On Holographic Stress Tensor of Critical Gravity

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- "Gravitational actions (in the absence of the cosmological term), which include terms quadratic in the curvature tensor are renormalizable.These theories suffer from ghost."[stelle (1977)]!

What happens if we include cosmological term?

The action of critical gravity is

$$S = \frac{1}{2k^2}\int_M d^4x\sqrt{-g}\,[R - 2\lambda + \alpha R^2 + \beta R^{\mu\nu}R_{\mu\nu}].$$

- It describes in general a massless graviton,a massive spin-2 field and a massive scalar.
- If we linearize the equations of motion around AdS_4, ($g_{\mu\nu} = \overline{g}_{\mu\nu} + h_{\mu\nu}$), and using the gauge condition $\nabla^{\nu} h_{\nu\mu} = \nabla_{\mu} tr(h)$

$$Tr.E.o.m \;\;\rightarrow\;\; \lambda[h - 2(\alpha + 3\beta)\Box h] = 0$$

If we set $\alpha = -3\beta$ then propagating massive scalar mode vanishes.

So we are left with the result that the variation of the field equations gives

$$(\Box - \frac{2\lambda}{3})(\Box - \frac{4\lambda}{3} - \frac{1}{3\beta})h_{\mu\nu} = 0,$$

$$"\beta = -\frac{1}{2\lambda}" \qquad \textit{Free Ghost}.$$

- Recently it was shown that at the critical point a new mode appears leading to logarithmic mode [(M.Alishahiha,R.Fareghbal)-(I.Gullu,M.Gurses, T.C.Sisman, B.Tekin)-(E.Bergshoeff,O.Hohm,J.Rosseel,P.K. Townsend)(2011)]

$$\mathrm{ds}^2 = \frac{\mathrm{d}\rho^2}{4\rho^2} + \frac{1}{\rho}\mathrm{g_{ij}}(\rho, \mathrm{x})\mathrm{dx^i dx^j},$$

$$\mathrm{g_{ij}}(\rho, \mathrm{x}) = \mathrm{b_{ij}^{(0)}} \log(\rho) + \mathrm{g_{ij}^{(0)}} + \mathrm{b_{ij}^{(3)}} \rho^{\frac{3}{2}} \log(\rho) + \mathrm{g_{ij}^{(3)}} \rho^{\frac{3}{2}} + \ldots$$

Critical Gravity may be dual to $LCFT_3$...

An AlAdS spacetime admits the following metric in a finite neighborhood of the conformal boundary, located at $\rho = 0$:[Fefferman-Graham (1985)]

$$ds^2_{(d+1)} = \frac{d\rho^2}{4\rho^2} + \frac{1}{\rho} g_{ij}(\rho, x) dx^i dx^j,$$

For Pure Einstein gravity:

$$g_{ij}(\rho, x) = g_{(0)ij} + \rho g_{(2)ij} + ... + \rho^{\frac{d}{2}} \log(\rho) b_{(d)ij} + \rho^{\frac{d}{2}} g_{(d)ij} + ...$$

which log-term exist only for $d = 2n$.

- $g_{(d)ij}$ is only partially determined by asymptotics and related to $< T_{ij} >$.
- $b_{(d)ij}$ is related to the Weyl anomaly of the boundary theory [Henningson, Skenderis(1998)].

Asymptotic expansion of metric is depend on the theory

- Pure Einstein gravity (d+1=4)

$$g_{ij}(\rho, x) = g_{ij}^{(0)} + g_{ij}^{(2)}\rho + \ldots$$

- Critical Gravity (d+1=4)

$$g_{ij}(\rho, x) = b_{ij}^{(0)} \log(\rho) + g_{ij}^{(0)} + b_{ij}^{(2)}\rho \log(\rho) + g_{ij}^{(2)}\rho + b_{ij}^{(3)}\rho^{\frac{3}{2}} \log(\rho) + g_{ij}^{(3)}\rho^{\frac{3}{2}} + \ldots$$

$b_{ij}^{(3)}$ is related to log divergence and sign of Weyl anomaly BUT it must be checked!

$$\delta \mathrm{S} = \frac{1}{2\mathrm{k}^2}\int_{\mathrm{M}} \mathrm{d}^4\mathrm{x}\sqrt{-\mathrm{g}}\,(\mathrm{e.o.m})_{\mu\nu}\delta \mathrm{g}^{\mu\nu} + \frac{1}{2\mathrm{k}^2}\,\delta \mathrm{S}_{\partial \mathrm{M}}$$

$$\delta S_{\partial M} = \int d^3x\sqrt{-\gamma}\Big(\mathcal{A}_{ij}\delta g^{ij} + \big[2 - 12\alpha - 48\beta\big]\rho g^{ij}\delta g'_{ij} + \widetilde{\mathcal{B}}^{ij}\delta g'_{ij} + \widetilde{\mathcal{C}}^{ijk}\delta g_{kj,i}\Big)$$

- $$\widetilde{\mathcal{B}}^{ij} \to 0 \quad (\rho \to 0)$$
- $\widetilde{\mathcal{C}}^{ijk}\delta g_{kj,i}$ is not important because boundary of boundary is empty.
- For $b_{ij}^{(0)} = 0$, Critical gravity has well-posed variational principle without need to add Gibbons-Hawking term.

$$\delta S = \frac{1}{2}\int d^3x\sqrt{-g_{(0)}}\, T_{ij}\, \delta g^{ij}_{(0)}$$

- $g^{ij}_{(0)}$ is the metric on the conformal boundary.
- In AdS/CFT correspondence T_{ij} is the expectation value of the stress tensor in a CFT defined on a space with metric conformal to $g^{ij}_{(0)}$.

Substitute this solution in the variation of Critical gravity action

$$g_{ij}(\rho, x) = g_{ij}^{(0)} + b_{ij}^{(2)} \rho \log(\rho) + g_{ij}^{(2)} + b_{ij}^{(3)} \rho^{\frac{3}{2}} \log(\rho) + g_{ij}^{(3)} \rho^{\frac{3}{2}} + \ldots$$

One must add proper counterterms

In Minimal subtraction scheme

- Critical Gravity $b_{ij}^{(0)} = 0$:

$$S_{c.t} = 0.$$

- Pure Einstein Gravity:

$$S_{c.t} = -\frac{1}{2k^2}\int_{\rho=\epsilon} d^d x\sqrt{-\gamma}\Big[2(1-d) + \frac{1}{d-2}R - \frac{1}{(d-4)(d-2)^2}\big(R^{ij}R_{ij} - \frac{d}{4(d-1)}R^2\big) + \ldots - a_{(d)}\log\epsilon\Big]$$

- On-shell action of Critical Gravity will be renormalized without need to any curvature tensor of boundary metric!.
- Although subleading log-term ($b_{ij}^{(3)}$) exist in asymptotic expansion of metric, but gravitational Weyl anomaly is zero.

Question: Is there any way to get ride of $b_{ij}^{(0)}$?

- It is recently shown that with proper Nuemann boundary condition Conformal gravity (Weyl squared term) is equivalent to Pure Einstein gravity [Maldacena(2011)].
- Critical gravity is combination of Conformal gravity with Pure Einstein gravity

$$\mathcal{L}_{crit} = \mathcal{L}_{E.H} - \mathcal{L}_{C.G}$$

- When $b_{ij}^{(0)} = 0$, only massless modes that remain have zero energy and resulting theory is somewhat trivial!

$$\text{Nuemann boundary condition} \leftrightarrow \; b_{ij}^{(0)} = 0 \quad ??$$

Thanks For Your Attention