Transport properties of high T_c superconductor at very low temperature and AdS/CFT

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Based on :

Work in progress with E. Kiritsis, C. Panagopoulos,
arXiv:1012.3464 with E. Kiritsis, C. Panagopoulos,
arXiv:1008.3286 with D. Yamada

- Holographic applications toward CM has been very active :
  - providing new directions to CM experiments,
    e.g. zero temperature entropy. Science 325, 1360 (2009)
  - building up intuitions for the strongly coupled field theories.
  - showing potentials for explaining some known parts of various strong coupling CM problems.
  - $\cdots$

- Can we provide a coherent picture of some challenging CM problems, such as high T_c superconductor?

Yes! normal-state transport properties of over-doped High T_c superconductor are captured by our holographic model .

- Comprehensive experimental transport data for normal-state HTS :

    i) Ohmic Resistivity,

    ii) Inverse Hall Angle : Ohmic resistivity / Hall resistivity,

    iii) Hall Coefficients : Resistivity / Magnetic field,

    iv) Magento-Resistance or (modified) Köhler ratio.

- We concentrate on one particular over-doped HTS
  $La_{2-x}Sr_xCuO_4$ (LSCO)
  and also $Tl_2Ba_2CuO_{6+d}$ (Tl2201) for further reference.

Schrödinger holography

- Geometric realizations with dynamical exponent $z \neq 1$ are interesting :
  - direct generalization of the AdS/CFT to the non-relativistic setup
  - potential applications to strongly coupled CM systems
  - conformal Schrödinger ($z = 2$) and Lifshitz with general z.

  Son[2008], Balasubramanian McGreevy[2008] , Kachru Liu Mulligan[2008]

$$ds^2 = r^2 \left(d\vec{x}^2 - 2dx^+ dx^- - r^2 dx^{+2} \right) + \frac{dr^2}{r^2} \ .$$

- Developments for Schrödinger background :

  Hořava Melby-Thompson[2009], Guica Skenderis Taylor van Rees[2010]

  Herzog Rangamani Ross[2008], Rangamani Ross Son Thompson[2008]

  Maldacena Martelli Tachikawa[2008], Adams Balasubramanian McGreevy[2008].

- but, complicated ....
- not well understood due to degenerate boundary ...

AdS in Light-Cone as Schödinger holography

Goldberger[2008], Barbon Fuertes[2008], Maldacena Martelli Tachikawa[2008], Ammon Hoyos O'Bannon Wu[2010], Kim Yamada[2010]

$$ds^2 = \left(\frac{r}{\ell}\right)^2 \left[\frac{1-h}{4b^2}(dx^+)^2 - (1+h)dx^+dx^- + (1-h)b^2(dx^-)^2 + dy^2 + dz^2\right] + \left(\frac{\ell}{r}\right)^2 \frac{1}{h}dr^2 \,, \qquad h = 1 - \frac{r_H^4}{r^4} \,.$$

I) Light-Cone coordinate with a particular normalization:

$$x^+ = b(t+x) \,, \qquad x^- = \frac{1}{2b}(t-x) \,,$$

with scaling dimensions $[b] = -1,\ [x^+] = -2,\ [x^-] = 0$ for $z = 2$.
II) Identification of x^+, light-like coordinate, as time!
III) Projection onto a fixed momentum in the x^- direction.

- thermodynamic and known transport data are identical to Schrödinger .
- advantage : simple and well defined holographic renormalization .

AdS in Light-Cone Transport Data

- Using Karch-O'Bannon probe brane technique Karch O'Bannon[2007]

$$A_+ = E_b y + h_+(r)\ , \quad A_- = h_-(r)\ , \quad A_y = h_y(r)\ ,$$

- DC conductivity :

$$\sigma_{LC} = \sigma_0 \sqrt{\frac{J^2}{t^2 A(t)} + \frac{t^3}{\sqrt{A(t)}}}, \qquad A(t) = t^2 + \sqrt{1+t^4}$$

where

$$t = \frac{\pi \ell T b}{\sqrt{2b\tilde{E}_b}}, \quad J^2 = \frac{64\sqrt{2}\langle \tilde{J}^+ \rangle^2}{(\tilde{\mathcal{N}} b \cos^3\theta)^2 (2b\tilde{E}_b)^3}, \quad \sigma_0 = \tilde{\mathcal{N}} b \cos^3\theta \sqrt{2b\tilde{E}_b}$$

- Drag limit : $t \ll 1$, $J \gg 1$,

$$\rho_{LC} = \frac{1}{\sigma} \approx \frac{t}{J\sigma_0} = \frac{\pi \ell b \sqrt{\tilde{E}_b b}}{\langle \tilde{J}^+ \rangle}\ T\ .$$

