Spontaneous Breaking of Conformal Symmetry and Trace Anomaly Matching

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Plan

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1)Setup

Consider a Conformal Field Theory at the quantum level i.e. the energy-momentum tensor obeys the operatorial identities:

$$\partial^m T_{mn} = 0$$ and

$$T^m{}_m = 0$$

the beta function being 0.

The Ward identities are put in evidence by coupling to an external metric:

$$g_{mn} = \eta_{mn} + h_{mn}$$

The metric transforms under Weyl transformations as:

$$g_{mn} \to e^{2\sigma(x)} g_{mn}$$

The generating functional W is invariant under Weyl transformations except trace anomalies reflecting violations of tracelessness in certain 3-point functions of the energy momentum tensor:

$$\delta_\sigma W = c \int d^4x \sqrt{g}\, \sigma\, C_{mnpq} C^{mnpq} - a \int d^4x \sqrt{g}\, \sigma\, E_4$$

A conformal theory can exist in two phases:

a)unbroken:the vacuum is invariant under the full group of SO(2,4) transformations

b)spontaneously broken to the Poincare group if some dimensionful scalar operator(s) get vacuum expectation values:

$$\langle 0|\mathcal{O}|0\rangle = v^{\Delta}$$

This phase has massless and massive states and due to Goldstone's theorem necessarily a massless dilaton with a linear coupling to the energy momentum tensor:

$$\langle 0|T_{mn}|\tau; q\rangle = \frac{1}{3} f q_m q_n$$

Anomaly Matching : the a and c coefficients are the same in the two phases the dilaton playing an essential role.

2)The proof of Anomaly Matching

Fast argument: a and c are dimensionless and therefore cannot depend on the dimensionful parameter v. However a singular behaviour at the limit is not a priori excluded and this argument still would leave the possibility of dependence on dimensionless ratios when there are “multiple breakings” .

Sketch of the general proof(for the a anomaly):
in the 3-point functions of energy momentum tensor the conservation and tracelessness Ward identities reduce at the symmetric point in phase space to :

$$A(q^2) = 0\,, \qquad A(q^2) - q^2 B(q^2) = 0$$

Since the imaginary part cannot be anomalous , in both phases:

$$B(q^2) = \frac{a}{q^2}$$

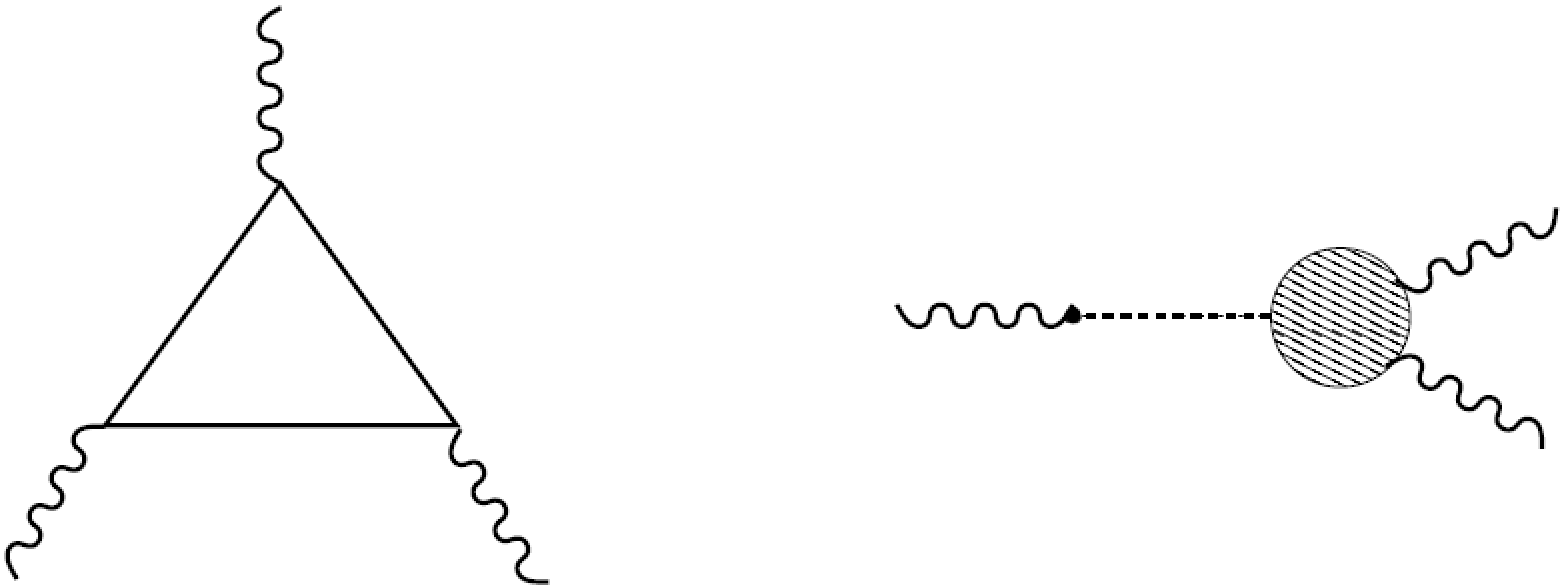
with possibly different coefficients in the two phases.
However in the broken phase for:

