Spin Models and Gravity

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- Paramagnet $T > T_c$ to ferromagnet transition $T < T_c$ as the system cools down.
- The $U(1)$ XY model in 2D (Kosterlitz-Thouless model) or 3D and the "$O(3)$ quantum rotor" in 3D, the Hubbard model. etc $\Rightarrow$ canonical models for super-fluidity/super-conductivity.

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- Non-trivial critical exponents at T_c only computable by Monte-Carlo for $D > 2$.

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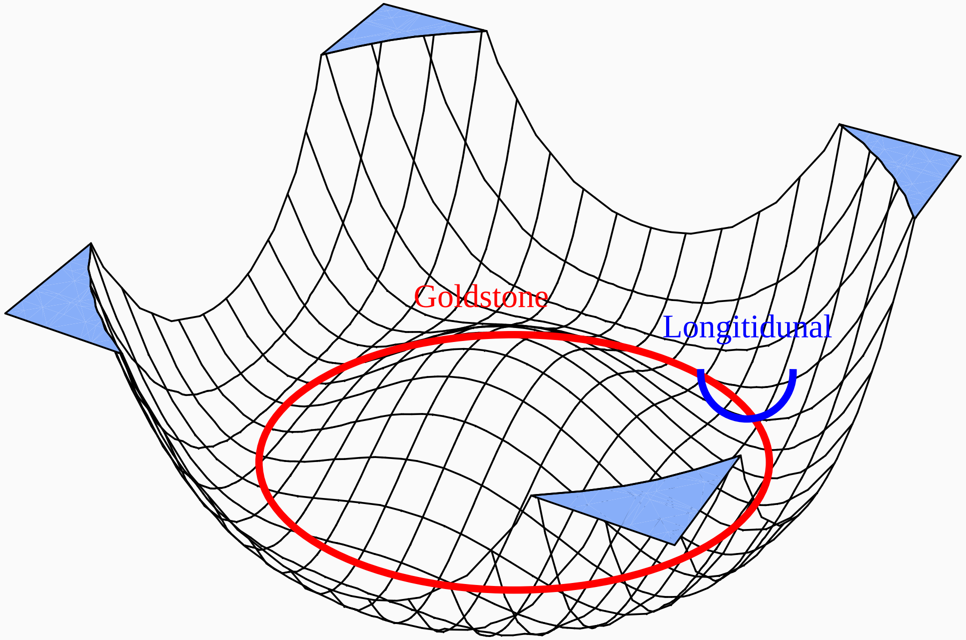
Zero modes of the system:

1. Goldstone mode (phase fluctuations) for $T < T_c$.
2. Longitudinal flat direction arises as $T \to T_c$



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- Gauge theories $\Rightarrow$ GR!
- A new approach to holographic super-fluids/super-conductors

Outline

- Mapping spin-models to gauge theories
- Realization of confinement-deconfinement transition in GR
- Continuous Hawking-Page transitions in GR
- Near transition region: linear-dilaton background
- Calculation of observables:
  - Second-sound
  - Spin-spin correlators from string theory
- Discussion

Lattice gauge theory and Spin-models

Polyakov ’78; Susskind ’79

Lattice gauge theory and Spin-models

Polyakov '78; Susskind '79

- Any LGT with *arbitrary* gauge group G in d-dimensions with arbitrary *adjoint matter*
- Integrate out gauge invariant states $\Rightarrow$ generate effective theory for the Polyakov loop
- $Z_{LGT}(P;T) \sim Z_{SpM}(\vec{s};T^{-1})$
- Ferromagnetic spin model $\mathcal{H} = -J\sum_{\langle ij\rangle} \vec{s}_i \cdot \vec{s}_j + \cdots$ in $d-1$ dimensions with spin symmetry $C = Center(G)$

- Inversion of temperature:
  Deconfined (high T) phase in LGT $\Leftrightarrow$ Ordered (low T) phase of Spin-model
  Confined (low T) phase in LGT $\Leftrightarrow$ Disordered (high T) phase of Spin-model

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$$\langle P^*(L)P(0)\rangle_{conf} \sim e^{-mL}, \qquad \langle P(x)\rangle = 0$$

$$\langle P^*(L)P(0)\rangle_{deconf} \sim 1 + e^{-mL}, \qquad \langle P(x)\rangle = 0$$

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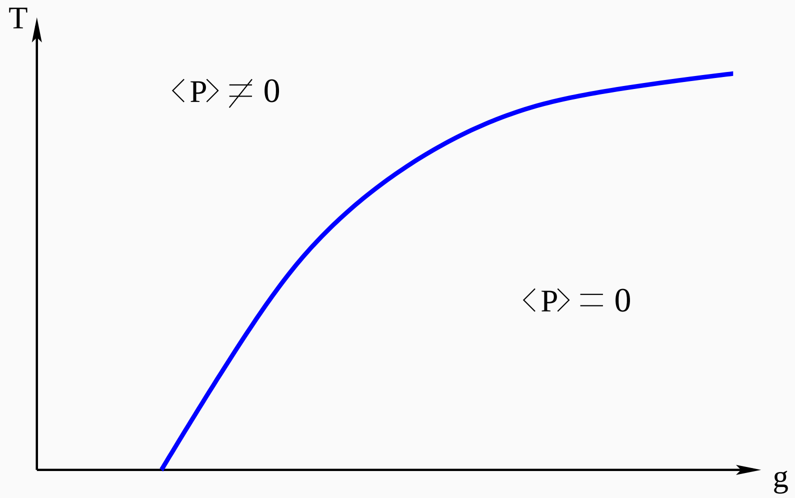

