Comments on AdS/CFT for higher derivative gravity

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Introduction

There are many ways we can use the AdS/CFT correspondence. Here I will focus on the set of lessons for conformal field theories.

The examples will involve higher derivative gravity in the bulk. Some of the physics only appears once higher curvature terms are included in the gravitational action.

Outline

Lovelock gravities: review

Gauss-Bonnet gravity Lagrangian:

$$\mathcal{L} = R + \frac{6}{L^2} + \lambda L^2 (R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd})$$

We will be interested in the $\lambda \sim 1$ regime.

More generally, we can add terms $\mathcal{O}(R^k)$ which are Euler densities in $2k$ dimensions:

$$\lambda_k L^{2k-2} \delta^{a_1 \dots b_k}_{c_1 \dots d_k} R^{c_1 d_1}{}_{a_1 b_1} \dots R^{c_k d_k}{}_{a_k b_k}$$

They become non-trivial for gravity theories in AdS_D with $D > 2k$.

Special properties

- AdS is a solution (therefore can study dual CFT).
- Equations of motion don't contain 3rd order derivatives g''' (holographic dictionary is not modified).
- Exact black hole solutions can be found (can study CFTs at finite temperature).

Implications for AdS/CFT

The UV behavior is modified. Consider conformal rescaling

$$g_{ab}^{CFT} \rightarrow \exp(2\sigma)\, g_{ab}^{CFT}$$

CFT action is anomalous; there are two terms in 3+1 dimensions:

$$\delta\mathcal{W} = aE_4 + cW^2$$

Enstein-Hilbert (E-H) implies $a = c$. Lovelock implies $a \neq c$. Of course, the IR behavior is modified as well. E.g. values of transport coefficients are different from E-H.

Finite T; metastable states

Consider propagation of gravitons in the black hole background.

(D=5 Gauss-Bonnet: Brigante, Liu, Myers, Shenker, Yaida)

$$ds^2 = -\frac{f(r)}{\alpha}dt^2 + \frac{dr^2}{f(r)} + \frac{r^2}{L^2}\left(\sum dx_i^2 + 2\phi(t,r,z)dx_1 dx_2\right)$$

Fourier transform: $\phi(t,r,z) = \int dwdqexp(-iwt+iqz)$
After substitutions and coordinate transformations, get
Schrodinger equation with $\hbar \to 1/\tilde{q} = T/q$:

$$-\frac{1}{\tilde{q}^2}\partial_y^2\Psi(y) + V(y)\Psi(y) = \frac{w^2}{q^2}\Psi(y)$$

Causality

Spectrum = states in finite T CFT.
In the $\tilde{q} \gg 1$ regime, there are stable states with $\partial w / \partial q > 1$ in some region of the parameter space. Causality places constraints on Lovelock couplings:

$$\sum_k [(d-2)(d-3) + 2d(k-1)]\lambda_k \alpha^{k-1} < 0$$

where α defines the AdS radius $L_{AdS}^2 = L^2/\alpha$ and satisfies $\sum_k \lambda_k \alpha^k = 0$.

This effect is absent at $T = 0$; appears as $\mathcal{O}(T/q)$ correction from the tails of black hole metric.

Positivity of energy flux = unitarity

Define $\varepsilon(\hat{n}) = \lim_{r\to\infty} r^2 \int dt T^0_i \hat{n}_i$
Conjecture (Hofman, Maldacena) $\langle\varepsilon(\hat{n})\rangle \geq 0$.
Consider a state created by $\epsilon^{ij} T_{ij}$

$$\langle\varepsilon(\hat{n})\rangle \sim 1 + t_2\left(\frac{\epsilon_{ij}\epsilon^{il}\hat{n}^j\hat{n}_l}{\epsilon_{ij}\epsilon^{ij}} - \frac{1}{d-1}\right) + t_4\left(\frac{(\epsilon^{ij}\hat{n}^i\hat{n}_j)^2}{\epsilon_{ij}\epsilon^{ij}} - \frac{2}{d^2-1}\right)$$

t_2 and t_4 are determined by the 2 and 3-point functions of T_{ab}.