$$q^2 \gg v^2$$

one should recover the amplitudes of the unbroken phase i.e. the coefficient should be the same .
The analytic structure at general momenta of the correlators will be completely different in the two phases(e.g. massive singularities in the broken phase) however at the special kinematical point which "carries" the info about the trace anomaly the amplitudes are the same. Related to that the

$$q^2 = 0$$

pole in the amplitude does not represent a physical particle in the unbroken phase but a collapsed , unfactorizable cut ; in the broken phase it represents the contribution of the dilaton:



The dilaton couplings to two energy momentum tensors are fixed by the anomalies

The Dilaton Effective Action

In order to fix the many ambiguities and arrive at a "minimal " solution we link the dilaton action to the well understood case of the spontaneous breaking of U(1) chiral symmetry ; if the system has N=1 unbroken SUSY one can use the N=1 superconformal structure to link the dilaton to the U(1) Goldstone boson if the conformal symmetry and the U(1)-R current are broken simultaneously. The effective action for the U(1) Goldstone boson is given by:

$$W(A,\beta) = -\int d^4x \left((\partial_m \beta - A_m)(\partial^m \beta - A^m) + a\beta F^{mn}\tilde{F}_{mn} \right)$$

it reproduces the chiral anomaly under the transformation:

$$A_m \to A_m + \partial_m \alpha \qquad\qquad \beta \to \beta + \alpha$$

In the supersymmetric setup the dilaton and the "axion" belong to a chiral supermultiplet Φ coupled to a supergravity multiplet parametrized by the superfields $W_{\alpha\beta\gamma}$, R and G_a

The effective action should reproduce the anomalies written in terms of the supergravity multiplet. The effective action which reduces for the U(1) component to the standard action is:

$$
\begin{aligned}
& f^2 \int d^8 z\, E^{-1} e^{-\Phi} e^{-\bar{\Phi}} \\
& + 2(c-a) \int d^8 z (E^{-1}/R)\, \Phi\, W^{\alpha\beta\gamma} W_{\alpha\beta\gamma} + c.c. \\
& - 2a \int d^8 z\, E^{-1} \Big\{ (\Phi + \bar{\Phi})(G^a G_a + 2R\bar{R}) - \frac{1}{2} G_{\alpha\dot{\alpha}}\, \bar{\mathcal{D}}^{\dot{\alpha}} \bar{\Phi}\, \mathcal{D}^{\alpha} \Phi \\
& \qquad - \frac{1}{4} \Big(R(\mathcal{D}\Phi)^2 + \bar{R}(\bar{\mathcal{D}}\bar{\Phi})^2 \Big) - \frac{i}{4} \mathcal{D}^{\alpha\dot{\alpha}} (\Phi - \bar{\Phi})\, \bar{\mathcal{D}}_{\dot{\alpha}} \bar{\Phi}\, \mathcal{D}_{\alpha} \Phi + \frac{1}{16} (\mathcal{D}\Phi)^2 (\bar{\mathcal{D}}\bar{\Phi})^2 \Big\}
\end{aligned}
$$

