Holographic Wilsonian RG —— flow diagrams, fermions

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based on

- Heemskerk-Polchinski arXiv:1010.1264
- Faulkner-Liu-Rangamani arXiv:1010.4036
- Elander-HI-Mandal arViv:1107.????

motivation

- Holographic WRG is an attempt to geometrize the Wilsonian approach to renormalization group, and relates IR(horizon) dynamics to UV(boundary) dynamics holographically

- One possible application is to explore IR effective holographic descriptions of strongly coupled systems —- phenomenological semi-holographic action for non-Fermi liquid [Faulkner-Polchinski], which consists of the action of the strongly interacting system with emergent dynamical fermions

example: free massive scalar

- background metric

$$ds^2 = g_{zz}(z)dz^2 + g_{\mu\nu}(z)dx^\mu dx^\nu$$

$$g_{zz}(z),\ g_{\mu\nu}(z) \sim z^{-2} \quad (z \sim 0)\ :\ \text{asymptotically AdS}_{d+1}$$

- action

$$S = \frac{1}{2}\int dz d^d x \sqrt{g}(g^{MN}\partial_M\phi\partial_N\phi + m^2\phi^2)$$

holographic WRG : canonical quantization in the radial direction

- We will formally write bulk path integrals as transition amplitudes with the radial coordinate regarded as the Euclidean time using the canonical quantization

- radial Hamiltonian & commutation relation

$$H_{\rm rad} = \int d^d x \left[\frac{1}{2\sqrt{g} g^{zz}} \pi^2 + \sqrt{g} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{m^2}{2} \phi^2 \right) \right]$$

$$[\phi, \pi] = i$$

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- these data reproduce the bulk path integral

$$\langle {\rm IR} | \widehat{\rm U}(\epsilon_{\rm IR}, \epsilon_0) | \phi_0 \rangle = \int [d\phi]_{\phi(\epsilon_0)=\phi_0,\, \phi(\epsilon_{\rm IR})=\phi_{\rm IR}} e^{-S}$$

$$\widehat{U}(\epsilon_2, \epsilon_1) := {\rm T} \exp - \int_{\epsilon_1}^{\epsilon_2} dz\, \widehat{H}_{\rm rad}$$

holographic WRG : assumptions

two assumptions

- radial coordinate $\epsilon \sim$ field theory energy scale Λ^{-1}
  (for pure AdS, near the boundary, $\epsilon = \Lambda^{-1}$)

  the precise relation between z and Λ in a general case is unknown, but that is not necessary in the following discussion

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- extend the GKP-W relation at the UV boundary ϵ_0 to that at a general cutoff ϵ:

$$\langle \mathrm{IR} | \widehat{U}(\epsilon_{\mathrm{IR}}, \epsilon) | \widetilde{\phi} \rangle = \left\langle \exp \int \widetilde{\phi} O_{\mathrm{s}} \right\rangle^{\mathrm{std}}_{\Lambda(\epsilon)}$$

holographic WRG : UV cutoff theory

- first, specify a UV initial state, equivalently, UV field theory

$$\langle \mathbf{IR}|\widehat{\mathbf{U}}(\epsilon_{\mathbf{IR}},\epsilon_{\mathbf{0}})|\Psi\rangle = \int [d\phi]\Big\langle \exp\int \phi_{\mathbf{0}} O\Big\rangle^{\mathbf{std}}_{\Lambda_{\mathbf{0}}} \langle \phi_{\mathbf{0}}|\Psi\rangle$$