(*) Polyakov loop

$P \propto \prod_{n=0}^{N_t-1} U_{\vec{r}+n\hat{t},0}$ is the order parameter

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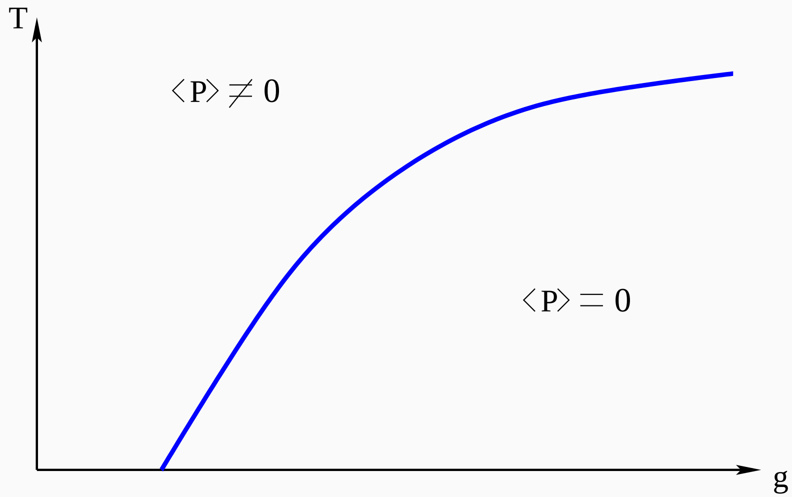

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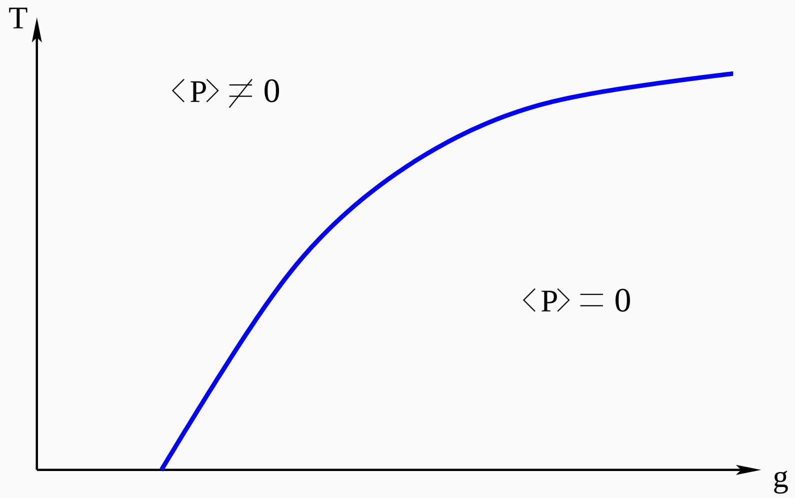

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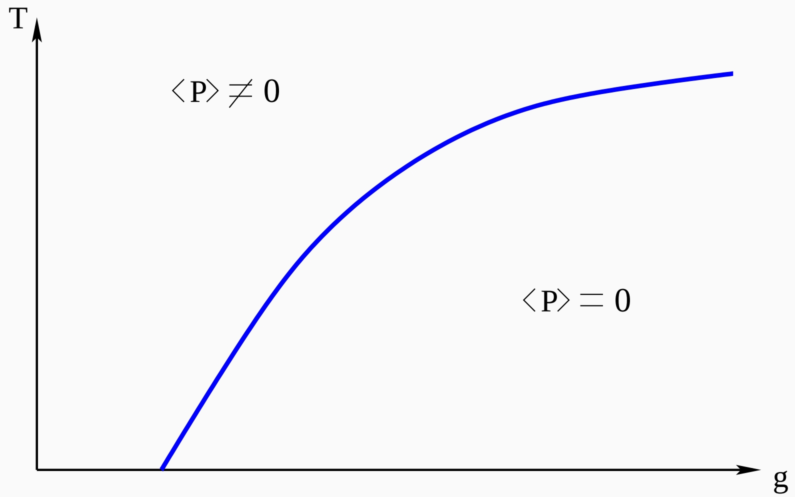

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- No long-range magnetic fluctuations $\Rightarrow$ integrate out $U_{\vec{r},j}$
- The resulting theory $\mathcal{L}[P]$ describes long-range fluctuations at criticality
- Polyakov '78; Susskind '79: Can be mapped onto a spin-model with $P \Leftrightarrow \vec{s}$ (explicitly shown in the limit $g \gg 1$)

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- Some examples:
  1. Pure $SU(2)$ in $d = 4$ second order transition with Z_2 (Ising) critical exponents,
  2. $SU(N)$ in d dimensions with $N > \frac{2d}{d-2}$,
     Spin model with Z_N fixed point flows to a $U(1)$ XY model
     $\mathcal{L} \sim |\partial\Phi|^2 + \Phi^N + \Phi^{*N} + |\Phi|^2$ so the mass-term is relevant for $N > 2d/(d-2)$: $Z_N \to U(1)$.
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- Focus on
  $SU(N)$ with $N \to \infty$,
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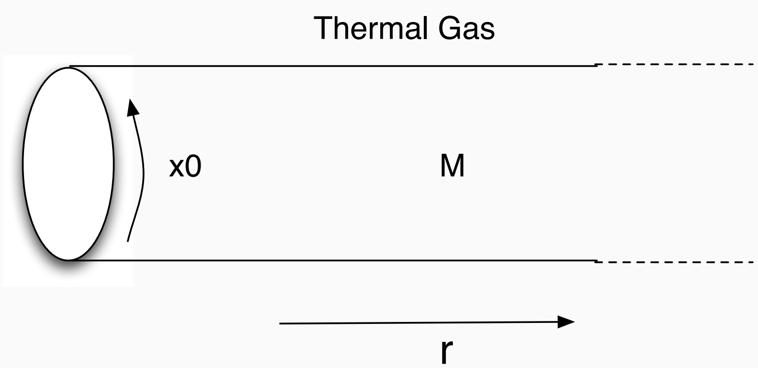

