Holographic (De)confinement Transitions in Cosmological Backgrounds

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June 20, 2011

ArXiv:1105.1776 (with J. Erdmenger and K. Ghoroku)

Outline

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Motivation

- AdS/CFT describes well QFTs at strong coupling and finite temperature (e.g. models of QGP)
- AdS/CFT can also describe strongly coupled QFTs in time-dependent background space-times, even with singularities (e.g. FRW)
- Properties of a strongly coupled (Q)GP at finite temperature in cosmological backgrounds (dS/AdS/matter or radiation domination) is of interest in cosmology (deconfinement, CSB enhancement/reduction)
- Interplay between internal confining dynamics and background properties (H, sgnλ)
- This talk: $\mathcal{N} = 4$ with gluon condensate & instanton density [Liu/Tseytlin]
  1. Construction of holographic backgrounds with a FRW boundary

     $$\boxed{\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \lambda\,, \quad \dot{\lambda} = 0}$$

     Finite Temperature = "Dark Radiation"/"Mirage Energy Density"
  2. Evaluate temporal Wilson loop to track the (de)confinement properties of the plasma

Outline

The Gravity Setup

- Liu-Tseytlin   D3-smeared D(-1)
  - **10D IIB Sugra** with Axio-Dilaton (χ, ϕ) and 5-Form F_5
  - **Freund-Rubin ansatz:** $\mathcal{M}_{10} = \mathcal{M}_5 \times S^5$ (N units of F_5 flux in S^5)

$$\boxed{S = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\left(R + 3\Lambda - \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}e^{2\phi}(\partial\chi)^2\right)}$$

  - **Supersymmetry:** $\boxed{\chi = -e^{-\phi} + \chi_0}$

- Effective Five-Dimensional EOMs:

$$\begin{aligned} R_{\mu\nu} &= -\Lambda g_{\mu\nu} \\ 0 &= \Box e^{\Phi} = \Box\chi \end{aligned}$$

- Ansatz from Brane World Cosmology: [Binetruy, Deffayet, Ellwanger, Langlois hep-th/9910219]

$$\boxed{ds^2 = -n(t,y)^2 dt^2 + a(t,y)^2 d\Omega_k^2 + dy^2}$$

Deriving the Gravity Background I

- Ansatz from Brane World Cosmology: [Binetruy, Deffayet, Ellwanger, Langlois hep-th/9910219]

$$\boxed{ds^2 = -n(t,y)^2 dt^2 + a(t,y)^2 d\Omega_k^2 + dy^2}$$

- Einsteins equations and Bianchi Identities are fulfilled up to tt/yy and ty:

$$\left(\frac{\dot{a}}{na}\right)^2 + \frac{k}{a^2} = -\frac{\Lambda}{4} + \left(\frac{a'}{a}\right)^2 + \frac{C}{a^4} \quad (1)$$

$$\frac{n'}{n}\frac{\dot{a}}{a} = \frac{\dot{a}'}{a} \quad (2)$$

The **Dark Radiation Contant** C corresponds to the **Black Hole Mass**

- Redefining of n and a solves (2): $n(t,y) = \frac{\dot{a}(t,y)}{\dot{a}_0(t)}\,,\ a(t,y) = a_0(t)A(t,y)$

$$\boxed{\left(\frac{\dot{a}_0}{a_0}\right)^2 + \frac{k}{a_0^2} = -\frac{\Lambda}{4}A^2 + (A')^2 + \frac{C}{A^2 a_0^4}}$$

Deriving the Gravity Background II

- Solve the remaining equation by separating LHS/RHS

$$\boxed{\left(\frac{\dot{a}_0(t)}{a_0(t)}\right)^2 + \frac{k}{a_0(t)^2} = \lambda(t) = -\frac{\Lambda}{4}A(t,y)^2 + (A'(t,y))^2 + \frac{C}{A^2(t,y)a_0^4(t)}}$$

- The LHS is the usual **Friedmann equation** with cosmological constant $\lambda(t)$.
- **The RHS determines the bulk geometry:** With $r/R = e^{y/R}$, $R^2 = 4/\Lambda$ and imposing asymptotically AdS boundary conditions,

$$\boxed{A = \frac{r}{R}\left(\left(1 - \frac{\lambda(t)R^2}{4}\frac{R^2}{r^2}\right)^2 + \frac{CR^2}{4a_0^4(t)}\frac{R^4}{r^4}\right)^{\frac{1}{2}}, \quad n = A + \frac{a_0\dot{A}}{\dot{a}_0}}$$