- Using O'Bannon probe brane technique O'Bannon[2007]

$$A_+ = E_b y + h_+(r)\ , \quad A_- = h_-(r)\ ,$$
$$A_y = h_y(r)\ ,\ A_z = B_b y + h_z(r)$$

- Hall conductivity :

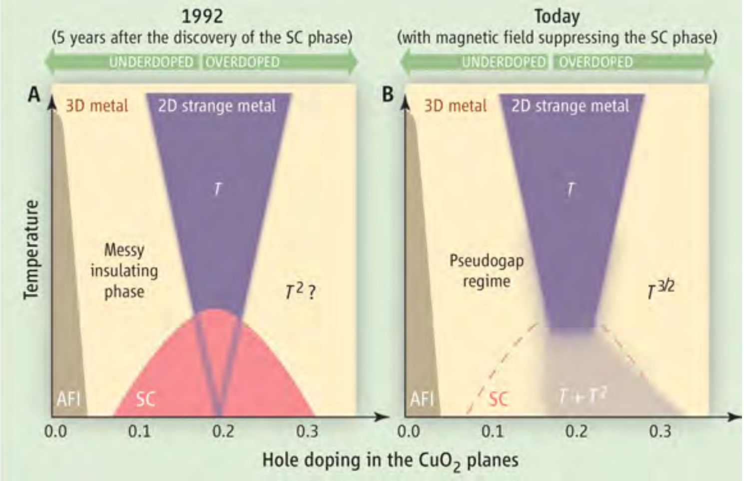
$$\sigma^{yy}_{LC} = \sigma_0 \frac{\sqrt{\mathcal{F}_+ J^2 + t^4\sqrt{\mathcal{F}_+\mathcal{F}_-}}}{\mathcal{F}_-}\ , \qquad \sigma^{yz}_{LC} = \bar{\sigma}_0 \frac{\mathcal{B}}{\mathcal{F}_-}\ \ ,$$

where

$$\mathcal{F}_\pm = \sqrt{\left(\mathcal{B}^2 + t^4\right)^2 + t^4} \mp \mathcal{B}^2 + t^4, \qquad \mathcal{B} = \frac{B_b}{2bE_b}, \qquad \bar{\sigma}_0 = \frac{\langle \tilde{J}^+ \rangle}{b\tilde{E}_b}\ .$$

- For $\mathcal{B} = 0$, the results reduce to the previous one.

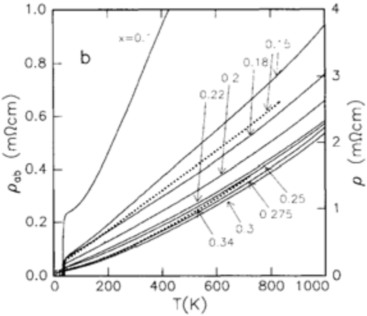
High T_c Superconductor : 25 years later



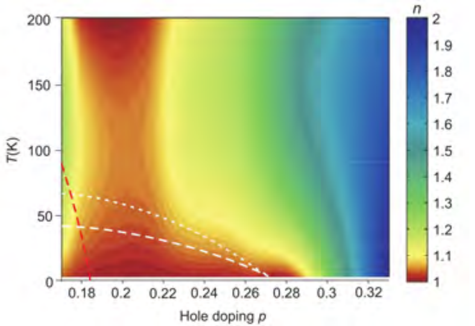
G.S. Boebinger, Science 323, 590 (**2009**)

- Suppress superconducting dome with *Zn*, *Co* or large magnetic field
- After 25 years, unexpected discovery at very low temperature!
- High T_c superconductor at over-doped side is very interesting!

Normal State LSCO : I. Resistivity $\rho \sim T^n$

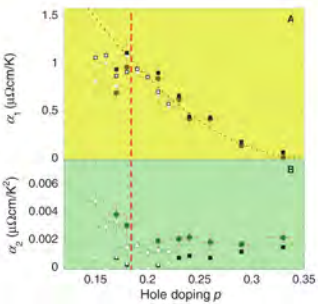


H. Takagi et al., PRL**69**, 2975 (1992)

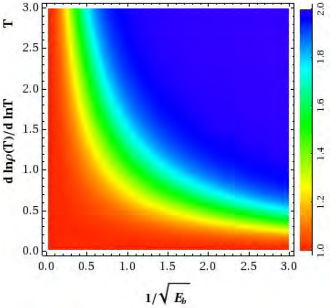


R.A. Cooper et al., Science 323, 603 (2009)

- 1992 : $T > 100K$, $\rho \sim T$ at optimal(OPD) and
  $\rho \sim T^2$ at over-doping(OD)
- Now : i) $\rho \sim T$: narrow region around OPD ($x = 0.17 \sim 0.19$)
  ii) $\rho \sim T + T^2$ at OD in the small temperature range
  iii) $\rho \sim T$ region is extended at very low temperature.



