

6th Crete Regional Meeting on String Theory

June 2011 - Milos - Greece

*On Holography
of
Julia-Zee Dyon*

*Davood Allahbakhshi
Sharif University of Technology & IPM*

Question

*What is the dual theory of the
gravitational 'tHooft-Polyakov monopole
or more generally
gravitational Julia-Zee dyon?*

Julia-Zee Dyon

$$S = \int d^4x \left\{ -\frac{1}{4}(\vec{F}_{\mu\nu})^2 - \frac{1}{2}(D_\mu \vec{\phi})^2 - \frac{\lambda}{4}(\vec{\phi}^2)^2 + \frac{1}{L^2}\vec{\phi}^2 \right\}$$

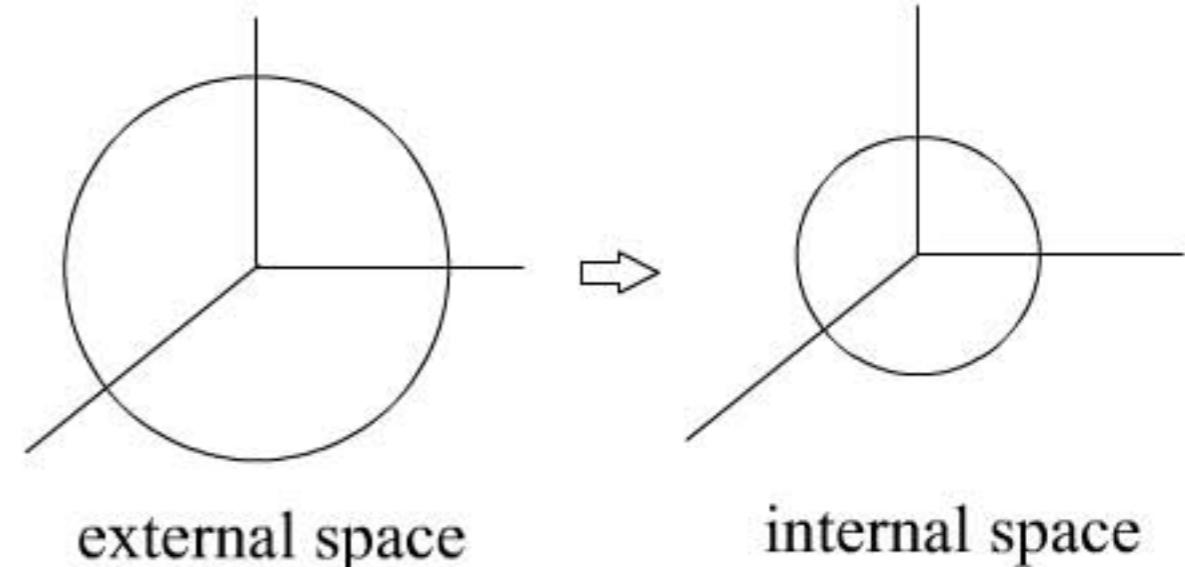
$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + e \epsilon^{abc} A_\mu^b A_\nu^c \quad D_\mu \phi^a = \partial_\mu \phi^a + e \epsilon^{abc} A_\mu^b \phi^c$$

$$\begin{aligned} \phi &= \vec{\tau} \cdot \vec{\phi} = \frac{H(r)}{er} \tau^r & A &= \vec{A} \cdot \vec{\phi} = \frac{J(r)}{er} \tau^r dt + \frac{2(1-K(r))}{e} (-\tau^\varphi d\theta + \tau^\theta \sin(\theta) d\varphi) \\ \tau^r &= \hat{r} \cdot \vec{\tau} & ; & \tau^\theta = \hat{\theta} \cdot \vec{\tau} & ; & \tau^\varphi = \hat{\varphi} \cdot \vec{\tau} \end{aligned}$$

$$F_{\mu\nu} = \hat{\phi} \cdot \vec{F}_{\mu\nu} + \hat{\phi} \cdot [D_\mu \hat{\phi} \times D_\nu \hat{\phi}]$$

$$\hat{\phi} = \frac{\vec{\phi}}{\sqrt{\vec{\phi} \cdot \vec{\phi}}}$$

$$B_i = \epsilon_{ijk} F^{jk} \rightarrow g = \oint_{S^2} B_i ds^i = \frac{n}{e}$$