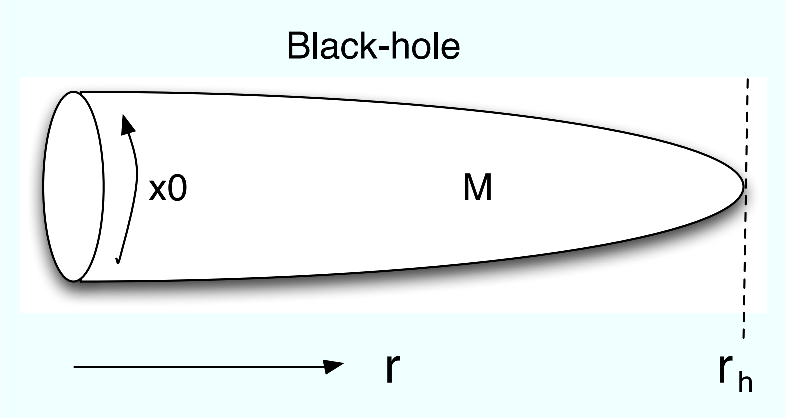

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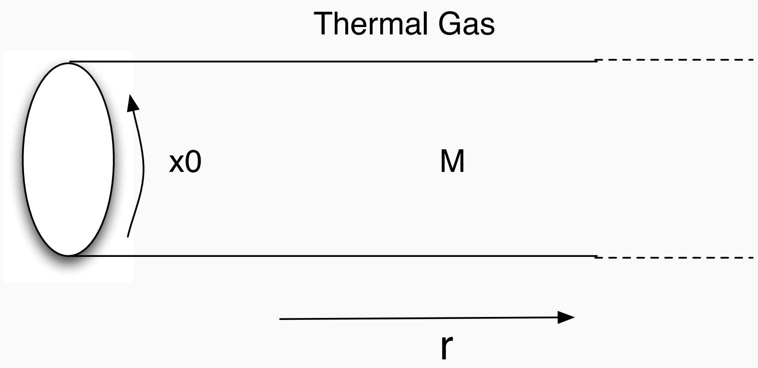

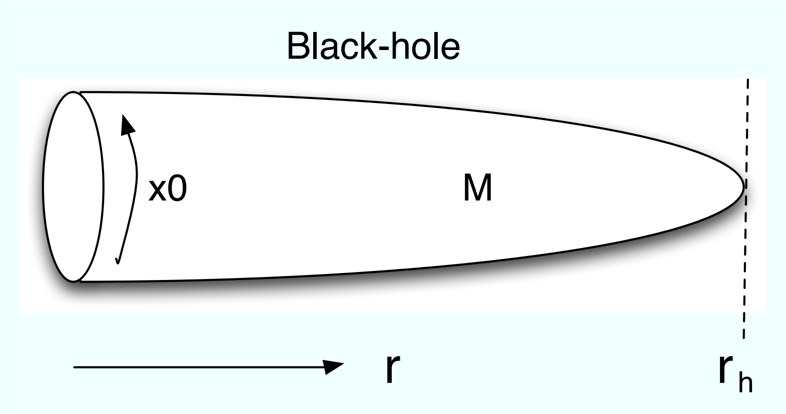

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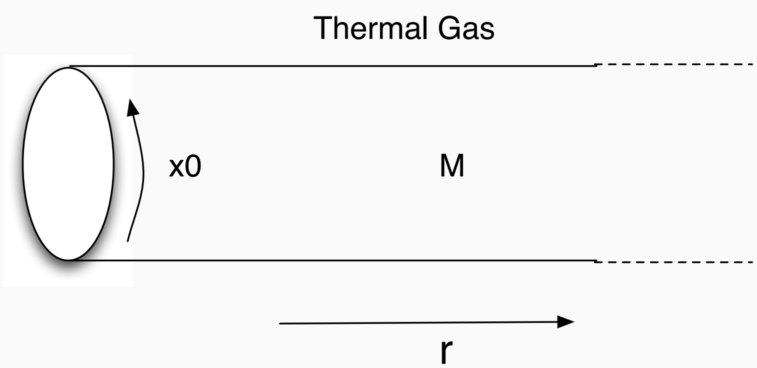

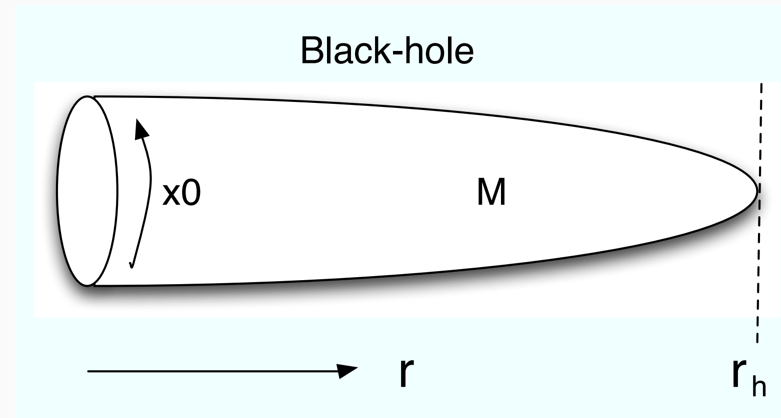

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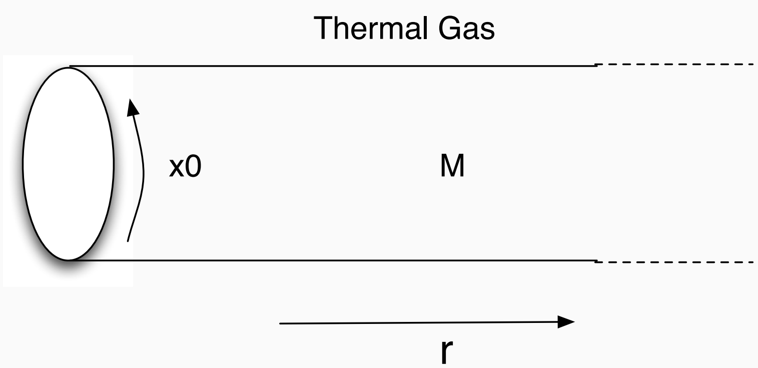