R.A. Cooper et al., Science 323, 603 (2009)



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- $\rho(T) = \alpha_0 + \alpha_1 T + \alpha_2 T^2$ for low T LSCO.

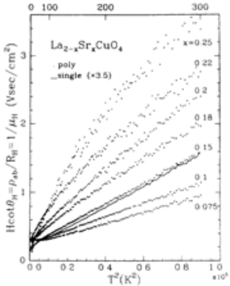
- $\rho_{LC,drag} \approx \frac{1}{J\sigma_0} t\sqrt{t^2 + \sqrt{1+t^4}}$,

  i) $\rho_1 = a_1 T = \frac{\sqrt{E_b}}{\ell\pi\sqrt{b}} \cdot a_2 T$, $\qquad \rho_2 = a_2 T^2 = \frac{\ell^2\pi^2 b^2}{\langle \tilde{J}^+ \rangle} T^2$

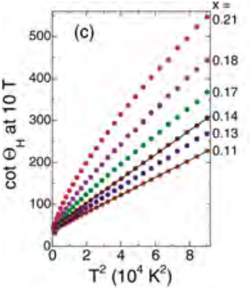
  ii) Input : a_2 : doping-independent,

  a_1 : rapidly decreasing with doping $\quad \Rightarrow \quad 1/\sqrt{E_b} =$ doping .

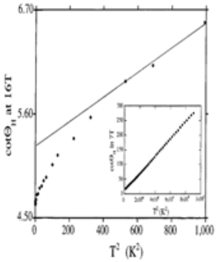
- This is further supported by long-range magnetic order in the UD side, which decrease as we increase doping. PRL **96**, 197001 (2006), Nature, **455**, 372 (1008)



H.Y. Hwang et al., PRL**72**, 2636 (1994)



Y.Ando et al., PRL**92**, 197001(2004)

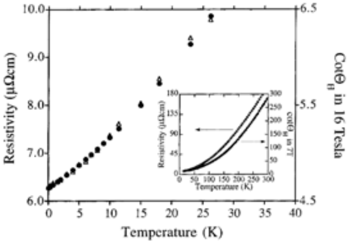


A.P.Mackenzie et al., PRB**53**,5848(1996)

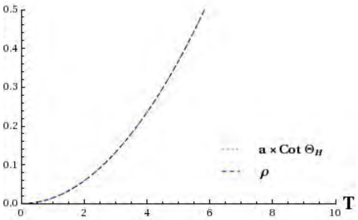
- 1994 & 2004 : $T > 100K$, $\cot \Theta_H \sim A + BT^2$ at UD,
  $\cot \Theta_H \nsim A + BT^2$ at OD

- At low temperature, systematic data of $\cot \Theta_H$ for LSCO is not available yet. Yet, these plots show more deviation from $\cot \Theta_H \sim T^2$

$$\Rightarrow \qquad \cot \Theta_H \sim T + T^2$$



A.P. Mackenzie et al., PRB **53**, 5848 (1996)



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- Very low temperature data for the $\cot\Theta_H$ available for over-doped TBCO with suppressed superconducting dome

$$\Rightarrow \qquad \cot\Theta_H \approx \rho \quad \sim \quad T + T^2 \; .$$

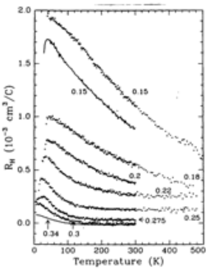
- $\cot\Theta_{H_{LC,drag}} \approx a \times \rho_{LC,drag} = \frac{B_b}{32\sqrt{2}\langle J^+\rangle}\rho_{LC,drag} \quad \sim \quad T + T^2$ is valid

  i) magnetic field in weak field, where $\frac{\Delta\rho}{\rho} \sim B_b^2$ and $\rho_{yz} \sim B_b$

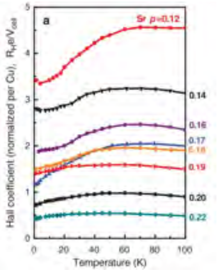
  $\Rightarrow \quad B_b \ll 1$ A.W. Tyler et al., PRB **57**, R728 (1998)

  ii) and in the small temperature regime

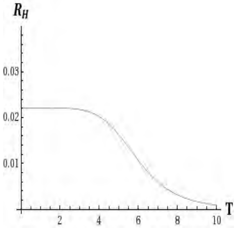
III. Hall Coefficient : $R_H = \rho_{xy}/B_b$



H.Y. Hwang et al., PRL**72**, 2636 (1994)



F.F.Balakirev et al.,PRL**102**,017004(2009)



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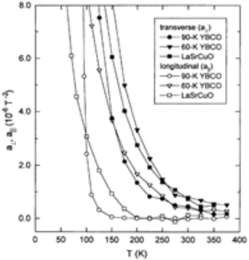
- In general, R_H depends on doping and on materials.

