + j)$$

- Correlation functions fall with N

$$\langle \chi_R(Z)\chi_S(Z)\chi_T(\overline{Z})\rangle = g(R,S;T) \prod_{i,j\in T} (N - i + j)$$

  ($g(R,S,T)$: *Littlewood-Richardson* coefficients)
- Two- and three-point functions: (here for antisymmetric)

$$\langle \chi_k^A(\bar{Z})\chi_k^A(Z)\rangle = \prod_{i=1}^{k}(N - i + 1),$$

$$\langle \chi_k^A(\bar{Z})\chi_{k-J}^A(Z)\chi_J^A(Z)\rangle = \prod_{i=1}^{k}(N - i + 1)$$

AdS duals for the Schurs?

- The description is simplest for the symmetric and antisymmetric cases
- Antisymmetric Schurs are nothing but determinant and subdeterminant operators

$$\chi_k^A(Z) = \det{}_k(Z)$$

- For $k \sim N$, these have been argued to be dual to giant gravitons on S^5 [Balasubramanian et al. '01]
- Satisfy the stringy exclusion principle
- Symmetric Schurs were shown to be dual to AdS_5 giant gravitons [Corley, Jevicki, Ramgoolam '01]

Giant Gravitons

[McGreevy, Susskind, Toumbas '00]

- D3-branes wrapped around (trivial) cycles in AdS_5 or S^5 and rotating along the S^5
- Stabilised by their angular momentum k
- Their radius increases with k through Myers effect
- As $k \to 0$, they reduce to pointlike gravitons
- Preserve $\frac{1}{2}$ Supersymmetry [Grisaru, Myers, Tafjord '00]
- Have been argued to be good duals to Schur polynomials for $k \sim N$
- Since $R \leq R_{S^5}$, we have a simple explanation of the stringy exclusion principle

Our goal

- Can we compute holographic correlation functions involving Schur polynomials?
- We are interested in the semiclassical limit, $k \sim N \gg 1$
- $\langle \overline{\chi}_k(\overline{Z})\chi_{k-l}(Z)\chi_l(Z)\rangle$ is beyond our reach. We would need the full semiclassical geometry
- Inspired by the progress in the semiclassical string context, we can try to compute a correlation function of two Schurs and one trace operator:

$$\langle \overline{\chi}_k(\overline{Z})\chi_{k-J}(Z)\mathrm{Tr}Z^J\rangle$$

- On the dual gravity side, this should correspond to a giant graviton emitting a light graviton

Gauge theory side

- We want the structure constant (here for symmetric):

$$C^S_{k,k-J,J} \equiv \frac{\langle \chi^S_k(\bar{Z})\chi^S_{k-J}(Z)\mathrm{Tr}Z^J\rangle}{\sqrt{\langle \chi^S_k(\bar{Z})\chi^S_k(Z)\rangle\langle \chi^S_{k-J}(\bar{Z})\chi^S_{k-J}(Z)\rangle\langle \mathrm{Tr}\bar{Z}^J\mathrm{Tr}Z^J\rangle}},$$

- We can simply use that:

$$\mathrm{Tr}Z^J = \sum_{R_J} \chi_{R_J}(\sigma_0)\chi_{R_J}(Z)$$

(σ_0 the cyclic permutation) to find

$$\begin{aligned}
\langle \chi^S_k(\bar{Z})\chi^S_{k-J}(Z)\,\mathrm{Tr}\,Z^J\rangle &= \prod_{j=1}^{k}(N-1+j), \\
\langle \chi^A_k(\bar{Z})\chi^A_{k-J}(Z)\,\mathrm{Tr}\,Z^J\rangle &= (-1)^{J-1}\prod_{i=1}^{k}(N-i+1)
\end{aligned}$$

Gauge theory result

- Normalise by dividing by the relevant norms
- We are interested in the limit

$$N, k \to \infty \quad \text{with} \quad \frac{k}{N} \text{ finite}, \quad J \ll k$$

- Result: The structure constants are:

$$\begin{aligned} C^S_{k,k-J,J} &= \frac{1}{\sqrt{J}} \left(1 + \frac{k}{N}\right)^{J/2}, \\ C^A_{k,k-J,J} &= (-1)^{(J-1)} \frac{1}{\sqrt{J}} \left(1 - \frac{k}{N}\right)^{J/2} \qquad k \leq N \end{aligned}$$

