Flavor Corrections in the Static Potential in Holographic QCD

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with D. Giataganas

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Outline

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AdS/CFT correspondence

- The AdS/CFT correspondence, in the original and best understood form, is a duality between the $\mathcal{N} = 4$ supersymmetric Yang-Mills and type IIB superstring theory on $AdS_5 \times S^5$.
- In this correspondence there exist a map between gauge invariant operators in field theory and states in string theory.
- Example:The Wilson loop, is a physical gauge invariant object and can measure the interaction potential between the external quarks and acts as an order of confinement.
- The Wilson loop operator in the fundamental representation is dual to a string worldsheet extending in the $AdS_5 \times S^5$ with boundary the actual loop placed on the AdS boundary. *(Maldacena; Rey, Yee)*

$$< W[C] >= e^{-S_{string}[C]}$$

- Search for more realistic gauge/gravity correspondence. For example:
- Less Supersymmetry.
- Broken conformal symmetry, confinement.
- Finite temperature.
- Inclusion of dynamical quarks.
- Observe phenomena like chiral symmetry breaking, deconfinement.

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Flavors in AdS/CFT

- In the AdS/CFT one can introduce fundamental matter by introducing (N_f) 'flavor branes' in the background (N_c color branes). In the probe limit ($N_f \ll N_c$) this is doable and well understood by now. *(Karch, Katz)*
- In this limit the glueballs affect the meson spectra but not the other way around. To find more realistic models we need to go beyond the probe approximation. The backreaction of the flavor branes need to be taken into account. This requires to solve second order, non-linear partial differential equations, which include Dirac delta functions due to the localization of the flavor branes. Very difficult to find solutions!
- The system of DE can be simplified significantly, if the flavor branes are smeared in the space or perturbative techniques are used. *(Nunez, Paredes, Ramallo; Sonnenschein)*

Motivation

- In the unquenched limit there are interesting phenomena to study, such as the modification of the string tension, the screening effects of the static force, the Lüscher term and the string breaking.
- There are difficulties to study these phenomena even in Lattice theory due to large computational time needed.
- These phenomena should be present in a fully flavor brane backreacted confining background. To study them one needs to know the background and use it to extract all the relevant information from the Wilson loop calculations.
- The Witten-Sakai-Sugimoto (WSS) background is the closest confining background so far to the QCD.
- Hence we use the WSS model where the backreaction of the flavor branes is taken into account and study the Wilson loop.

Outline

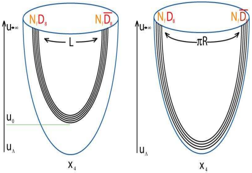
Witten-Sakai-Sugimoto background

Near horizon limit of N_c D4-branes (01234) wrapping a circle in the x_4 direction and intersected by N_f $D8$-branes($x_4 = 0$) and N_f $\bar{D}8$ branes ($x4 = \pi L_8$),(0123 $u\Omega_4$) with the probe limit metric:

$$ds^2 = \left(\frac{u}{R}\right)^{3/2} \left(\eta_{\mu\nu} dx^\mu dx^\nu + f(u) dx_4^2\right) + \left(\frac{R}{u}\right)^{3/2} \left(\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right),$$

$$f(u) = 1 - \frac{u_k^3}{u^3}, \; R := \pi g_s N_c l_s^3,$$

The theory is confining! Chiral symmetry breaking is also present:
At the boundary D8 and $\bar{D}8$ branes stay separated ($U(N_f)_L \times U(N_f)_R$) but in the bulk they meet (group brakes to $U(N_f)$).



(hep-th/0604161)

- To avoid singularity at the tip it is required

$$x^4 \sim x^4 + 2\pi\rho \ , \qquad \rho := \frac{2}{3}\frac{R^{3/2}}{u_k^{1/2}} \ .$$

  where ρ is the periodicity of the circle where the $4+1$ dimensional gauge theory lives on.
- The gravity approximation in valid when $\lambda_5 \gg \rho$.
- At energies$< M_{KK} \sim 1/\rho$ the theory is effectively 4-dim confining. In the phase diagram of the WSS model, the 5-dimensional $SU(N_c)$ gauge theory passes, without crossing a sharp phase transition from a Coulomb phase to a 4-dimensional confined phase.