A.M.Polyakov, JTEP Lett. 20 (1974) 194

G.'tHooft, Nucl. Phys. B 79 (1974) 276

B. Julia and A.Zee , Phys. Rev. D 11 (1975) 760

J. Arafune , P.G.O.Freund and C.J.Goebel, Lect. Notes Phys. 39 (1975) 240

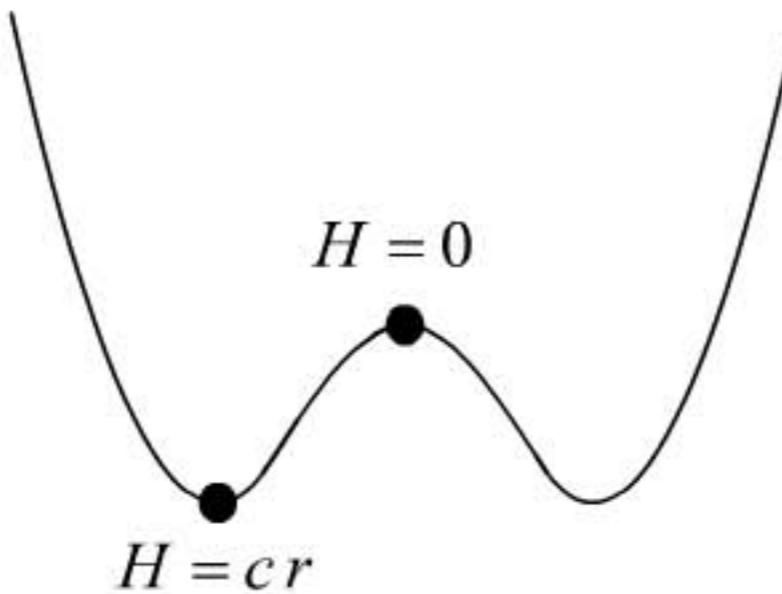
Julia-Zee Dyon

$$S = \int \sqrt{-g} d^4x \left\{ \frac{1}{16\pi G} \left(R + \frac{6}{L^2} \right) - \frac{1}{4} (\vec{F}_{\mu\nu})^2 - \frac{1}{2} (D_\mu \vec{\phi})^2 - \frac{\lambda}{4} (\vec{\phi}^2)^2 + \frac{1}{L^2} \vec{\phi}^2 \right\}$$

$$ds^2 = -e^{X(r)} dt^2 + e^{Y(r)} dr^2 + r^2 d\Omega_2^2$$

$$H = \sqrt{\frac{2}{\lambda L^2}} r, K = 0, J = \mu r - \rho, g_{\mu\nu} \rightarrow AdS - RN$$

$$H = 0, K = 0, J = \mu r - \rho, g_{\mu\nu} \rightarrow AdS - RN$$



In flat back ground, $H=0$ solution is **unstable** but, in AdS back ground, such solutions can be **stable**.

Dynamical Stability

Dynamical stability of $H=0$ solution:

- $\left\{ \begin{array}{l} T > T_c : \text{the quasi-normal modes decay and the background is stable} \\ T < T_c : \text{the quasi-normal modes blow up and the background is unstable} \end{array} \right.$

In the space of the parameters of the model for the $H=0$ solution, there is a "wall of marginal stability".

By crossing this wall a "Phase transition" occurs in the dual field theory.

$4\pi T$	ω
0.212	$0.764969 + 1.66858 i$
0.212	$4.93743 + 3.35654 i$
0.812	$10.133 + 36.3126 i$
0.812	$0.669683 + 9.05345 i$
1.212	$9.9903 + 34.2773 i$
2.211	$0.0148391 + 35.9313 i$
3.211	$12.5418 + 52.0683 i$
4.210	$4.98854 \times 10^{-15} - 1.30046 i$
6.210	$5.26857 \times 10^{-14} - 2.09467 i$
8.209	$2.36149 \times 10^{-14} - 3.01488 i$
10.21	$1.26155 \times 10^{-9} - 4.13518 i$
16.21	$2.16507 - 7.78775 i$
16.21	$8.66376 - 7.8904 i$
20.20	$3.7928 - 9.49205 i$

Vacuum Expectation Values

The dual field theory contains:

$$O^a \leftrightarrow \phi^a$$

$$V_\mu^a \leftrightarrow A_\mu^a$$

Suppose the asymptotic expansion of the fields are:

$$\begin{cases} \vec{\phi} \approx \vec{\phi}_S r^{\Delta_+} + \vec{\phi}_C r^{\Delta_-} + \dots \\ \vec{A}_i \approx \vec{A}_{Si} r^{\eta_+} + \vec{A}_{Ci} r^{\eta_-} + \dots \\ \vec{A}_t \approx \vec{A}_{St} r^{\delta_+} + \vec{A}_{Ct} r^{\delta_-} + \dots \end{cases}$$