Gravity side

- Now compute the same object in the dual $\mathrm{AdS}_5 \times \mathrm{S}^5$ theory, following the approach of [Zarembo '10]
- As discussed, we need to evaluate the following object:

$$\frac{\langle \mathcal{W}\mathcal{O}_I(y)\rangle}{\langle \mathcal{W}\rangle} = \lim_{\epsilon\to 0} \frac{\pi}{\epsilon^{\Delta_I}} \sqrt{\frac{2}{\Delta_I - 1}} \left\langle \phi_I(y,\epsilon) \frac{1}{Z_{\mathrm{D3}}} \int DX e^{-S_{D3}[X]} \right\rangle_{\mathrm{bulk}}$$

- S^E_{D3} is the Euclidean D-brane action

$$S^E_{D3} = \frac{N}{2\pi^2} \int d^4\sigma \left(\sqrt{g} - iP[C_4]\right),$$

where $g_{ab} = \partial_a X^M \partial_b X_M, \quad a, b = 0, \cdots 3$. X^M are the brane embedding coordinates

Giant graviton in S^5

- Global metric for $AdS_5 \times S^5$:

$$ds^2 = -\cosh^2\rho\, dt^2 + d\rho^2 + \sinh^2\rho\, d\widetilde{\Omega}_3^2 + d\theta^2 + \sin^2\theta\, d\phi^2 + \cos^2\theta\, d\Omega_3^2.$$

- Giant graviton ansatz

$$\rho = 0, \quad \sigma^0 = t, \quad \phi = \phi(t), \quad \sigma^i = \chi_i, \qquad C_{\phi\chi_1\chi_2\chi_3} = \cos^4\theta\, \mathrm{Vol}(\Omega_3)$$

- Action

$$S = -N \int dt \left[\cos^3\theta \sqrt{1 - \dot{\phi}^2 \sin^2\theta} - \dot{\phi}\, \cos^4\theta\right]$$

- Angular momentum

$$k = \frac{\delta L}{\delta \dot{\phi}} = \frac{N\dot{\phi}\sin^2\theta\cos^3\theta}{\sqrt{1 - \dot{\phi}^2\sin^2\theta}} + N\cos^4\theta.$$

- The energy $E = \dot{\phi}k - L$ is minimized by

$$\cos^2\theta = \frac{k}{N}, \qquad E_{\text{min.}} = k, \qquad S_{\text{min.}} = 0 \quad \Rightarrow \dot{\phi} = 1$$

Giant graviton in S^5 (cont.)

- We will need the fluctuations of the sugra mode [Kim, Romans, van Nieuwenhuizen '85, Lee, Minwalla, Rangamani, Seiberg '98, Berenstein, Corrado, Fischler, Maldacena '98]

$$\delta g_{\mu\nu} = \left[-\frac{6\,\Delta}{5}\, g_{\mu\nu} + \frac{4}{\Delta+1}\, \nabla_{(\mu}\nabla_{\nu)} \right] s^{\Delta}(X)\, Y_{\Delta}(\Omega),$$

$$\delta g_{\alpha\beta} = 2\,\Delta\, g_{\alpha\beta}\, s^{\Delta}(X)\, Y_{\Delta}(\Omega),$$

$$\delta C_{\mu_1\mu_2\mu_3\mu_4} = -4\,\epsilon_{\mu_1\mu_2\mu_3\mu_4\mu_5} \nabla^{\mu_5}\, s^{\Delta}(X)\, Y_{\Delta}(\Omega),$$

$$\delta C_{\alpha_1\alpha_2\alpha_3\alpha_4} = 4\epsilon_{\alpha\alpha_1\alpha_2\alpha_3\alpha_4} s^{\Delta}(X) \nabla^{\alpha} Y_{\Delta}\,(\Omega)\,,$$

- $Y_{\Delta}(\Omega)$ correspond to the $[0, \Delta, 0]$ representation

$$Y_{\Delta}(\Omega) = \frac{\sin^{\Delta}\theta e^{\Delta t}}{2^{\Delta/2}} \quad \Leftrightarrow \quad \mathcal{O} = \mathrm{Tr} Z^{\Delta}$$

- s^{Δ} will be replaced by the bulk-to-boundary propagator

$$s^{\Delta} \to \sqrt{\frac{\alpha_0}{B_{\Delta}}} \frac{z^{\Delta}}{((x-x_B)^2+z^2)^{\Delta}} \simeq \sqrt{\frac{\alpha_0}{B_{\Delta}}} \frac{z^{\Delta}}{x_B^{2\Delta}},$$

Giant graviton in S^5 (cont.)