Perturbative Backreaction of WSS

- We choose to place the $D8, \bar{D}8$ branes lie at antipodal points in the compactified circle. This is a stable configuration. (Sakai, Sugimoto)
- In order to find the $D8$ brane configuration we substitute the induced metric and the dilaton in the DBI action.
- In order to calculate the backreaction, we need to expand the Massive IIA Einstein and dilaton equations in a perturbative parameter q_f. These equations can be decoupled and solved exactly in some limits. (Burrington, Kaplunovsky, Sonnenschein)

- The backreacted metric close to the tip u_k can be found by making the ansatz

$$ds^2 = e^{2A_1(u,x_4)}\left(-dt^2 + dx_i^2\right) + e^{2A_2(u,x_4)}dx_4^2 + e^{2A_3(u,x_4)}du^2 + e^{2A_4(u,x_4)}d\Omega_4^2,$$
$$A_i(u,x_4) = A_{u,i}(u) + q_f A_{d,i}(u,x_4)$$

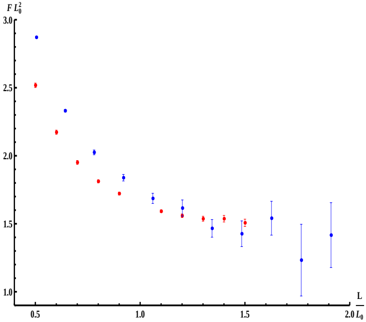
which leads to:

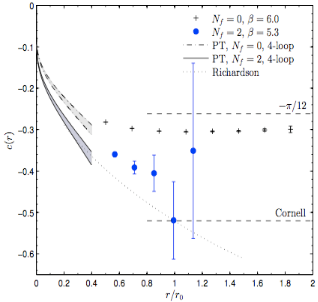
$$g_{00} = -\left(\frac{u}{R}\right)^{3/2} e^{\left(\frac{q_f \sin\left(\frac{x_4}{\rho}\right)\sqrt{u^3-u_k^3}\left(u_k^3\left(7+2\cos\left(\frac{2x_4}{\rho}\right)\right)+4\sin\left(\frac{x_4}{\rho}\right)^2 u^3\right)}{27u_k^{7/2}}\right)},$$
$$g_{00} = -g_{xx}$$
$$g_{44} = \left(\frac{u}{R}\right)^{3/2} e^{\left(q_f \frac{8}{5}\sin\left(\frac{x_4}{\rho}\right)\sqrt{\frac{u^3-u_k^3}{u_k}}\right)\left(1-\frac{u_k^3}{u^3}\right)},$$
$$g_{uu} = \left(1-\frac{u_k^3}{u^3}\right)^{-1}\left(\frac{u}{R}\right)^{-3/2} e^{\left(q_f \frac{8}{5}\sin\left(\frac{x_4}{\rho}\right)\sqrt{\frac{u^3-u_k^3}{u_k}}\right)}.$$

- Next we use this backreacted solution to calculate the static potential.

Lattice Static Potential/Force data

From the Alpha Collaboration, hep-lat/0108008, 1012.3037, 1012.1141, Necco, Sommer; M. Donnellan, F. Knechtli, Leder, Sommer; Knechtli, Leder





red: $N_f = 0$ data, blue: $N_f = 2$ QCD data

Static potential

- The static potential measures the interaction between two heavy quarks. The $SU(N_c)$ pure gauge theory at zero temperature has a static potential of the form

$$E = \mathrm{const.} + \sigma L + \frac{c_1^{\mathrm{gauge}}}{L} + \cdots$$

- The constant is non-physical and one can get rid of it by considering the static force.
- The string tension σ for small N_c, can be computed in lattice Monte Carlo simulations as the slope of the static potential, as a function of the static quark separation L. It is usually expressed in terms of the lattice spacing or in terms of some other physical dimensionful scale.

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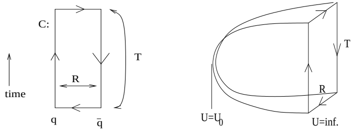
- The Coulomb like term (Lüscher term), is of special importance because its coefficient is dimensionless. In the UV, or small L, it is related to the coupling of the gauge theory and is running. Towards the IR, it stops running roughly around a particular scale, the Sommer scale and takes a constant value predicted and successfully confirmed in a Monte Carlo simulation to be

$$c_1^{\mathrm{gauge}} = -(d-2)\pi/24$$

  in d-dimensions and for any N_c (Lüscher, Symanzik, Weisz).
- This result can be also reproduced using the gauge/gravity duality. (Aharony, Karzbrun, Field,...)