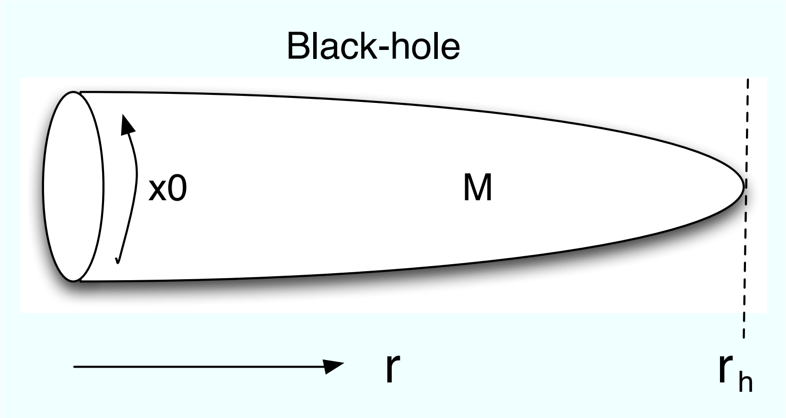

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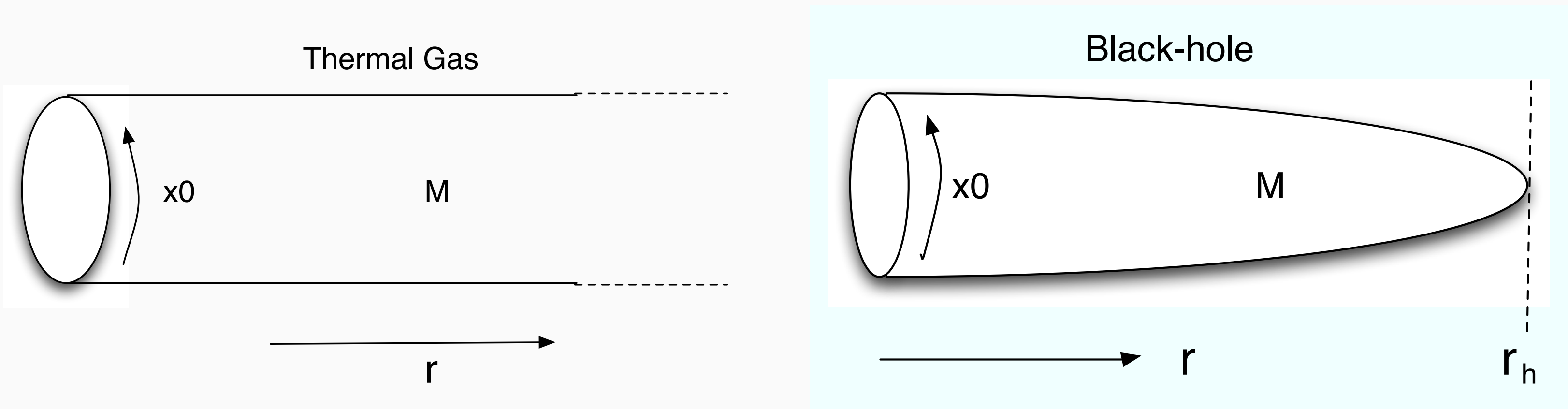

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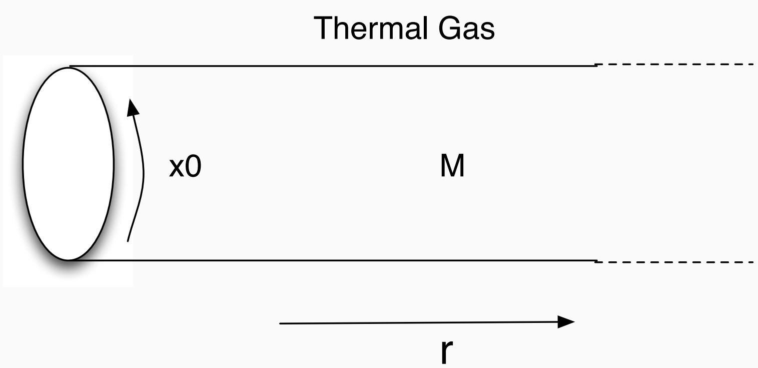