- The **Boundary Metric** is

$$\boxed{ds_0^2 = -dt^2 + a_0(t)^2 d\Omega_k^2}$$

The Holographic Stress Energy Tensor

- Fefferman-Graham gives the **VEV of the Stress Energy Tensor**

$$\langle T^{\mu}{}_{\nu}\rangle = \mathrm{diag}(-\rho, p, p, p)\,, \quad \alpha = \frac{4R^3}{16\pi G_N^{(5)}}\,,$$

$$\rho = 3\alpha\left(\frac{C}{4R^2 a_0^4(t)} + \frac{\lambda(t)^2}{16}\right)\,, \quad p = \alpha\left(\frac{C}{4R^2 a_0^4(t)} - \frac{3\lambda(t)^2}{16}\right)$$

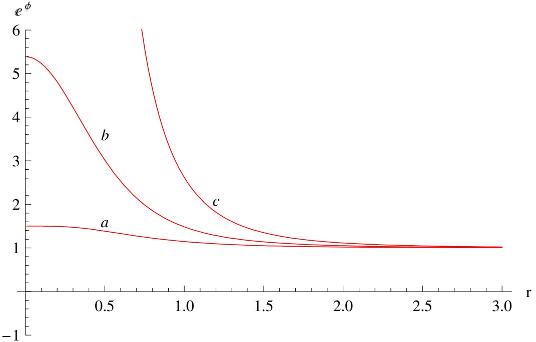
- The dark radiation part $\propto C$ is **relativistic radiation at finite "temperature"**, and is conformal. [Kraus, Kehagias/Kiritsis, Langlois et. al., Tetradis et. al.]
- The **conformal anomaly contribution** is $\propto \lambda^2$.
- **Conservedness of the stress energy tensor implies a time-independent boundary cosmological constant:**

$$\boxed{\nabla_\mu \langle T^{\mu}{}_{\nu}\rangle = 0 \Leftrightarrow 0 = \dot{\rho} + 3H(\rho + p) \Rightarrow \dot{\lambda}(t) = 0}$$

- This condition ensures **regularity of the bulk geometry.** [Li, Pang 1105.0038]

The Dilaton Solution

- The dilaton equation of motion $\Box e^{\Phi} = 0$ is straightforwardly integrated. For the solution see eq. (41) of 1105.1776.



Three cases: (a,b) $\lambda \leq -\frac{2\sqrt{C}}{Ra_0^2(t)}$ $\rightarrow$ Area law

(c) $\lambda > -\frac{2\sqrt{C}}{Ra_0^2(t)}$: Divergence at $r = R\sqrt{\frac{\sqrt{C}R^2}{2a_0^2} + \frac{\lambda R^2}{4}}$, Perimeter law

Outline

Calculating the Tension

- $V_{\bar{q}q}(L)$ is calculated from the (renormalized) energy of a **Nambu-Goto string** drooping into the bulk from the temporal Wilson loop at the boundary
- L: **Proper distance** between string quark and anti-quark:

$$\boxed{L = 2\int_{\tilde{\sigma}_{min}}^{\tilde{\sigma}_{max}} d\tilde{\sigma} = 2a_0(t)\int_{\sigma_{\min}}^{\sigma_{max}} \frac{d\sigma}{1+k\sigma^2/4}}$$

- With the Ansatz $r(\tilde{\sigma})$, the **energy of the string** is

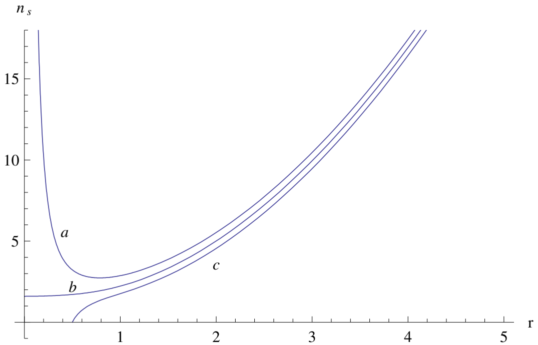
$$E = \frac{1}{2\pi\alpha'}\int d\tilde{\sigma}|n_s|\sqrt{1+\left(\frac{R^2\partial_{\tilde{\sigma}}r}{r^2\bar{A}}\right)^2}\,,\qquad n_s = e^{\phi/2}\frac{r^2}{R^2}|\bar{A}\bar{n}|$$

$$\bar{A} = \left(\left(1-\frac{\lambda(t)R^2}{4}\frac{R^2}{r^2}\right)^2+\frac{CR^2}{4a_0^4(t)}\frac{R^4}{r^4}\right)^{\frac{1}{2}}\,,\quad \bar{n} = \frac{\left(1-\frac{\lambda(t)R^2}{4}\frac{R^2}{r^2}\right)^2-\frac{CR^2}{4a_0^4(t)}\frac{R^4}{r^4}}{\bar{A}}$$