Static Potential in AdS/CFT

The static potential can be measured by introducing two infinitely heavy probe quarks on the boundary of the space. This corresponds to a Wilson loop of the following shape:



(pic taken from 0712.0689)

The normalized expectation value of the Wilson loop which involves the minimal surface of the particular world-sheet minus the infinite quark mass is

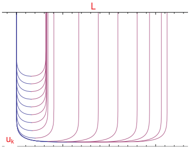
$$W[C] \sim e^{-(S - mass_Q)} \sim e^{-V_{Q\bar{Q}} T}$$

Static Potential in WSS model

- In our model we consider a string world-sheet(w-s) of the following form.

$$t = \tau, \qquad x_1 = \sigma, \qquad x_4 = \pi\rho/2, \qquad u = u(\sigma)\ .$$

  Using the Nambu-Goto action and solving the eoms we obtain the following picture of the string



- For large seperation of $Q\bar{Q}$ the string w-s lies almost whole of it near u_k! The main contributions to the WL expectation value come from this region.
- Hence we can use the perturbative backreacted metric to calculate the leading flavor contributions to the static potential!

To find the static potential one needs to derive from the eoms the length L of the WL in terms of its turning point u_0 and invert the expression $L(u_0)$ to $u_0(L)$. Then express the minimal surface ($\sim$static potential) in terms of L. Generally the action reads

$$S = -\frac{1}{2\pi\alpha'}\int d\sigma d\tau \sqrt{-g_{00}\left(g_{11} + g_{44}x_4'^2 + g_{uu}u'^2\right)} \, ,$$

The eom for x_4 is satisfied for $x_4 = \pi\rho/2$. The Hamiltonian is equal to

$$H = \frac{g_{00}g_{11}}{\sqrt{D}}$$

and is a constant of motion c_0. Solving for u' we get the turning point equation

$$u' = \pm\sqrt{-\frac{(g_{00}g_{11} + c_0^2)g_{11}}{c_0^2 g_{uu}}} \, ,$$

which is solved for

$$g_{00}g_{11} = -c_0^2 \, .$$

Its solution specify how deep the world-sheet goes into the bulk, and we call the value of the turning point u_0.

The length of the two endpoints of the string on the brane is given by

$$L = 2\int_{\infty}^{u_0} \frac{du}{u'} = 2\int_{u_0}^{\infty} du \sqrt{\frac{-g_{uu}c_0^2}{(g_{00}g_{11} + c_0^2)g_{11}}} .$$

Which should be inverted as $u_0(L)$. The normalized energy of the string is

$$\begin{aligned} 2\pi\alpha' E &= 2\left(\int_{u_0}^{\infty} d\sigma \mathcal{L} - \int_{u_0}^{\infty} du\sqrt{g_{00}g_{uu}}\right) \\ &= c_0 L + 2\left[\int_{u_0}^{\infty} du\sqrt{-g_{uu}g_{00}}\left(\sqrt{1 + \frac{c_0^2}{g_{11}g_{00}}} - 1\right) - \int_{u_k}^{u_0} du\sqrt{-g_{00}g_{uu}}\right] . \end{aligned}$$

This does not always implies that exist a term linear in L, since eventually c_0 is a function of L.

Static Potential in the UV ($N_f = 0$)

$$\hat{L} = \frac{L}{3\rho}, \qquad \hat{E} = \frac{2\pi\alpha'}{u_k}E, \qquad A = \frac{u_k}{u_0}$$

$$\hat{L} = \frac{\sqrt{\pi}}{3}\sum_{k=0}^{\infty} c_k \frac{\Gamma(k+2/3)}{\Gamma(k+7/6)} A^{3k+1/2},$$

$$\hat{E} = \mathrm{const.} + \frac{2\sqrt{\pi}}{3}\sum_{k=0}^{\infty} c_k \frac{\Gamma(k-1/3)}{\Gamma(k+1/6)} A^{3k-1}.$$

$$\hat{L} = \sum_{k=0}^{\infty} a_k x^{6k+1}, \qquad \hat{E} = \sum_{k=0}^{\infty} b_k x^{6k-2}$$

$$a_k = c_k \frac{\sqrt{\pi}}{3}\frac{\Gamma(k+2/3)}{\Gamma(k+7/6)}, \qquad b_k := c_k \frac{2\sqrt{\pi}}{3}\frac{\Gamma(k-1/3)}{\Gamma(k+1/6)}, \qquad x = \sqrt{A},$$

Static Potential in the UV ($N_f = 0$)

$$\hat{E} = \frac{b_0}{(x_c + d_1 \hat{l} + d_2 \hat{l}^2 + \cdots)^2} + b_1 (x_c + d_1 \hat{l} + d_2 \hat{l}^2 + \cdots)^4 + \cdots$$

$$\hat{l} = \hat{L} - \hat{L}_c, \qquad \hat{L}_c = \sum_{k=0}^{\infty} x_c^{6k+1} a_k$$

$$d_1 = \frac{1}{\sum_{k=0}^{\infty} (6k+1) x_c^{6k} a_k}\,, \qquad d_2 = -\frac{1}{2} \frac{\sum_{k=0}^{\infty} 6k(6k+1) x_c^{6k-1} a_k}{\left(\sum_{k=0}^{\infty} (6k+1) x_c^{6k} a_k\right)^3}$$