The dilaton component of the supersymmetric action , which by our definition is the “minimal” solution is given by:

$$W_{\rm loc} = -f^2 \int d^4x \sqrt{g} e^{-2\tau} \left(\partial^m \tau \partial_m \tau - \frac{1}{6} R \right) - \int d^4x \sqrt{g} \Big\{ a\tau E_4 + c\tau C^{mnpq} C_{mnpq} + 2a \left(R^{mn} - \frac{1}{2} R g^{mn} \right) \partial_m \tau \partial_n \tau + 4\,a\,(\partial^m \tau \partial_m \tau)\,\Box\tau - 2\,a\,(\partial^m \tau\, \partial_m \tau)^2 \Big\}$$

It is given in terms of the “Goldstone coupling” and the trace anomaly coefficients . It contains a “Wess-Zumino term ” i.e. a self interaction term of the dilaton when the external metric is flat.

Integrating out the dilaton one obtains an effective action in terms of the metric only , reproducing the trace anomalies : it is related to various effective actions in the literature which therefore describe always the broken phase.

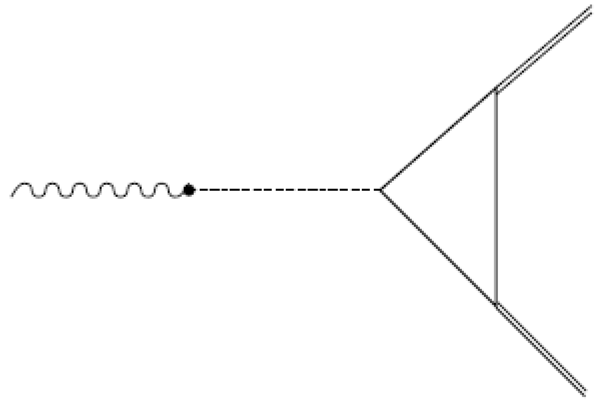
4)Checks of Trace Anomaly Matching.

The “Coulomb branch “ of N=4 SUSY YM offers the possibility to check the matching both at weak coupling using perturbation theory and at strong coupling using AdS/CFT .

On the Coulomb branch some of the scalars can get expectation values without breaking N=4 SUSY . The simplest pattern is when the global(R) symmetry is broken from SO(6) to SO(5) .

a) Weak Coupling

The dilaton is one of the scalars which stays massless .It has couplings to the scalars and fermions which became massive. Through the massive loops the dilaton develops couplings to two energy momentum tensors such that its contribution to the anomaly comes from the diagram:



The coupling to the two energy momentum tensors behaves at 0-momentum as :

$$g^2 v \frac{\bar{c}}{m^2} = g^2 v \frac{\bar{c}}{g^2 v^2} = \frac{\bar{c}}{v}$$

and the linear coupling to the energy momentum tensor cancels the denominator . The coefficient matches the anomaly coefficient since the diagram can be thought of as the infrared regulator of the anomaly through the massive loop.

b)Strong Coupling

Bianchi,Freedman,Skenderis(hep-th/0105276) showed that the Coulomb branch of N=4 corresponds to a domain wall solution in AdS.Their explicit calculation of the trace anomalies is consistent with Anomaly Matching.

Using the domain wall solution we calculate the Dilaton Effective Action which matches the expression obtained above.