For Einstein-Yang-Mills-Higgs action, the 1-point functions are:

$$\begin{cases} \langle \vec{O} \rangle = \lim_{r \rightarrow \infty} \frac{\delta S_{o.s.}}{\delta \vec{\phi}_S} \propto \left(\lim_{r \rightarrow \infty} r^{\Delta_+} \sqrt{-g} D^r \vec{\phi} \right)_{\text{finite term}} \\ \langle \vec{V}_i \rangle = \lim_{r \rightarrow \infty} \frac{\delta S_{o.s.}}{\delta \vec{A}_{Si}} \propto \left(\lim_{r \rightarrow \infty} r^{\eta_+} \sqrt{-g} \vec{F}^{ri} \right)_{\text{finite term}} \\ \langle \vec{V}_t \rangle = \lim_{r \rightarrow \infty} \frac{\delta S_{o.s.}}{\delta \vec{A}_{St}} \propto \left(\lim_{r \rightarrow \infty} r^{\delta_+} \sqrt{-g} \vec{F}^{rt} \right)_{\text{finite term}} \end{cases}$$

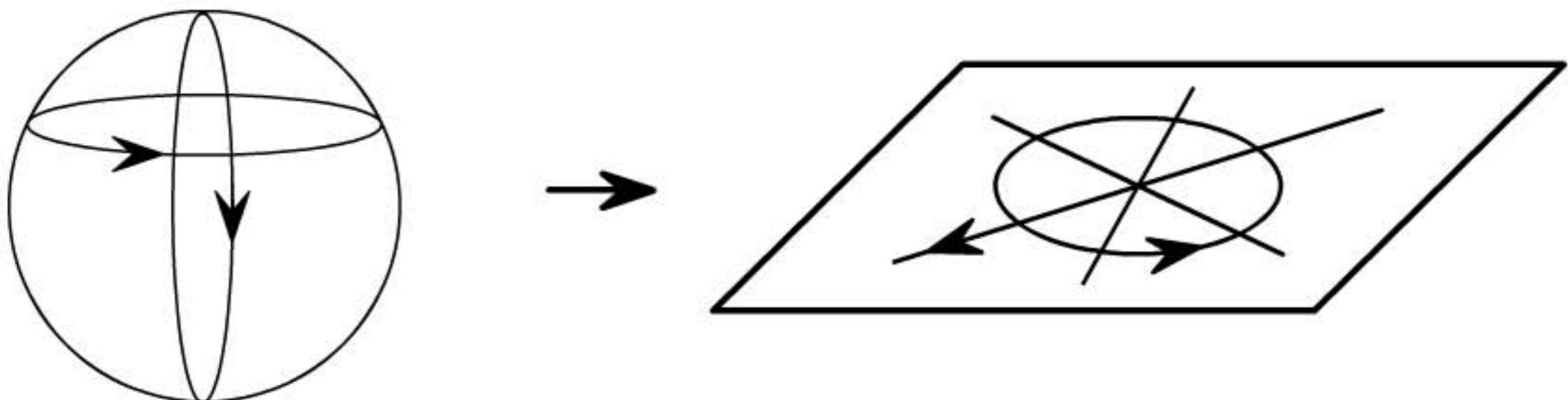
Vacuum Expectation Values

For the Julia-Zee dyon we have:

$$\begin{cases} \langle O^r \rangle \propto \left(\lim_{r \rightarrow \infty} r^{\Delta_+} \sqrt{-g} (D^r \phi)^r \right)_{\text{finite term}} & \rightarrow \langle O^r \rangle \\ \langle \vec{V}_i \rangle \propto \left(\lim_{r \rightarrow \infty} r^{\eta_+} \sqrt{-g} \vec{F}^{ri} \right)_{\text{finite term}} & \rightarrow \langle V_\theta^\varphi \rangle, \langle V_\varphi^\theta \rangle \\ \langle \vec{V}_t \rangle \propto \left(\lim_{r \rightarrow \infty} r^{\delta_+} \sqrt{-g} \vec{F}^{rt} \right)_{\text{finite term}} & \rightarrow \langle V_t^r \rangle \end{cases}$$

And all other v.e.v.s are zero.

By stereographic projection the profile of the vectors looks like a vortex



The dual field theory has a vortex condensate.