- If $n_s(r)$ has a finite, positive minimum at $r_* \geq 0$, the **string tension** for large L is $\boxed{\tau_{\bar{q}q} = \frac{n_s(r_*)}{2\pi\alpha'}}$

Qualitative Behaviour of the Wilson Loop

- If $n_s(r)$ has a finite, positive minimum at $r_* \geq 0$, the **string tension** for large L is $\boxed{\tau_{\bar{q}q} = \frac{n_s(r_*)}{2\pi\alpha'}}$

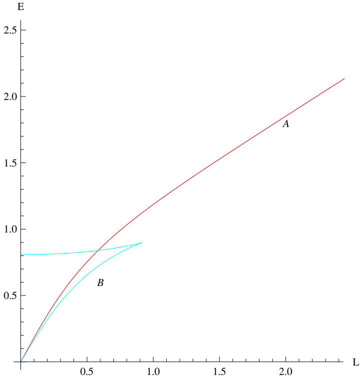


Three cases: (a,b) $\lambda \leq -\frac{2\sqrt{C}}{Ra_0^2(t)} \rightarrow$ Area law, Confinement

(c) $\lambda > -\frac{2\sqrt{C}}{Ra_0^2(t)} \rightarrow$ Perimeter law, Deconfined

The Quark-Antiquark Potential

- The full **energy of the string** can be evaluated numerically



(A) $\lambda \leq -\frac{2\sqrt{C}}{Ra_0^2(t)}$ $\rightarrow$ Confinement

(B) $\lambda > -\frac{2\sqrt{C}}{Ra_0^2(t)}$ $\rightarrow$ Deconfined, Screening, String Breaking

Outline

Classification of (De)confining Behaviour

- We found that the Wilson loop shows Area Law or Perimeter Law for

$$\lambda \leq -\frac{2\sqrt{C}}{Ra_0^2(t)} \quad \text{and} \quad \lambda > -\frac{2\sqrt{C}}{Ra_0^2(t)}$$

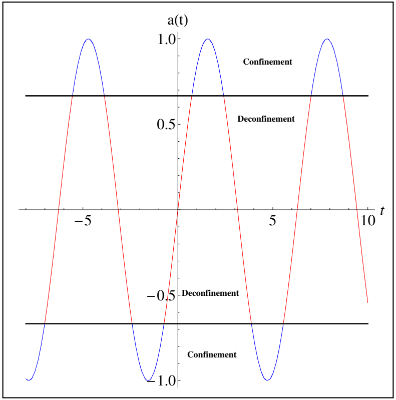
- The dark radiation constant C corresponds to a finite temperature state of the field theory.
- The system behaves differently depending on the sign of λ:
  1. $\lambda > 0$: De Sitter like, deconfined for $C \geq 0$.
  2. $\lambda = 0$: Deconfined for $C > 0$, confined for $C = 0$.
  3. $\lambda < 0$: Anti de Sitter like, either confined (for large a_0) or deconfined (for small a_0)
- De Sitter like expansion screens the quark-antiquark potential at any temperature.
- The last case is particularly interesting, as temperature and negative cosmological constant (Anti de Sitter like contraction) compete.

A cosmological confinement-deconfinement transition

- The case $k = -1$ and $\lambda < 0$ allows for **oscillating cosmology**,

$$\boxed{a_0(t) = \frac{\sin\sqrt{|\lambda|}t}{\sqrt{|\lambda|}}}$$

For small radiation constants, $C \leq R^2/4$, the system undergoes **periodic confinement-deconfinement transitions:**



Blue Region: $\lambda \leq -\frac{2\sqrt{C}}{Ra_0^2(t)}$

Red Region: $\lambda > -\frac{2\sqrt{C}}{Ra_0^2(t)}$

Summary

- Holographic backgrounds to 5D axio-dilaton gravity with a boundary FRW evolution in the presence of a cosmological constant were constructed.
- The construction follows [Liu-Tseytlin] and incorporates a gluon condensate and an instanton density.
- The dark radiation constant appears as an integration constant analoguously to the mass of a black hole, putting the field theory at finite temperature.
- The quark-antiquark potential as extracted from the Wilson loop shows interesting patterns of confinement-deconfinement transitions.
- Positive cosmological constant screens the quark-antiquark potential even at zero temperature, inspite of the presence of a (confining) gluon condensate.
- Negative cosmological constants allow for oscillating cosmologies undergoing periodic transitions between deconfined states near the big bang/crunch singularities and confinement for larger scale factors.