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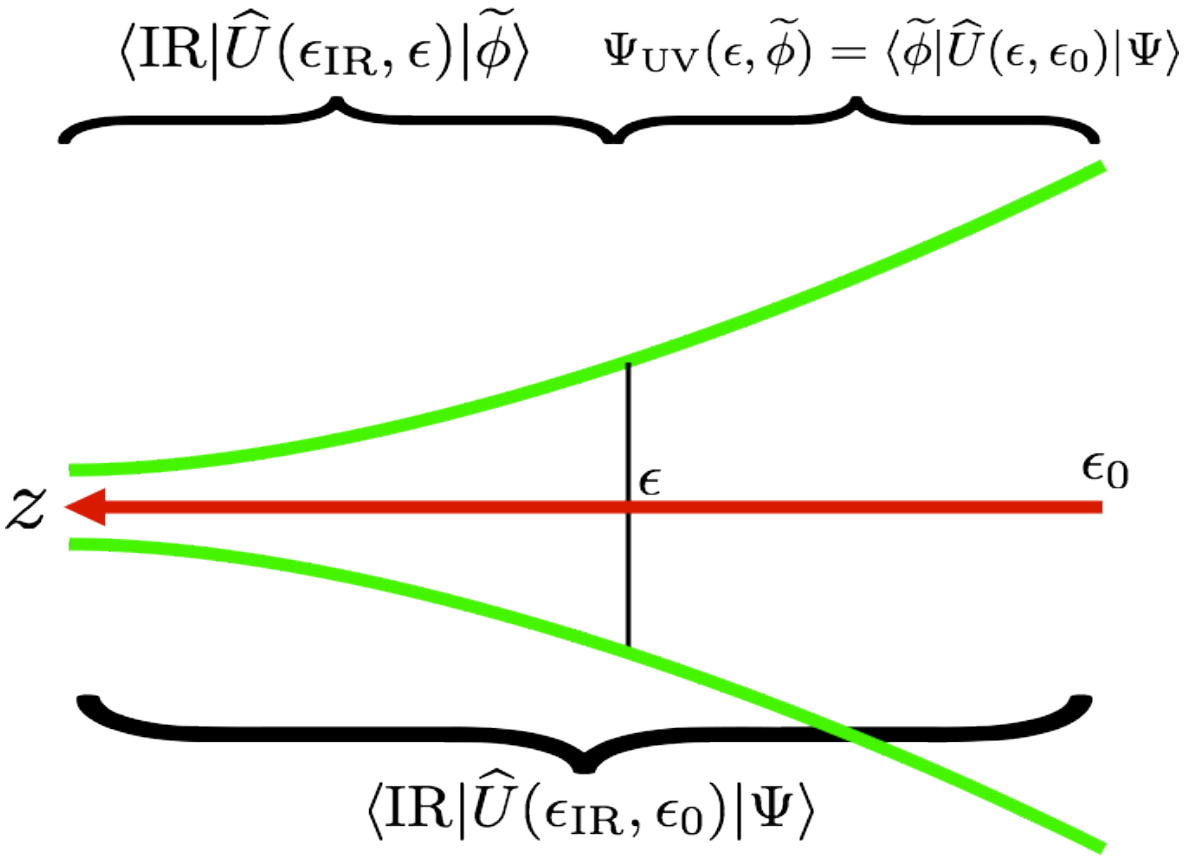
- Ψ specifies UV field theory
  - standard quantization : $|\Psi\rangle = |\phi\rangle \qquad \widehat{\phi}|\phi\rangle = \phi|\phi\rangle$
  - alternative quantization : $|\Psi\rangle = |\pi\rangle\rangle \qquad \widehat{\pi}|\pi\rangle\rangle = \pi|\pi\rangle\rangle$
  - multi-trace deformations, inclusion of bulk counter terms

holographic WRG : divide the full transition amplitude

- divide the full partition function (UV boundary theory) at an intermediate cutoff by inserting the completeness relation

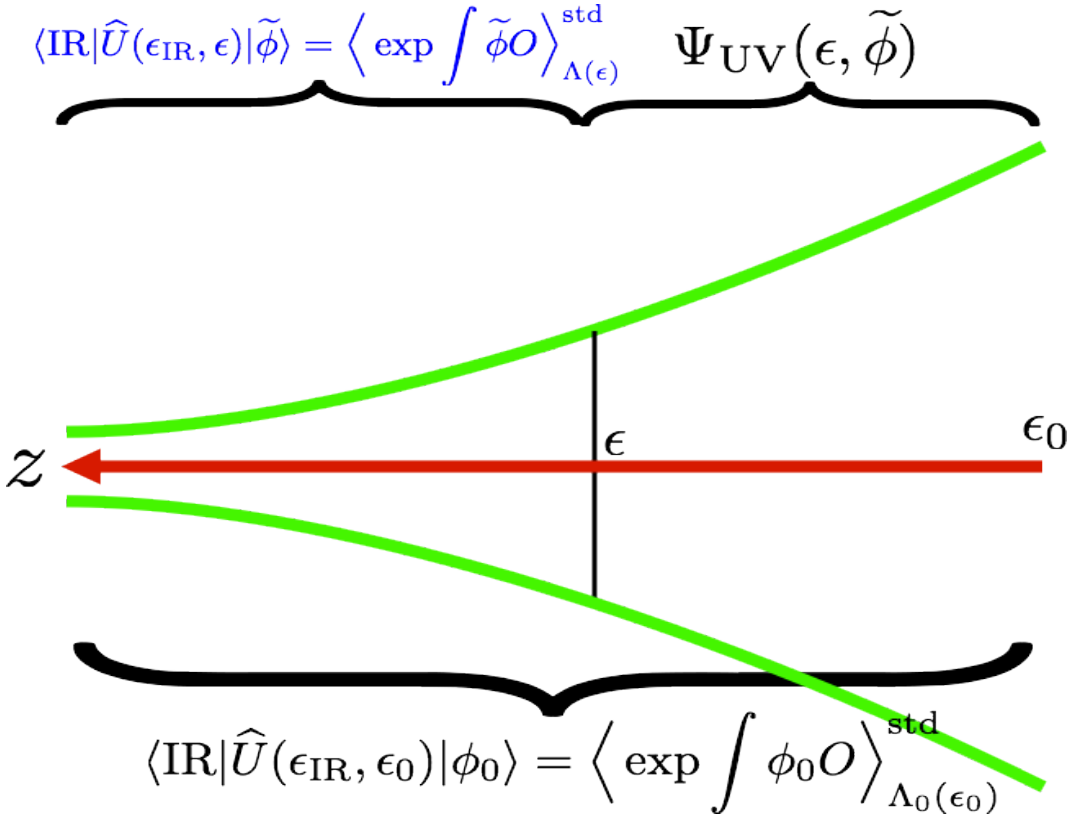
$$\langle \mathbf{IR}|\widehat{U}(\epsilon_{\mathbf{IR}},\epsilon_0)|\Psi\rangle = \int [d\widetilde{\phi}]\langle \mathbf{IR}|\widehat{U}(\epsilon_{\mathbf{IR}},\epsilon)|\widetilde{\phi}\rangle\langle\widetilde{\phi}|\widehat{U}(\epsilon,\epsilon_0)|\Psi\rangle$$