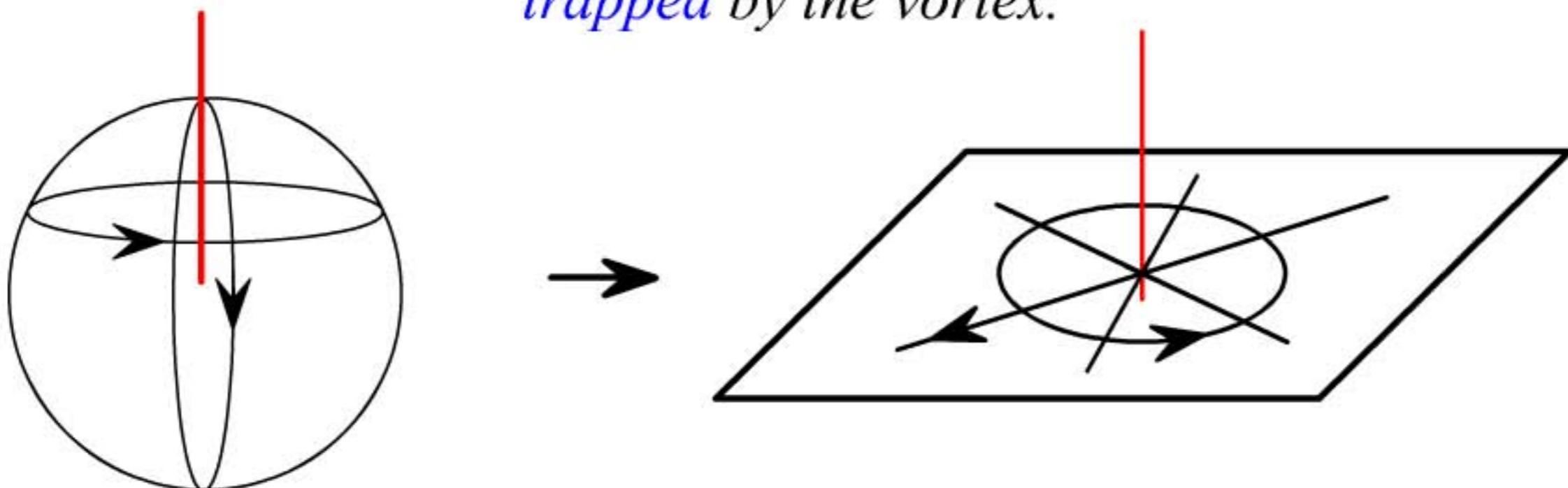
Holography of Magnetic Charge

In the abelian gauge both scalar and vector fields are in the 3rd direction of the gauge space.

In this gauge, there is a singularity (Dirac