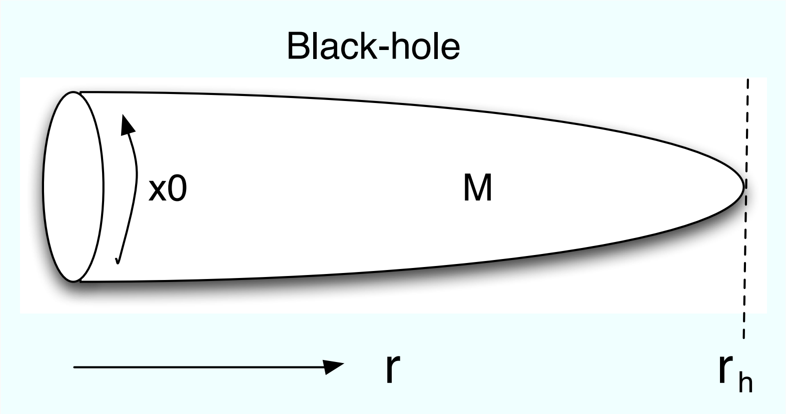

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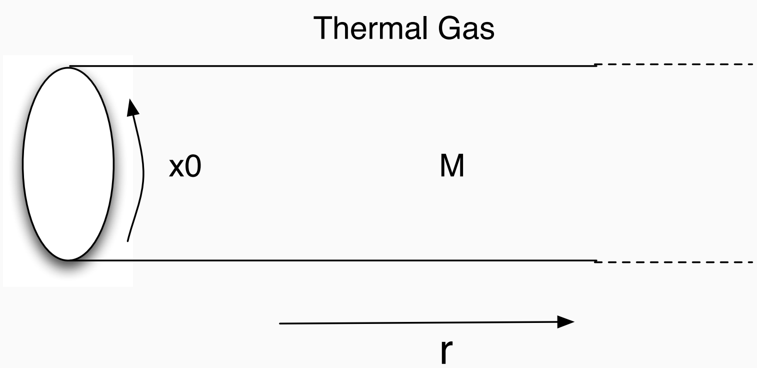

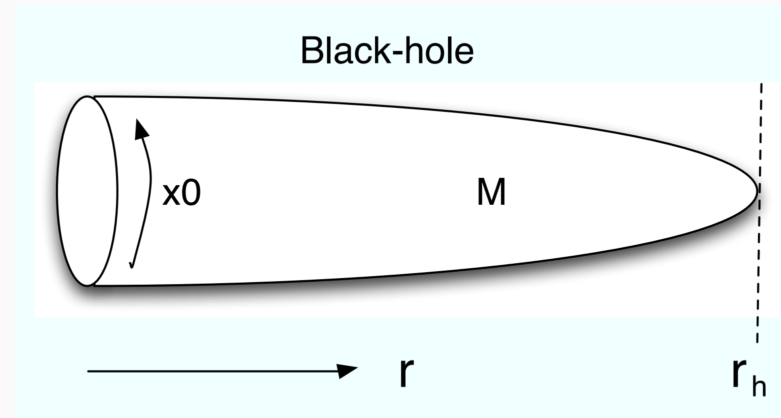

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- Fluctuations $\delta\Psi \Leftrightarrow$ Goldstone mode in the dual spin-model

Identification of the symmetries

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	Gravity	Gauge theory		Spin model	T
T_c	$BH, \not{U}(1)_B$	$Deconf.\ \not{U}(1)_{\mathcal{C}}$		$S.fluid\ \not{U}(1)_S$	T_c^{-1}
T	$TG, U(1)_B$	$Conf.\ U(1)_{\mathcal{C}}$		$Normal\ U(1)_S$	

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GRAVITY/SPIN-MODEL CORRESPONDENCE

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Requirements for a second order Hawking-Page transition:

i.) There is a finite T_c at which:

ii.) $\Delta F(T_c) = 0$. TG(BH) dominates for $T < T_c$ $(T > T_c)$.

iii.) $\Delta S(T_c) = 0$

iv.) Make sure that this happens between the thermodynamically favored BH and TG branches.

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- Similar to “Little string theory in a double scaling limit” Giveon, Kutasov ’99

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- Can be done because this regime is governed by a linear-dilaton CFT on the world-sheet!

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A consistent truncation of IIB with single scalar! Pilch-Warner '00
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An analytic kink solution from asymptotically AdS at $r=0$, $\Phi=\Phi_0$:

$$\begin{aligned} ds^2_{TG} &= e^{-\frac{4}{3}\Phi_0}\,\frac{\cosh^{\frac{2}{3}}(\frac{3r}{2\ell})}{\sinh^2(\frac{3r}{2\ell})}\left(dt^2+dx^2_{d-1}+dr^2\right), \\ e^{\Phi(r)} &= e^{\Phi_0}\,\cosh(\frac{3r}{2\ell}). \end{aligned}$$

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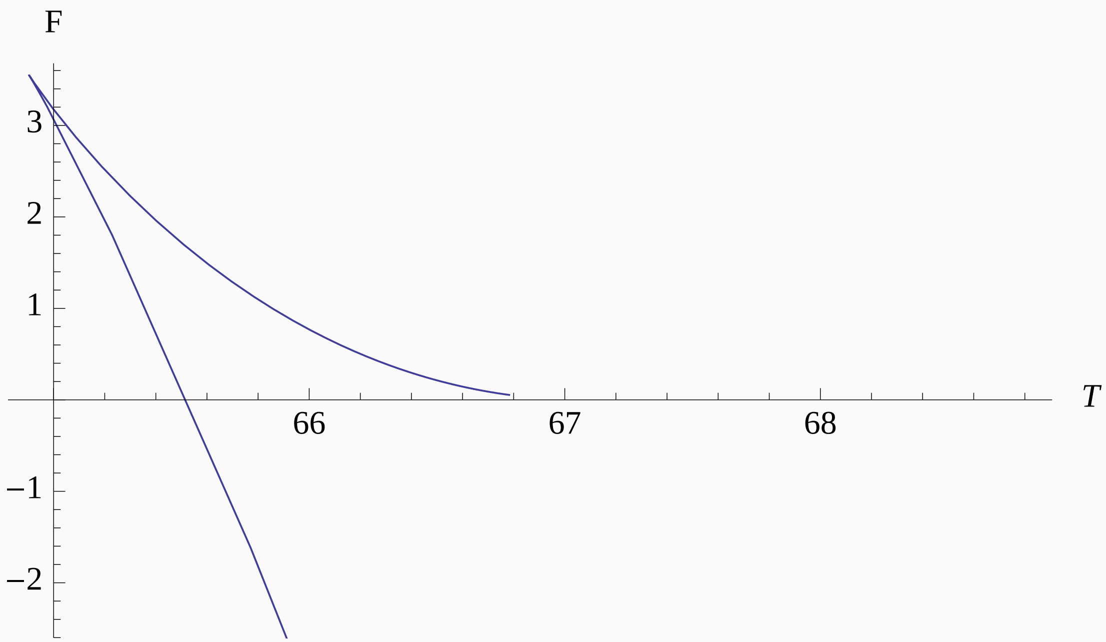