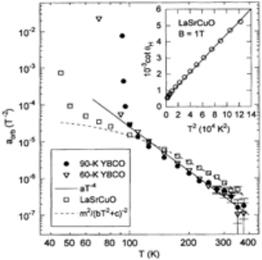
1995 : at $p = 0.22$(OD), R_H gradually decreases and becomes constant.
2009 : at $p = 0.22$(OD), R_H is constant for $T < 100K$.

- Typical plot for $R_{H_{LC,Drag}}$, which starts with some finite value at low T, decreases and then approaches to a constant value at high T.

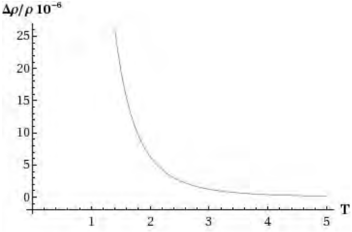
IV. Magnetoresistance : $\frac{\Delta\rho}{\rho} = \frac{\rho_{xx}(B_b) - \rho_{xx}(0)}{\rho_{xx}(0)}$



J.M. Harris et al., PRL**75**, 1391 (1995)



J.M. Harris et al., PRL**75**, 1391 (1995)



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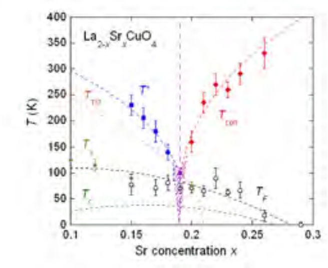
$\Uparrow$

- Magento-resistance data is available for $T > T_c$ with superconducting dome.
- $\left(\frac{\Delta\rho}{\rho}\right)_{YBCO} \sim \frac{1}{T^4}, \quad \left(\frac{\Delta\rho}{\rho}\right)_{LSCO} \sim \frac{1}{(aT^2+b)^2}.$

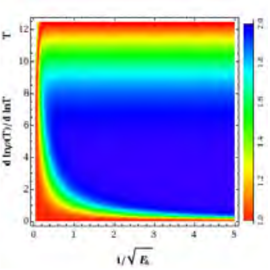
$\Leftarrow$

- $\left(\frac{\Delta\rho}{\rho}\right)_{LS,Drag} \sim \frac{1}{T^4+T^2}$ in weak field regime.

High temperature Transport Phase Diagram?



Hussey et al., Phil.Trans.RSoc.**A369**, 1687 (2010)



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- $\rho \sim T$ at high and low temperatures have different origins.
  - low T : associated with superconducting pairing mechanism,
  - high T : onset of incoherence due to quasi-particle.

• Plot generated with equal contribution of drag and pair-creation terms. These two different origins can be nicely connected to the Drag and pair-creation contributions in the probe brane approach.

Comparison to AdS$_5$ transport?

$$\sigma^2_{AdS} = \frac{\langle \tilde{J}^t \rangle^2}{\tilde{E}^2 + \pi^4 T^4} + \frac{N_f^2 N_c^2}{16\pi^2} \sqrt{\tilde{E}^2 + \pi^4 T^4} \cos^6\theta \ .$$

$$\sigma^2_{LC} = \frac{2\langle \tilde{J}^+ \rangle^2 / (\ell^2 \pi^2 T^2 b^2)}{\ell^2 \pi^2 T^2 b^2 + \sqrt{4\tilde{E}_b^2 b^2 + \ell^4 \pi^4 T^4 b^4}} + \frac{\ell^3 \tilde{\mathcal{N}}^2 \pi^3 T^3 b^5 \cos^6\theta}{32\sqrt{2\left(\ell^2 \pi^2 T^2 b^2 + \sqrt{4\tilde{E}_b^2 b^2 + \ell^4 \pi^4 T^4 b^4}\right)}} \ .$$

Conclusion and Outlook

- AdS in LC is an appropriate candidate for the Schrödinger holography : simple and well-defined holographic renormalization.
- We show that transport properties of AdS in LC fall into an universality class of over-doped high T_c superconductors, especially LSCO and TBCO.

- Under investigation of the Superconductiong phase transition for the AdS in LC.
- Constructing a holographic model capturing the transport properties of the under-doped side would be an exciting challenge.
- Coupling our model with fermions and investigating its properties will be interesting.
- Many more .....