holographic WRG : connecting UV with IR

- replace the full and the IR transition amplitudes by generating functional

$$\langle \mathbf{IR}|\widehat{U}(\epsilon_{\mathbf{IR}},\epsilon_0)|\Psi\rangle = \int [d\widetilde{\phi}]\left\langle \exp\int \widetilde{\phi} O_{\mathbf{s}}\right\rangle_{\Lambda}^{\mathbf{std}} \Psi_{\mathbf{UV}}(\epsilon,\widetilde{\phi})$$



holographic WRG : connecting UV with IR

- for example, when UV is in the standard quantization $|\Psi\rangle = |\phi_0\rangle$,

$$\left\langle \exp \int \phi_0 O_s \right\rangle_{\Lambda_0}^{\mathrm{std}} = \int [d\widetilde{\phi}] \left\langle \exp \int \widetilde{\phi} O_s \right\rangle_{\Lambda}^{\mathrm{std}} \Psi_{\mathrm{UV}}(\epsilon, \widetilde{\phi})$$

- UV transition amplitude on the bulk side
  $\longleftrightarrow$ integration from UV cutoff to lower cutoff on the boundary side

UV wave functional and flows

- If the bulk action is quadratic, the UV amplitude generally becomes a Gaussian :

$$\Psi_{\mathrm{UV}}(\epsilon, \widetilde{\phi}) = \exp \int_k \sqrt{\gamma} \left[-\frac{1}{2} F(\epsilon) \widetilde{\phi}^2 + B(\epsilon) \widetilde{\phi} \right] + \int_k C(\epsilon)$$

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- Then, the integration over $\widetilde{\phi}$ at the intermediate cutoff gives rise to (up to O-independent terms)

$$\left\langle \exp \int \frac{1}{2\sqrt{\gamma} F(\epsilon)} O_{\mathrm{s}}^2 + \frac{B(\epsilon)}{F(\epsilon)} O_{\mathrm{s}} \right\rangle_{\Lambda(\epsilon)}^{\mathrm{std}}$$

this defines a flow of equivalent low energy effective theories, which we call holographic Wilsonian RG flow.

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- UV theory with single-trace deformation
  = IR theory with single- and double-trace deformations

  $F \sim$ double-trace deformation $\qquad B \sim$ single-trace deformation

  multi-trace deformation : $\widetilde{\phi}^m \longleftrightarrow O^m$

flow equations and beta functions

- the wavefunctional satisfies the Schrödinger equation with the radial Hamiltonian H_{rad}

$$\partial_\epsilon \Psi_{\mathrm{UV}}(\epsilon) = -H_{\mathrm{rad}} \Psi_{\mathrm{UV}}(\epsilon)$$

that gives rise to flow equations for coefficients F, B, C

$$\frac{1}{\sqrt{g}} \partial_\epsilon (\sqrt{\gamma} F) = -F^2 + g^{\mu\nu} k_\mu k_\nu + m^2$$

$$\frac{1}{\sqrt{g}} \partial_\epsilon (\sqrt{\gamma} B) = -BF$$

$$\frac{1}{\sqrt{g}} \partial_\epsilon C = \frac{1}{2} \int_k J^2 + O(\kappa^2)$$

flow equations and beta functions

- consider pure AdS$_{d+1}$: $g_{zz} = z^{-2}\,, \quad g_{\mu\nu} = z^{-2}\eta_{\mu\nu}$