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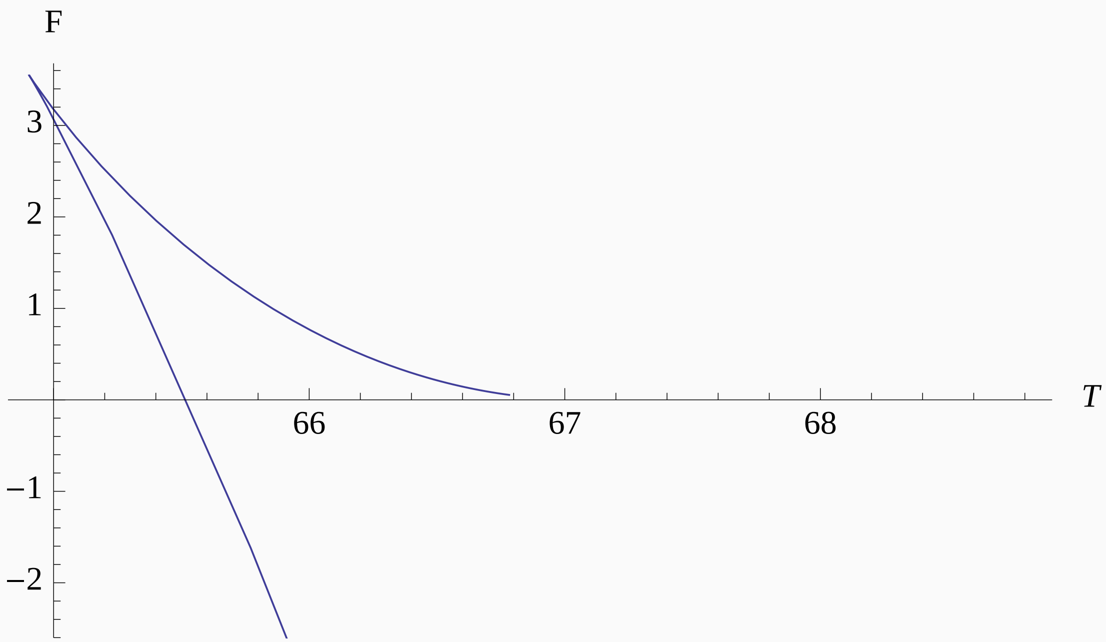

Very generic in Einstein-scalar system $\Leftrightarrow$ Confinement-deconfinement transition is generically first order in gauge theories.

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- One finds $c_\psi^2 \propto e^{-\sqrt{V_\infty} r_h} \sim (T - T_c)$.
- Second sound indeed vanishes at T_c with the mean-field exponent!

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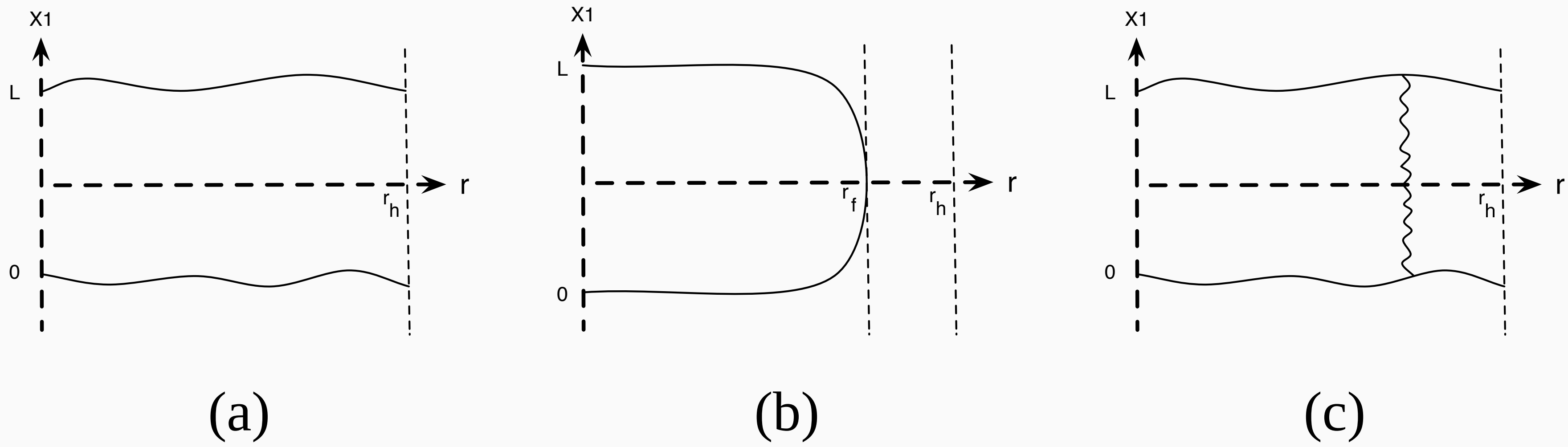