- Now we need to vary the action
- DBI part

$$\delta S_{DBI} = \frac{N}{2}\cos^2\theta \int dt\, Y_\Delta(\Omega)\left(\frac{4}{\Delta+1}\partial_t^2 - \frac{2\Delta(\Delta-1)}{\Delta+1} - 8\Delta\sin^2\theta + 6\Delta\right)s^\Delta$$

- Wess-Zumino part

$$\delta S_{WZ} = -2^{-\frac{\Delta}{2}+2} N\Delta \int dt\, e^{\Delta t}\sin^\Delta\theta\cos^4\theta s^\Delta$$

- Substituting s^Δ, with $z = R/\cosh t$, we finally find

$$\delta S = -\left(\frac{2R}{x_B^2}\right)^\Delta \sqrt{\Delta}\cos^2\theta\sin^\Delta\theta$$

to conclude that

$$\mathcal{C}^A_{k,k-J,J} = \sqrt{J}\frac{k}{N}\left(1-\frac{k}{N}\right)^{J/2}$$

Giant graviton in AdS_5

- Now the graviton wraps an $\mathrm{S}^3 \subset \mathrm{AdS}_5$, ($\mathrm{S}^3 : \vartheta, \phi_1, \phi_2$)
- We take the following ansatz

$$\rho = \text{const.}, \quad \sigma^0 = t, \quad \sigma^i = \widetilde{\chi}_i, \quad \phi = \phi(t), \quad \theta = \frac{\pi}{2}$$

to obtain

$$S = \int dt\, L = -N \int dt \left[\sinh^3\rho \sqrt{\cosh^2\rho - \dot{\phi}^2} - \sinh^4\rho\right]$$

- More complicated bulk-to-boundary propagator:

$$s \to \frac{\Delta+1}{4\sqrt{\Delta} N x_B^{2\Delta}} \frac{R^\Delta e^{\Delta t}}{(\cosh\rho\cosh t - \cos\vartheta \sin\phi_1 \sinh\rho)^\Delta}$$

- The final result is

$$\delta S = -\left(\frac{2R}{x_B^2}\right)^\Delta \frac{1}{\sqrt{\Delta}} \left(\cosh^\Delta\rho - \cosh^{-\Delta}\rho\right)$$

or

$$C^S_{k,k-J,J} = \frac{1}{\sqrt{J}} \left[\left(1+\frac{k}{N}\right)^{J/2} - \left(1+\frac{k}{N}\right)^{-J/2}\right]$$

Summary of Results

- Antisymmetric (S^5) case
  - Gauge theory

$$\mathcal{C}_{k,k-J,J} = (-1)^{J-1}\frac{1}{\sqrt{J}}\left(1-\frac{k}{N}\right)^{J/2}$$

  - Gravity

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- Symmetric (AdS_5) case
  - Gauge theory

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- There do not seem to be any subtle $1/N$ enhancements
- Were we correct in identifying Schur polynomials with giant gravitons?

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- But the Schurs don't reduce to single traces in any limit!

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- So the Schurs cannot be dual to giant gravitons for any k. Even at $k \sim N$, we cannot trust the very subleading terms in their OPE to reflect giant graviton physics

$$\chi_k(\bar{Z}(0))\,\chi_{k-J}(Z(x)) = \ldots + C^{gauge}_{k,k-J,J}\mathrm{Tr}\bar{Z}^J(0)\,x^{-2J} + \ldots.$$

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- The chiral primary $\mathrm{Tr}Z^J$ is sensitive to corrections of this order
- Need the true basis of operators, which interpolates between the Schurs and the trace operators

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- Other directions
  - ABJM theory (in progress)

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