string) in the whole positive semi axis in the third direction of space.

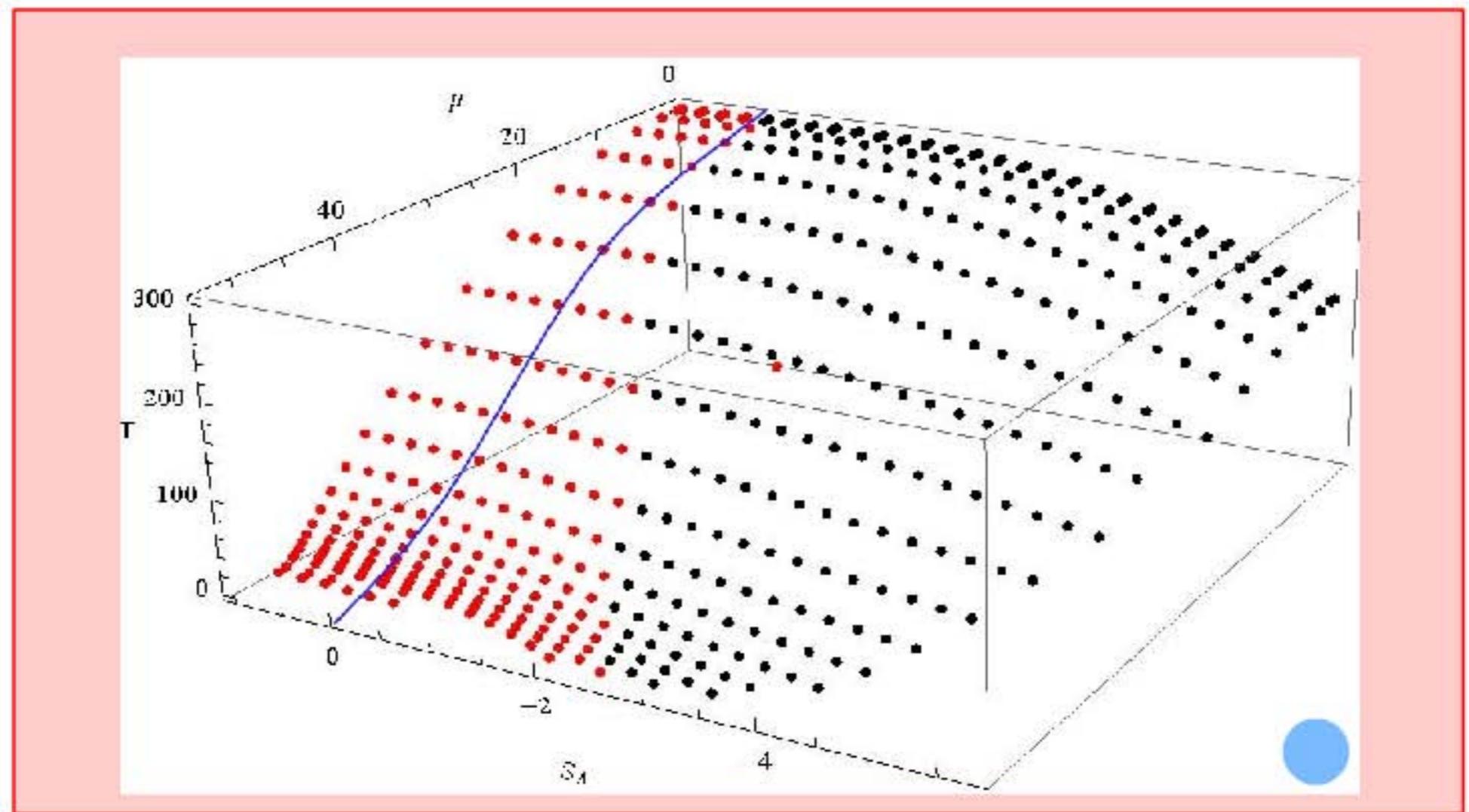
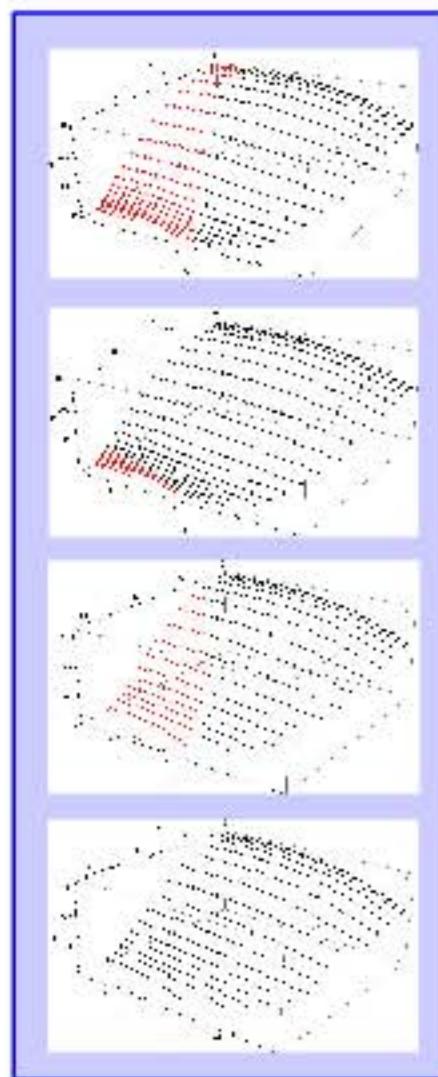
In this gauge the magnetic charge comes from the Dirac string.