- flow equation for F

$$\epsilon\partial_\epsilon F = -F^2 + dF + \epsilon^2 k^2 + m^2 \quad (k^2 := \eta^{\mu\nu}k_\mu k_\nu)$$

- interpret F as a collection of double-trace coupling constants

$$F(\epsilon, k) = \sum_{n=0}^{\infty} f_n(\epsilon)(\epsilon k)^{2n} \qquad f_n(\epsilon) \longleftrightarrow O\Box^n O$$

- flow equations for coefficients f_n

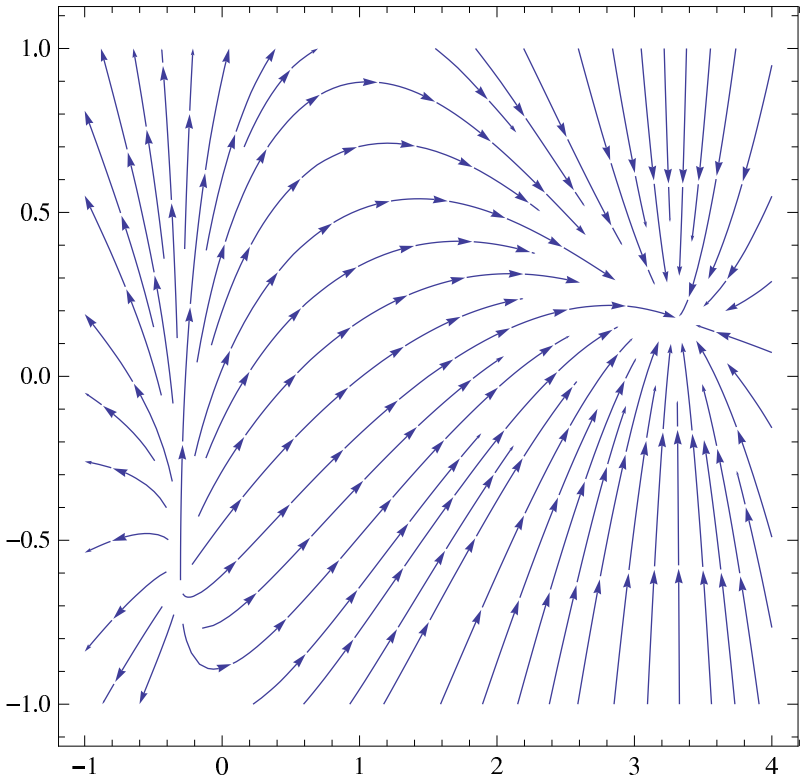
$$\begin{aligned}
\epsilon\partial_\epsilon f_0 &= -f_0^2 + df_0 + m^2 \\
\epsilon\partial_\epsilon f_1 &= (d-2)f_1 - 2f_0 f_1 + 1 \\
&\cdots
\end{aligned}$$

flow diagram f_0-f_1

the flow equation for f_0 exhibits two fixed points

- UV $\quad f_0 = \Delta_-,\ J \sim \epsilon^{\Delta_+}$: alternative quantization
- IR $\quad f_0 = \Delta_+,\ J \sim \epsilon^{\Delta_-}$: standard quantization

where $\Delta_\pm := \frac{d}{2} \pm \nu\,,\ \nu := \sqrt{\left(\frac{d}{2}\right)^2 + m^2}$



extremal black hole background

- metric : charged black hole

$$g_{ii} = \frac{1}{z^2}, \quad g_{tt} = \frac{H}{z^2}, \quad g_{zz} = \frac{1}{z^2 H},$$
$$H(z) = 1 + 3\left(\frac{z}{z_*}\right)^4 - 4\left(\frac{z}{z_*}\right)^3$$

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- expected fixed points
  - fixed points near the boundary $\longleftrightarrow$ AdS$_{d+1}$
  - fixed points near the horizon $\longleftrightarrow$ near horizon AdS$_2$ $\times\, \mathbb{R}^{d-1}$

black hole : flow equations and beta functions

- flow equation for F

$$\epsilon\partial_\epsilon(f\sqrt{H}) = d\, f\sqrt{H} - f^2 + \epsilon^2(k_i k_i + \omega^2/H) + m^2$$