- energy flux positivity in CFTs dual to Lovelock gravities is equivalent to causality at finite temperature!
- also equivalent to the absence of ghosts. This can be proven for arbitrary CFT with field theory methods.

Entanglement entropy: review

Consider two systems A, B with Hilbert spaces consisting of two states $\{|1\rangle, |2\rangle\}$. Reduced density matrix of A is obtained by tracing over B; entanglement entropy is the resulting VN entropy.

$$\rho_A = \mathrm{tr}_B \rho; \qquad S_A = -\mathrm{tr}_A\, \rho_A \log \rho_A$$

Product state:

$$|1_A 1_B\rangle \Rightarrow S_A = 0$$

Pure (non product) state:

$$\frac{1}{\sqrt{2}} \left(|1_A 2_B\rangle - |2_A 1_B\rangle \right) \Rightarrow S_A = \ln 2$$

Holographic EE

Consider EE in CFT dual to Lovelock gravity in AdS. A proposal for holographic EE (Fursaev).

$$S(V) = \frac{1}{G_N^{(5)}} \int_\Sigma \sqrt{\sigma}\left(1 + \lambda_2 L^2 R_\Sigma\right)$$

Σ is the minimal surface ending on (∂V) which satisfies the e.o.m. derived from this action. R_Σ is the induced scalar curvature on Σ. Consider the case of a ball, bounded by the two-sphere of radius R. It is not hard to solve EOM near the boundary of AdS and extract the log-divergent term:

$$S \simeq \frac{R^2}{\epsilon^2} + \frac{a}{90} \ln R/\epsilon + \dots$$

where the expression for a in terms of λ_{GB} was substituted.

Holographic EE

Similarly, for a cylinder of radius R and length l

$$S \simeq \frac{Rl}{\epsilon^2} + \frac{c}{720}\frac{l}{R}\ln R/\epsilon + \ldots$$

This structure repeats for the six-dimensional CFT dual to AdS_7. For the ball of radius R the log-term is proportional to the A-type anomaly coefficient. For the cylinder, the log-term is proportional to the linear combination of the B-type anomalies.

We can also compute EE in Lifshitz geometries, which are solutions of gravity EOM if $\lambda_{GB} = 1/4$, with the results similar to the ones discussed above.

a-theorem and AdS/CFT

Zamolodchikov's c-theorem in 1+1 dimensions: there is a c-function [made out of $\langle T_{ab} T_{ab} \rangle$] which is

- positive
- decreases along the RG flows
- equal to the central charge c at fixed points

Is there an analogous quantity in 3+1 dimensions? Conjecture: yes, and it is equal to a at fixed points.

Holographic a-theorem (Myers, Sinha) Consider a background which holographically describes the RG flow:

$$ds^2 = \exp(2A(r))(dx_\mu)^2 + dr^2$$

a-theorem and AdS/CFT

Then the quantity

$$a(r) = \frac{1}{l_p^3 A'(r)^3} \left(1 - 6\lambda L^2 A'(r)^2\right)$$

is equal to a at fixed points and satisfies

$$a'(r) = -\frac{1}{l_p^3 A'(r)^4} \left(T_0^0 - T_r^r\right)$$

The right hand side of this equation is proportional to the null energy condition.

a-theorem in other dimensions?

One can hope that the off-shell generalization of the A-type anomaly coefficient, a, plays a role of the c-function in even dimensions.

In 4d such quantity is related to the 3-point functions of T_{ab}. In 6d a is related to the 4-point function, and cannot be written in terms of the coefficients which determine the three-point function alone.

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- Black hole solutions with novel properties
- Phenomenological applications in AdS/QCD, AdS/CMT and AdS/???