Since the string passes through the center of the vortex, the magnetic charge of the monopole is the (projected) magnetic flow which is trapped by the vortex.



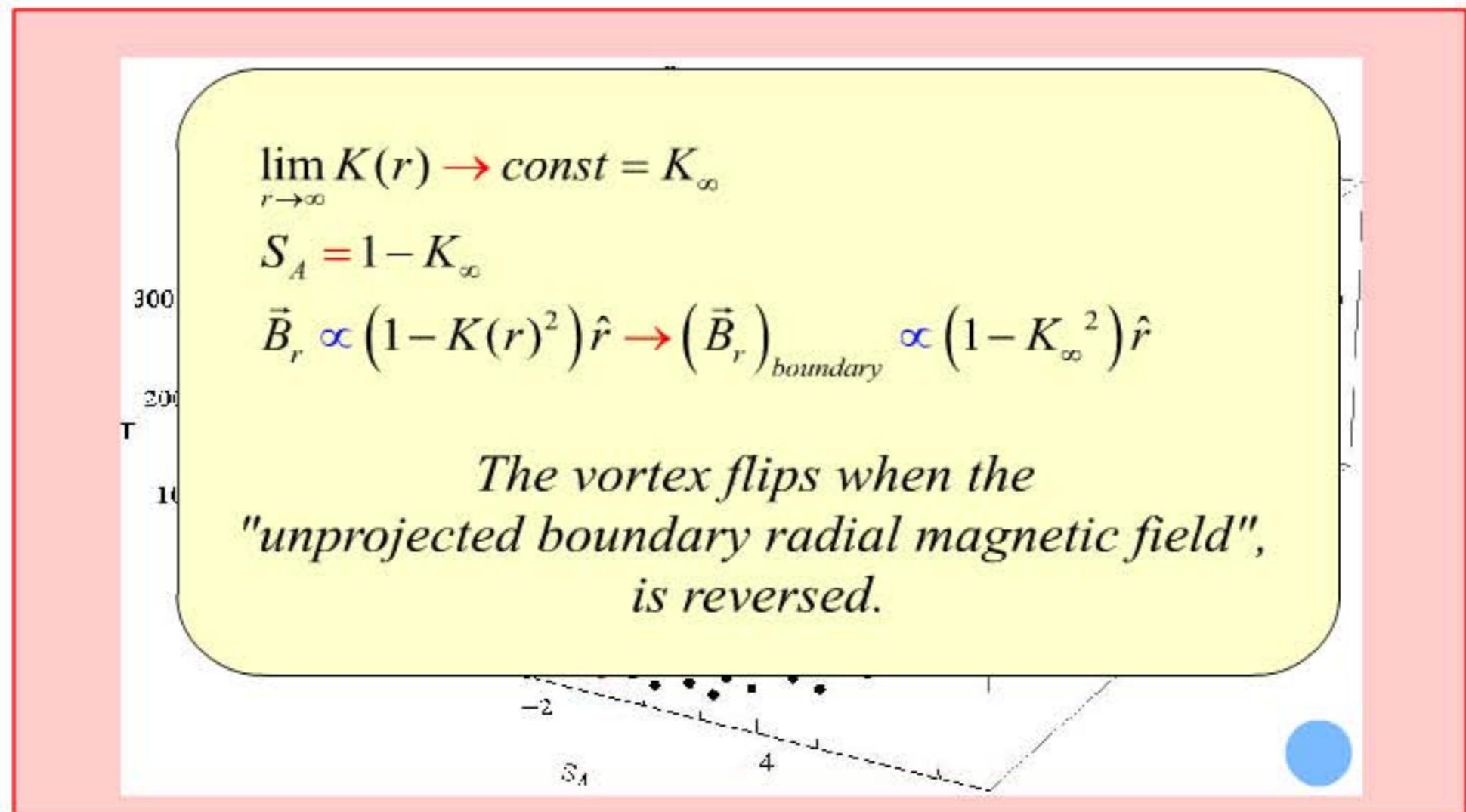
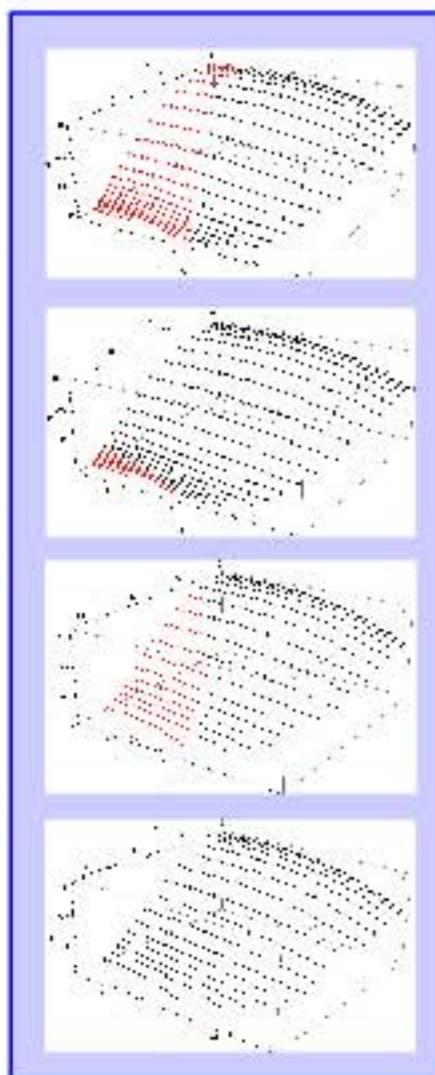
Vacuum Expectation Values

- † The phase space is a 3-dim hypersurface in the space of temperature, chemical potential, scalar and vector sources.
- † Vacuum expectation values of the **vector** and **scalar** operators have **different signs** in different regions of the phase space, and for each operator there is a **two dimensional surface of vanishing v.e.v.s.**