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- derivative expansion of F

$$F(k,\epsilon) = \sum_{n,m} \bar{f}_{n,m}(\epsilon)(\epsilon^2 k_i k_i)^n (\epsilon\omega)^{2m}$$

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- regard the red shift factor H as an additional coupling to eliminate explicit ϵ-dependence from the RHS
(equivalent to regard H as a flow parameter)

$$\epsilon\partial_\epsilon \bar{f}_{0,0} = \left[d - \frac{\beta(H)}{2H}\right]\bar{f}_{0,0} + \frac{m^2 - \bar{f}_{0,0}^2}{\sqrt{H}}$$

$$\epsilon\partial_\epsilon H = \beta(H)$$

where $\beta(H)$ satisfies

$$(1 - H + \beta(H)/3)^3 = (1 - H + \beta(H)/4)^4$$

$f_{0,0}$-H diagram

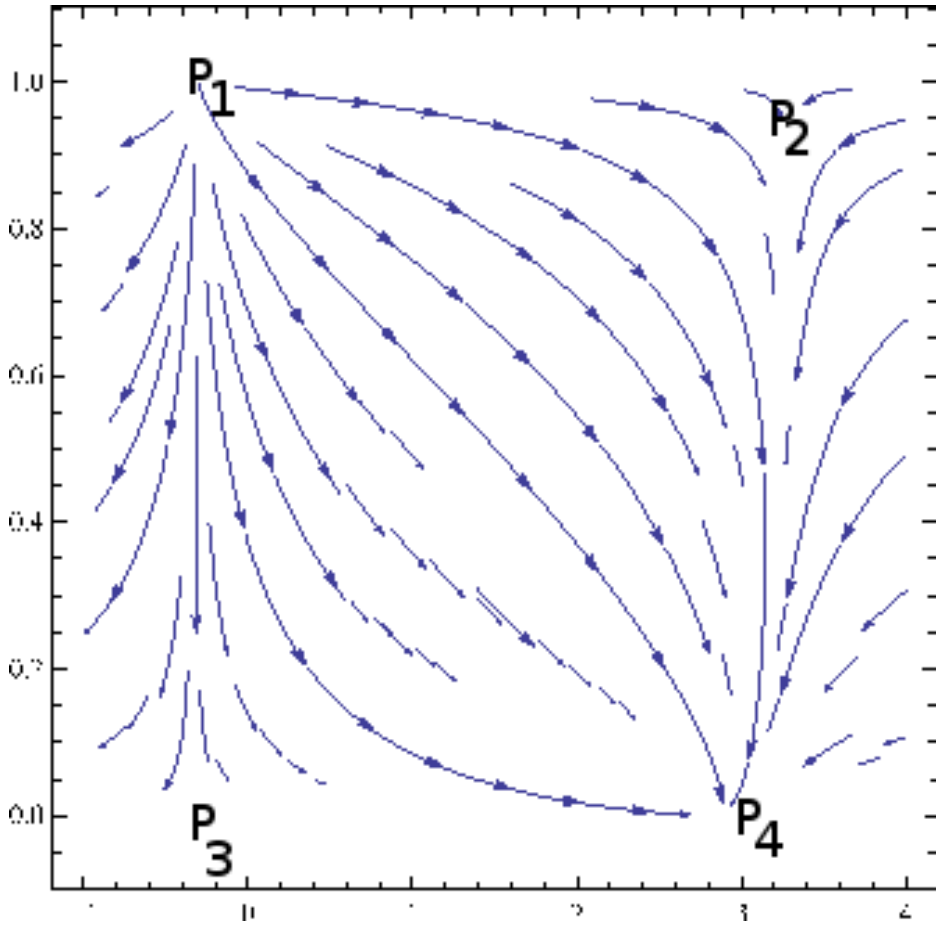

Figure: RG flow diagram in the $\bar{f}_{0,0}$-H plane which is plotted for $d = 3, m = 1$. P1 and P2 are UV fixed points, P3 and P4 are IR fixed points.

four fixed points

- at the UV boundary ($d = 3$) : $H = 1$

$$f_{0,0} = \frac{3}{2} \pm \sqrt{\left(\frac{3}{2}\right)^2 + m^2 R_{AdS_4}^2}$$

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$$f_{0,0} = \frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 + \frac{m^2 R_{AdS_4}^2}{6}}$$

the last expression coincides with $f_{0,0}$ at two fixed points in near horizon $\mathrm{AdS}_2 \times \mathbb{R}^2$ geometry