For $x_c = 0$:

$$\begin{aligned} E &= -\frac{4}{27} \left(\frac{\sqrt{\pi}\,\Gamma(2/3)}{\Gamma(7/6)} \right)^3 \frac{R^3}{2\pi\alpha'} \frac{1}{L^2} \\ &= -\frac{2}{27} \left(\frac{\sqrt{\pi}\,\Gamma(2/3)}{\Gamma(7/6)} \right)^3 \frac{g_5^2 N_c}{4\pi^2} \frac{1}{L^2} . \end{aligned}$$

Static Potential in the IR

In our case the situation is more complicated since the integral and the inversions are not doable in general. However, by expanding in q_f and keeping only $\mathcal{O}(q_f)$ terms we obtain

$$L = L_0 + q_f L_f \ ,$$

Additionally by keeping only the leading terms for $u_0 \simeq u_k$ in the integral we can do it and finally invert it to get

$$u_0 = u_k + \frac{4q_f^2 u_k^3}{108\text{LambertW}\left[\pm\frac{q_f u_k}{3^{7/4}} e^{-\frac{B}{2}}\right]^2} \ ,$$

where

$$B = \frac{11}{16} - \frac{\pi}{2\sqrt{3}} - \frac{3L\sqrt{u_k}}{2R^{3/2}} \ .$$

The Lambert W function can be Taylor expanded in q_f giving

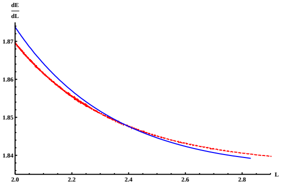
$$u_0 = u_k \left(1 + \sqrt{3}e^B\right) \pm 2 \cdot 3^{-5/4} q_f u_k^2 e^{\frac{B}{2}} \ .$$

Performing the integrals in the energy in a similar way and expressing it in terms of L we obtain:

Static Force

$$2\pi\alpha' E = C + \left(\frac{u_k}{R}\right)^{3/2} L - \sqrt{3} u_k e^B + \mathcal{O}\left(q_f^2 e^{cB}\right) \quad ,$$

For the static force we find:

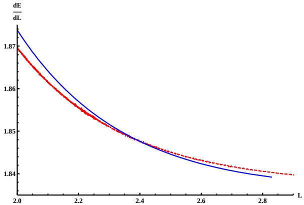


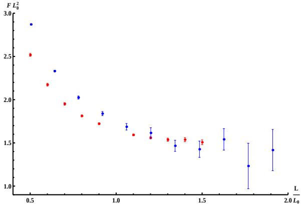
The blue line is the static force in the backreacted background and with red the static force in the probe approximation.

Screening effects are visible for large L($\sim$turning point of the string w-s close to u_k) where our approximation is valid!

Static Force and Lattice data

For a qualitative comparison we use again the lattice data for $N_f = 0$ and $N_f = 2$. (Taken from hep-lat/0108008, 1012.3037, 1012.1141, Necco, Sommer; M. Donnellan, F. Knechtli, Leder, Sommer; Knechtli, Leder)





• Provided that our approximation is still valid in this region and there is not a subtlety with the fixed scale chosen, there is obvious qualitative agreement!

Outline

Conclusions

- We managed to examine in holographic QCD the flavor effects to the static potential, choosing a particular configuration which allows us to use the only known perturbative backreacted solution of the WSS model.
- We saw screening of the static force in the flavored case.
- We saw qualitative agreement with the Lattice data.
- Other questions that we studied but not mentioned today are:
  -How the string w-s is modified in the flavored background.
  -Manage to examine shorter WL analytically in some limits.
  -We found some exact coefficients of the $V_{Q\bar{Q}}(L)$ expression in the non flavored case.

Further directions

- Calculate the full backreacted solution of WSS model with the technics of smeared flavor branes and find the static potential. Does the background has clear information for the string breaking? Comparison with our results at certain limits?
- The fact that we have only perturbative flavor effects plus that the quark masses are zero, does not allow us to extract information for the string breaking. Solution to that could be
  • to include a scalar bi-fundamental tachyon field in the theory
  • and calculate the full backreacted (or perturbative) solution possibly with smearing of the flavor branes.
  More realistic string breaking scale and modification of the string tension or the Lüsher term?

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