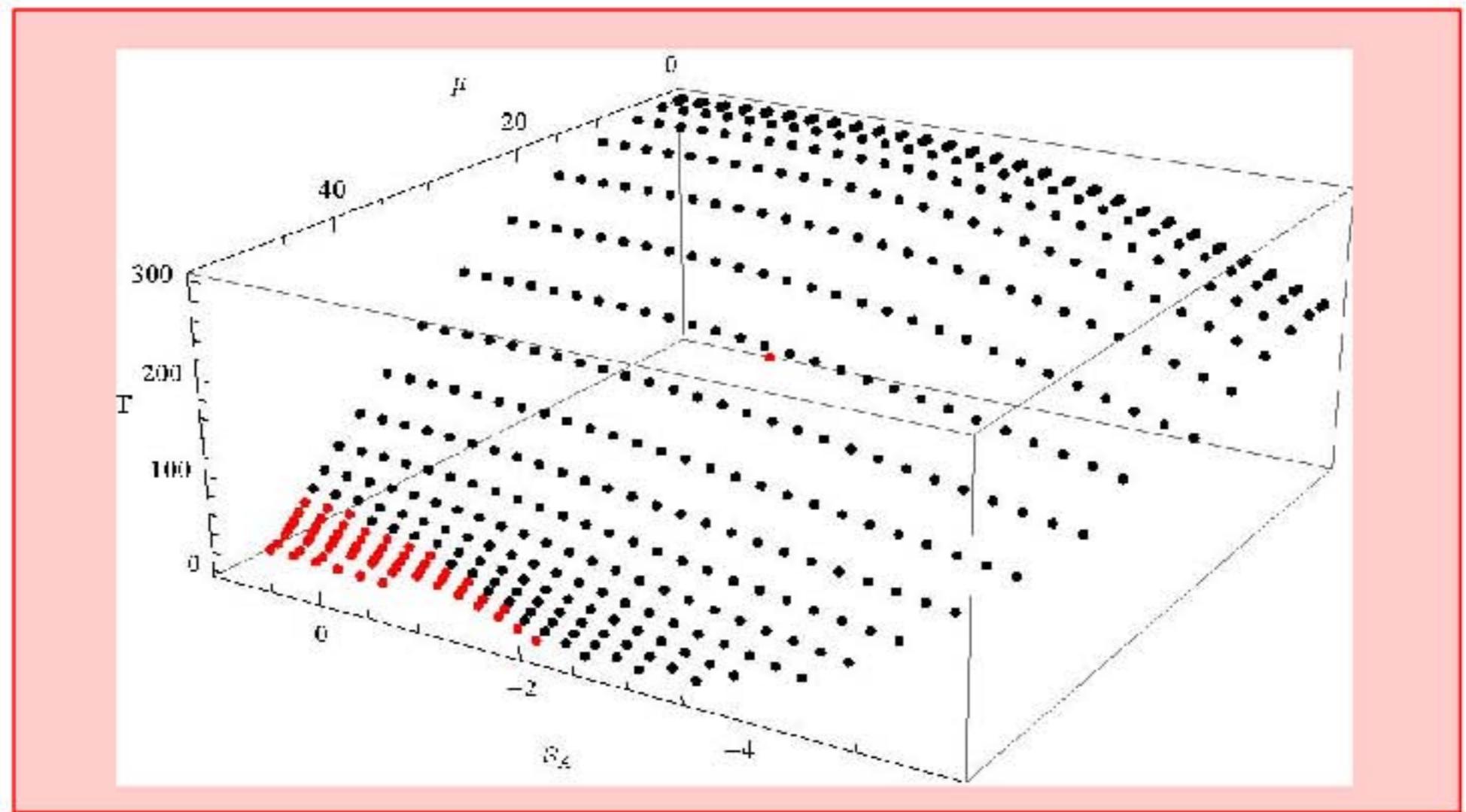
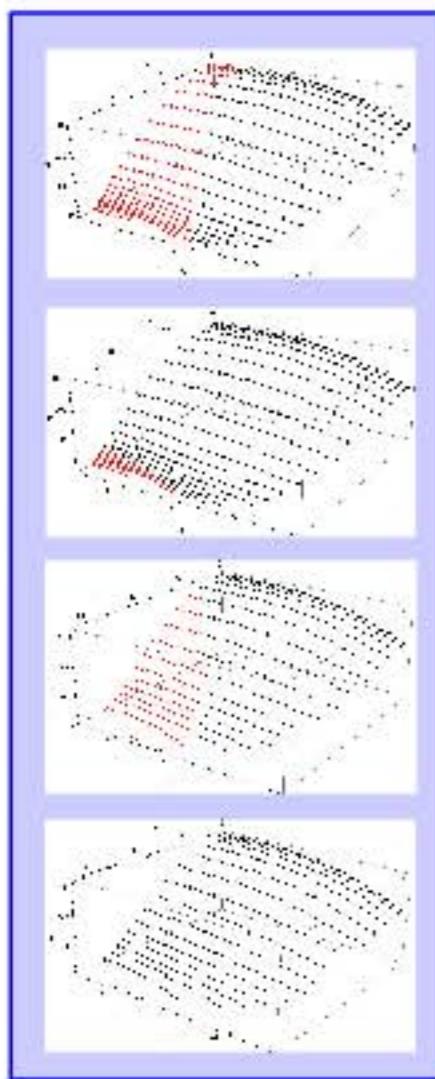
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