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- note 1: $f_{0,0} = \frac{d}{2} \pm \sqrt{\left(\frac{d}{2}\right)^2 + m^2 R^2_{AdS_{d+1}}}$ for AdS_{d+1}
- note 2: $R_{AdS_2} = \frac{R_{AdS_{d+1}}}{\sqrt{d(d-1)}}$ as the NH geometry of AdS_{d+1} BH

FERMIONS

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- Main difference from the scalar case is that the Dirac action is in first order and thus $\Psi, \overline{\Psi}$ are both the coordinates and the momenta. In order to solve this second-class constraint, it is convenient to decompose $\Psi, \overline{\Psi}$ in terms of the chirality.

$$\Psi = \begin{pmatrix} \Psi_+ \\ \Psi_- \end{pmatrix}, \quad \overline{\Psi} = \begin{pmatrix} \overline{\Psi}_+ & \overline{\Psi}_- \end{pmatrix}$$

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- canonical quantization with the radial coordinate as the Euclidean time

$$\{\chi_\pm, \overline{\chi}_\pm\} = \pm 1 \,, \qquad \chi := (g g^{zz})^{1/4} \Psi$$

$$H_{\rm rad} := \int_k \sqrt{g_{zz}} \left[i \overline{\chi}_+ \gamma^\mu k_\mu \chi_- + i \overline{\chi}_- \gamma^\mu k_\mu \chi_+ + m \chi_+ \overline{\chi}_+ - m \overline{\chi}_- \chi_- \right]$$

initial states for standard quantization

- standard quantization : fix non-normalizable modes, which are

$$\overline{\chi}_+ \sim z^{\frac{d}{2}-m}\overline{\chi}_+^0, \quad \chi_- \sim z^{\frac{d}{2}-m}\chi_-^0$$

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- Thus, following the scalar case, we choose an eigenstate of $(\overline{\chi}_+, \chi_-)$ as an initial state

$$\widehat{\overline{\chi}}_+|\overline{\chi}_+, \chi_-\rangle = \overline{\chi}_+|\overline{\chi}_+, \chi_-\rangle$$
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- With this choice, a transition amplitude starting from the initial state correctly reproduce the correct boundary action for the standard quantization:

$$\langle \mathbf{IR}|\widehat{U}(\epsilon_{\mathbf{IR}}, \epsilon)|\overline{\chi}_+, \chi_-\rangle = \int [d\chi] \exp\left(S[\chi] + \int_{\mathbf{bdy}} \overline{\chi}_+\chi_+\right)$$

GKP-W relation

- Thus, the GKP-W relation for the standard quantization is

$$\langle \mathbf{IR}|\widehat{U}(\epsilon_{\mathbf{IR}},\epsilon)|\overline{\chi}_+,\chi_-\rangle = \left\langle \exp\int \overline{O}_-\chi_- + \overline{\chi}_+ O_+ \right\rangle^{\mathbf{std}}_{\Lambda(\epsilon)}$$

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- for alternative quantization, we just have to replace non-normalizable modes by normalizable modes

$$(\overline{\chi}_+,\chi_-) \longmapsto (\chi_+,\overline{\chi}_-) \qquad (O_+,\overline{O}_-) \longmapsto (\overline{O}_+,O_-)$$

UV amplitude and flow equations

- divide the full partition function

$$\langle \mathbf{IR}|\widehat{U}(\epsilon_{\mathbf{IR}},\epsilon_0)|\Psi_0\rangle$$
$$= \int [d\chi]\, \langle \mathbf{IR}|\widehat{U}(\epsilon_{\mathbf{IR}},\epsilon)|\overline{\chi}_+,\chi_-\rangle e^{\int \overline{\chi}_+\chi_+ + \overline{\chi}_-\chi_-}\, \Psi_{\mathbf{UV}}(\epsilon;\chi_+,\overline{\chi}_-)$$

where the UV amplitude is

$$\Psi_{\mathbf{UV}}(\epsilon;\chi_+,\overline{\chi}_-) := \langle \chi_+,\overline{\chi}_-|\widehat{U}(\epsilon,\epsilon_0)|\Psi_0\rangle$$

and the completeness relation inserted is

$$1 = \int [d\chi]|\overline{\chi}_+,\chi_-\rangle e^{\int \overline{\chi}_+\chi_+ + \overline{\chi}_-\chi_-}\langle \chi_+,\overline{\chi}_-|$$