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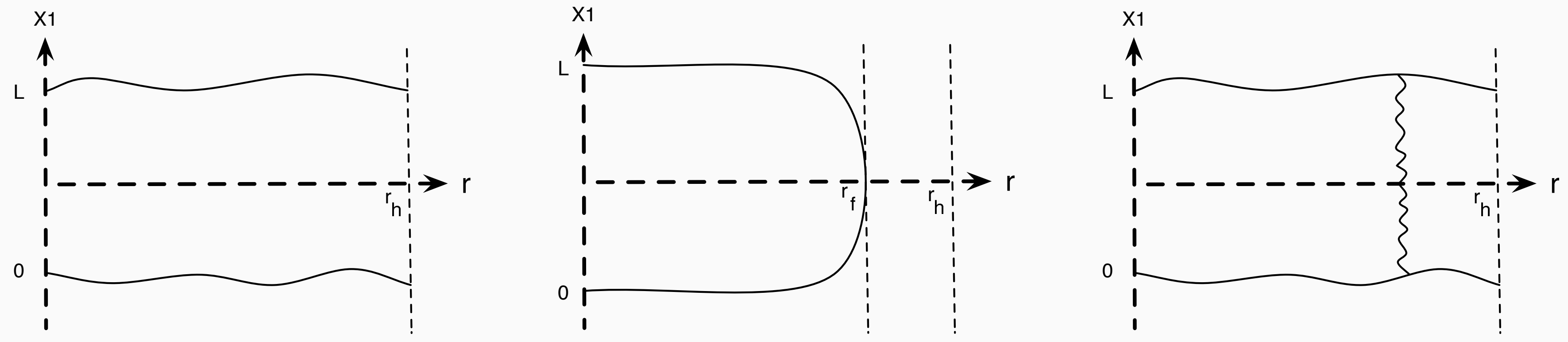

(a) (b) (c)

(a) $\langle \vec{m}(L) \cdot \vec{m}(0) \rangle_a = |\vec{M}|^2$. Finite in BH, 0 for TG.

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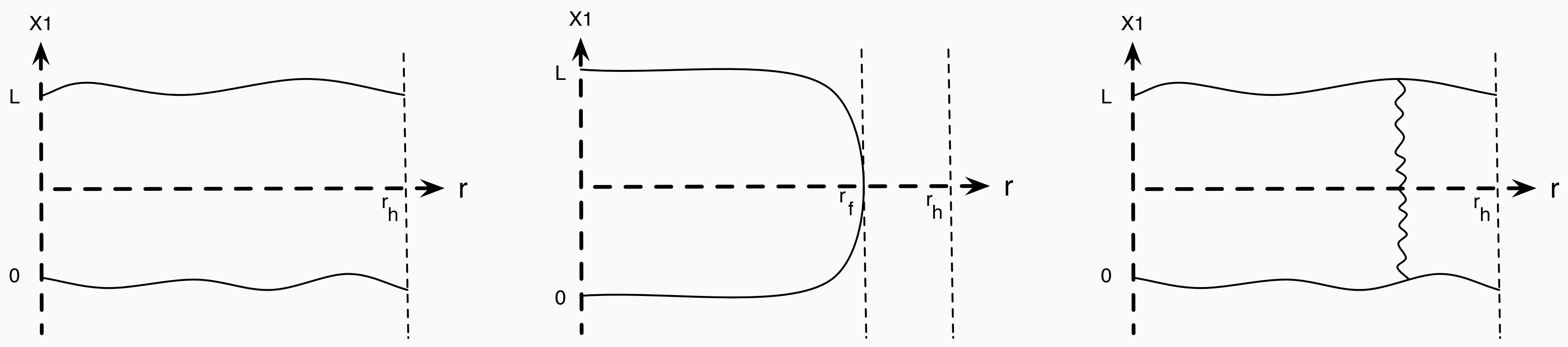

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(b) $S_{F1} \to m_T L + \cdots$

4 $\langle \vec{m}(L) \cdot \vec{m}(0) \rangle_b \sim e^{-m_T L + \cdots}$ for $L \gg 1$.

Two-point function, cont'ed

(c) bulk exchange diagrams:

$$\langle \vec{m}_{||}(L) \cdot \vec{m}_{||}(0) \rangle_c \propto \langle ReP[L] ReP[0] \rangle \sim \frac{e^{-m_+ L}}{L^{d-3}}$$

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Precisely the expected behavior from the XY model,

with $\xi_{\|}^{-1} \to min(m_T, m_+)$ for $L \gg 1$.

Correlation length ξ

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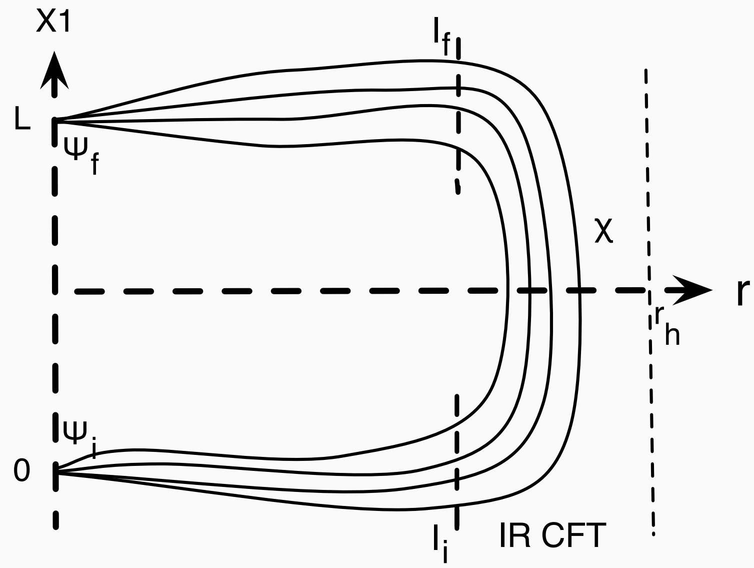
(b) Connected paths:



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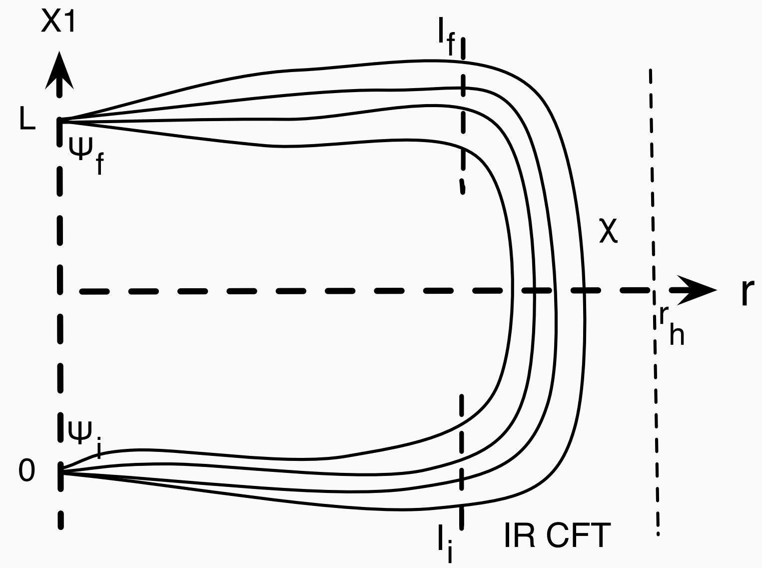

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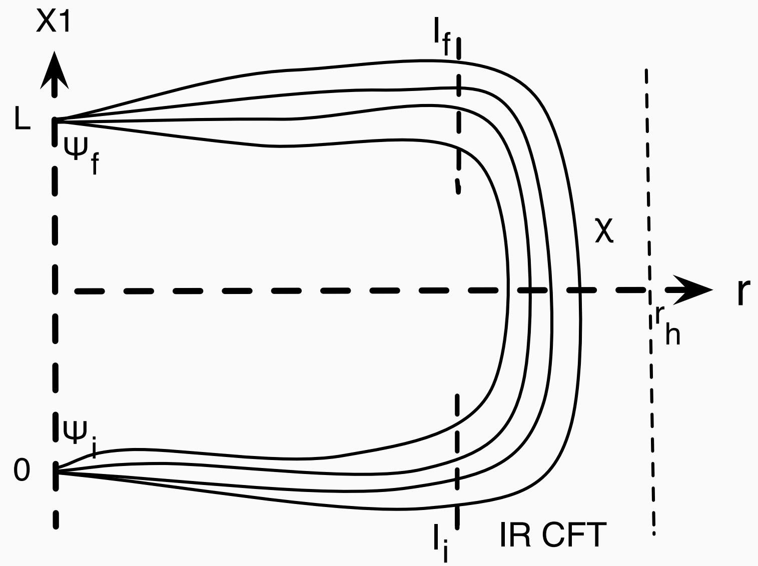

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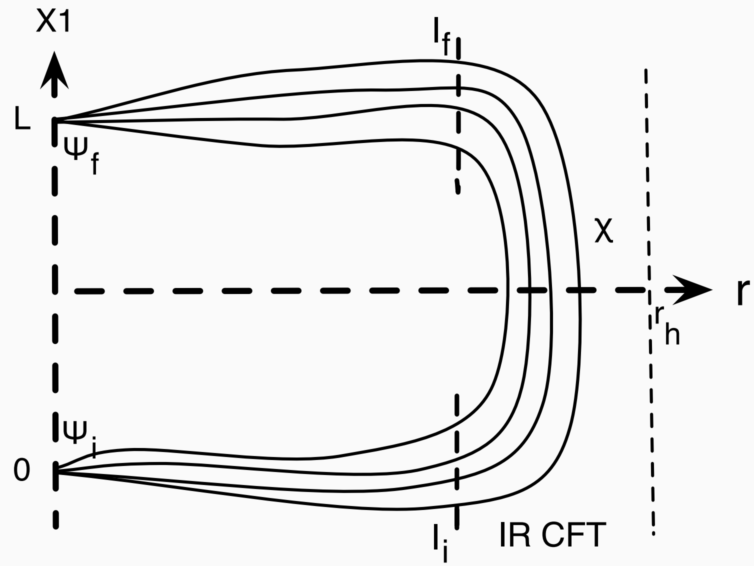

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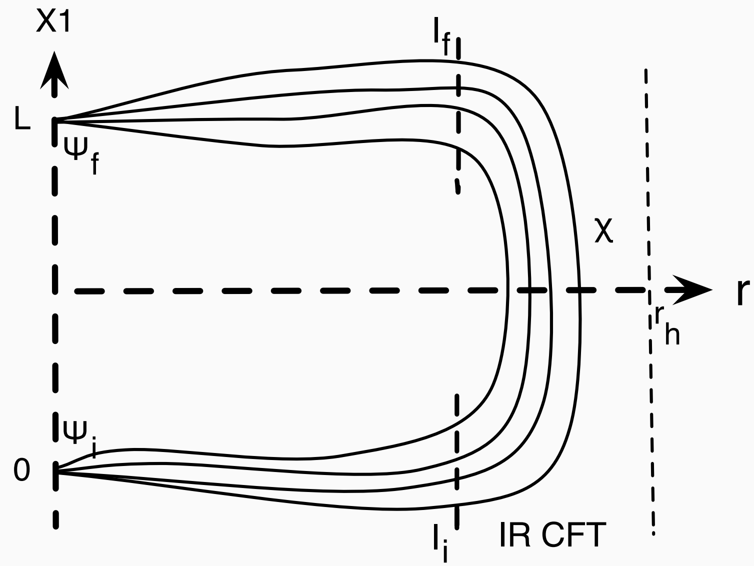

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Mean-field scaling again!

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THANK YOU !

Solving for the conditions

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- Then condition ii) is automatic.

Solving the conditions, cont'ed

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$$V(\Phi) \to V_\infty \, e^{2\sqrt{\frac{\xi}{d-1}}\Phi} \left(1 + V_{sub}(\Phi)\right), \qquad \Phi \to \infty$$