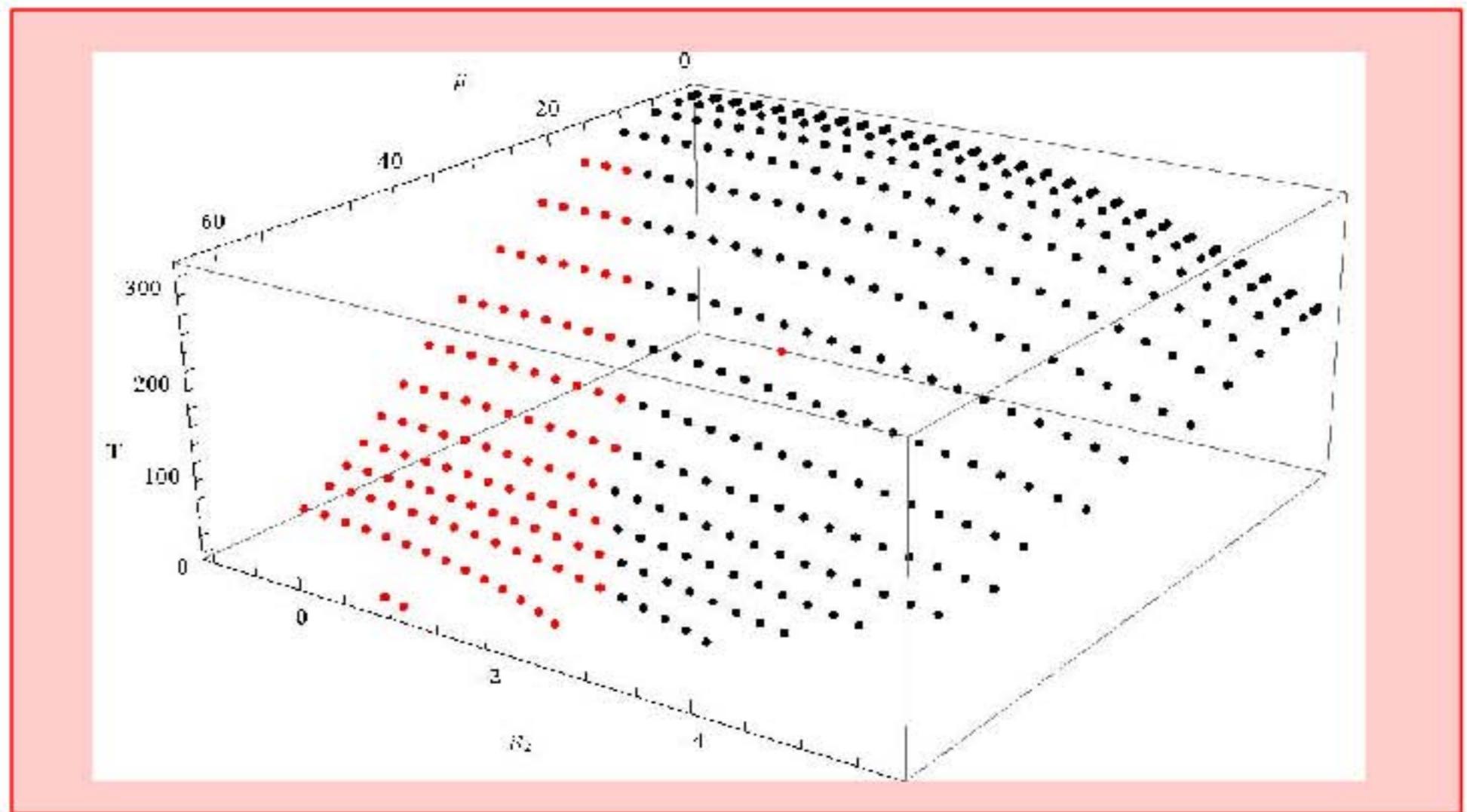
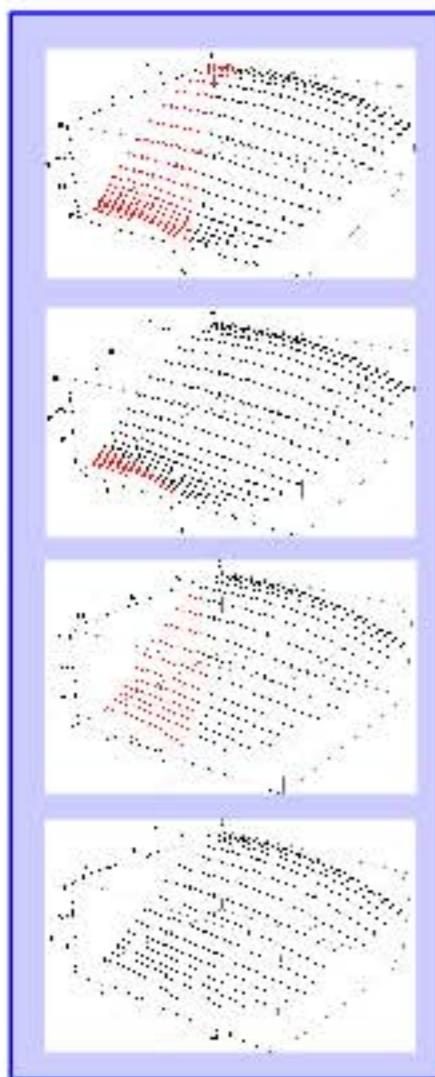
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