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- the UV amplitude is generally given by

$$\Psi_{\mathbf{UV}} = \exp \int \overline{\chi}_- F(\epsilon)\chi_+ + \overline{B}_+(\epsilon)\chi_+ + \overline{\chi}_- B_-(\epsilon) + C(\epsilon)$$

UV amplitude and flow equations

- a flow of physically equivalent effective theories

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- flow equations from the Schrödinger equation $-\partial_\epsilon \Psi_{\mathrm{UV}} = \widehat{H}_{\mathrm{rad}} \Psi_{\mathrm{UV}}$

$$\begin{aligned}
\sqrt{g^{zz}}\partial_\epsilon F &= F(i\gamma^\mu k_\mu)F + i\gamma^\mu K_\mu - 2mF\,, \\
\sqrt{g^{zz}}\partial_\epsilon B_- &= F(i\gamma^\mu k_\mu)B_- - mB_-\,, \\
\sqrt{g^{zz}}\partial_\epsilon \overline{B}_+ &= \overline{B}_+(i\gamma^\mu k_\mu)F - m\overline{B}_+\,, \\
\sqrt{g^{zz}}\partial_\epsilon C &= \overline{J}_+(i\gamma^\mu k_\mu)J_- + O(\kappa^2)\,,
\end{aligned}$$

flow equations and beta functions

- expand F in momentum (derivative expansion)

$$F(\epsilon, k) = \sum_{n=0}^{\infty} f_n(\epsilon)(\epsilon\gamma^{\hat{\mu}} k_\mu)^{n-1}$$

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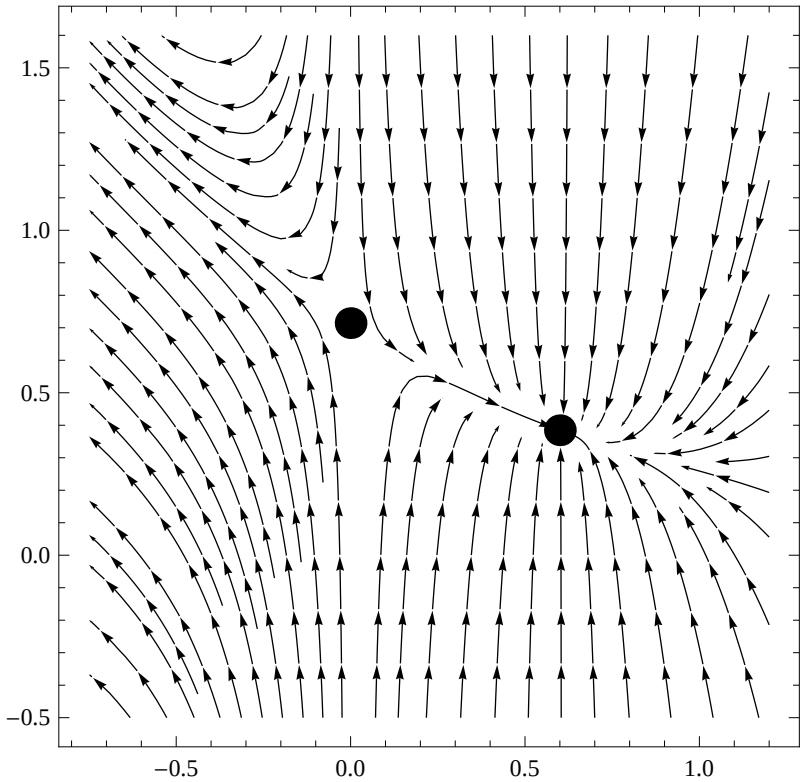
- flow equations for f_0-f_1

$$\epsilon\partial_\epsilon f_0 = (1-2m)f_0 - f_0^2$$
$$\epsilon\partial_\epsilon f_1 = 2f_0 f_1 - 2m f_1$$
$$\cdots$$

flow diagram : pure AdS



fixed points (in KW window : $0 < m < 1/2$)

- UV : $f_0 = 0$: standard quantization
- IR : $f_0 = 1 - 2m$: alternative quantization

flow diagram : extremal charged black hole

In the same way as the bosonic case, we regard the red shift factor H as a new coupling and draw a flow diagram in f_{00}-H

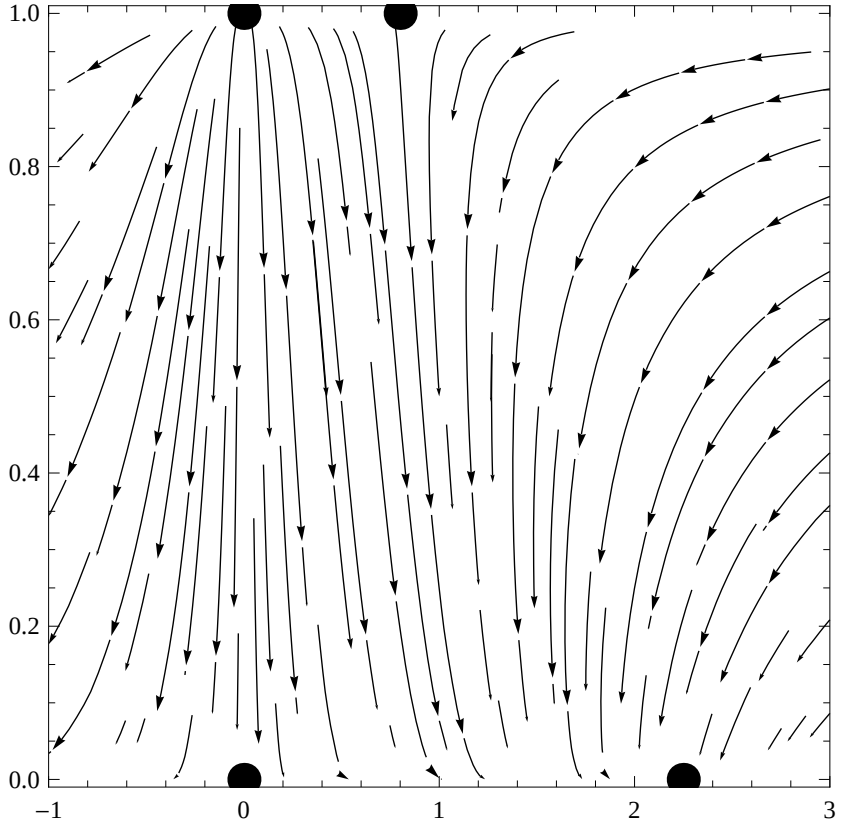


Figure: $f_{0,0}$-H diagram with $m = 0.2$

conclusion and open problems

what we have done

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- analyzed flow equations for extremal charged black holes and checked that the possible four fixed points give us the correct mass dimensions.
- extended holographic WRG to fermions by introducing generalized coherent states.

conclusion and open problems

We can partially understand the emergence of the semi-holographic action

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$$\int [d^4\chi]\, \langle \mathrm{IR}|\widehat{U}(\epsilon_{\mathrm{IR}},\epsilon)|\overline{\chi}_+,\chi_-\rangle e^{\int \overline{\chi}_+\chi_+ + \overline{\chi}_-\chi_-} \langle \chi_+,\overline{\chi}_-|\widehat{U}(\epsilon,\epsilon_0)|\Psi\rangle$$

$$= \int [d^2\chi] \left\langle \exp \int \overline{\chi}_+ O_+ + \overline{O}_- \chi_- \right\rangle^{\mathrm{std}}_{\Lambda(\epsilon)} \exp - \int \overline{\chi}_+ F^{-1} \chi_- + \cdots$$

$$= \left\langle \exp \int \overline{O}_- F O_+ + \cdots \right\rangle^{\mathrm{std}}_{\Lambda(\epsilon)}$$

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$$= \int [d^2\chi] \left\langle \exp \int \overline{\chi}_+ O_+ + \overline{O}_- \chi_- \right\rangle^{\mathrm{std}}_{\Lambda(\epsilon)} \exp - \int \overline{\chi}_+ F^{-1} \chi_- + \cdots$$

$$= \left\langle \exp \int \overline{O}_- F O_+ + \cdots \right\rangle^{\mathrm{std}}_{\Lambda(\epsilon)}$$

3. the last expression becomes non-local around k_F. In order to avoid this, keep χ of momentum in the neighbourhood of k_F <u>*un-integrated out*</u>

$$\left\langle \exp \int_{k\in S_F} \overline{\chi}_+ O_+ + \overline{O}_- \chi_- \right\rangle^{\mathrm{std}}_{\Lambda(\epsilon)} \exp - \int_{k\in S_F} \overline{\chi}_+ F^{-1} \chi_-$$

$$= \int [dM] \exp S[M] + \int_{k\in S_F} \overline{\chi}_+ O_+ + \overline{O}_- \chi_- - \overline{\chi}_+ \gamma^{\hat{\mu}} (k-k_F)_\mu \chi_-$$

$\chi_-, \overline{\chi}_+$ can be interpreted as emergent dynamical fermions

conclusion and open problems

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- hWRG is now in the level of re-interpretation...
  Can hWRG really shed new light on issues which cannot be understood by conventional holographic techniques ??