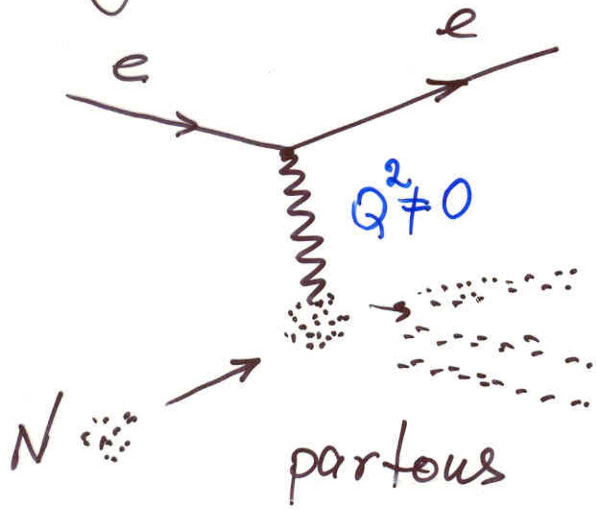
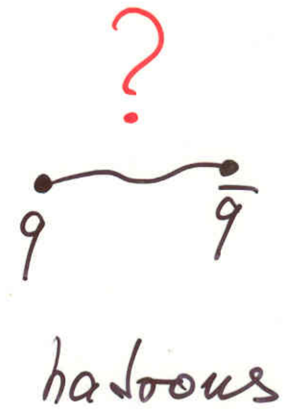
QCD

1973

Asymptotic freedom $\longleftrightarrow$ Confinement

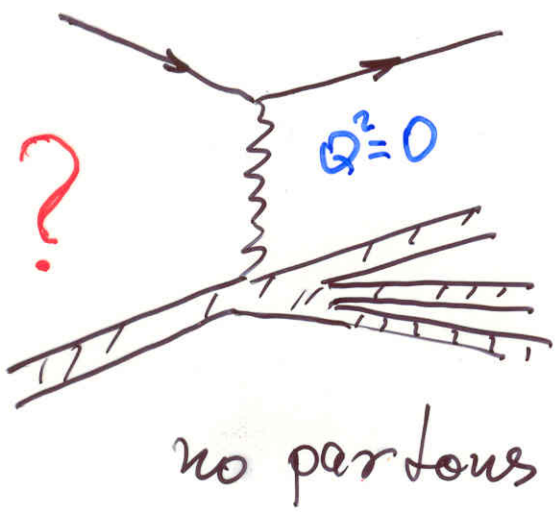


$\longleftrightarrow$

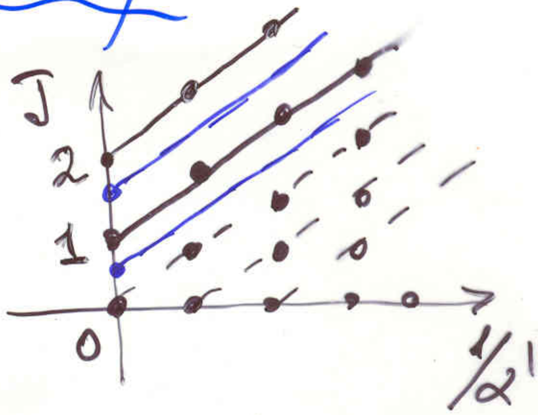


String theory

1968
1971



$\longleftrightarrow$



$$T = \frac{1}{2\pi\alpha'} \neq 0$$

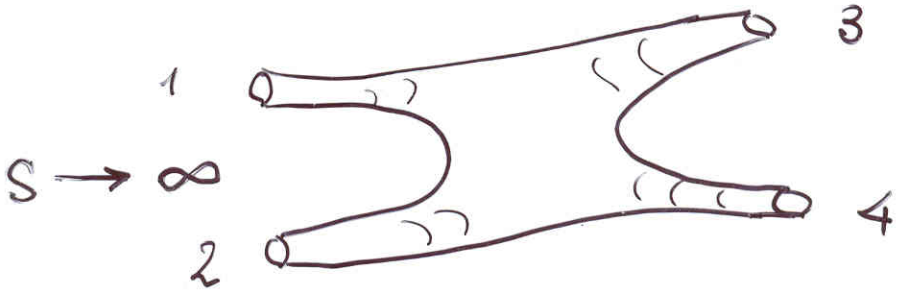
$D_c = 10\,,\ J = 2$

gauge theory amplitudes are <u>hard</u>
while string theory amplitudes are <u>soft</u>

High energy fixed angle scattering

Amati- Ciafoloni- Veneziano (1987)
Gross- Mende (1987)

$\theta < \theta_M$



Amplitudes decay exponentially

$$A \simeq e^{-\alpha' s \ln s - \alpha' t \ln t - \alpha' u \ln u}$$

in all $(s\, t\, u)$ invariants in sharp contrast to the power law fall off in all known QFT.

- Gross (1988)

$$\alpha' \to \infty \qquad M_n^2 = \frac{1}{\alpha'}(n-1) \to 0$$

unbroken phase of string theory

high symmetries?

- $\alpha' \to 0$ FT limit $M_n^2 \to \infty$

Duality map (Maldacena)

Gauge Fields $\longleftrightarrow$ Strings IIB

$SU(N)$, $\mathcal{N}=4$

Classical Supergravity approximation $AdS_5 \times S_5$

$\lambda = g_{YM}^2 N$

$$\sqrt{\lambda} = \frac{R^2}{\alpha'}$$

R^2; α'

- Strong Coupling

$\lambda \gg 1$ $\qquad$ $\alpha' \to 0$ $\qquad$ $\frac{R^2}{\alpha'} \gg 1$

- free gauge theory (Witten, Sundborg)

$\lambda \ll 1$ $\qquad$ $T \equiv \frac{1}{\alpha'} = \frac{\sqrt{\lambda}}{R^2_{AdS}} \to 0$

$\alpha' \to \infty$

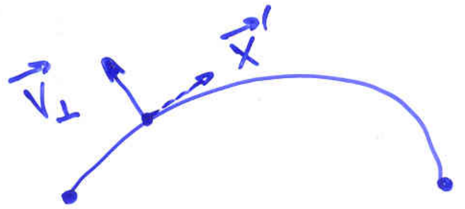
tensionless limit

Nambu-Goto Action

$$S = T \int \sqrt{-\dot{X}^2 X'^2 + (\dot{X} X')^2}\, d\sigma\, d\tau$$

in time like gauge $X^0 \sim \tau$

$$= T \int \sqrt{1 - \vec{V}_\perp^2(s,t)}\, ds\, dt$$

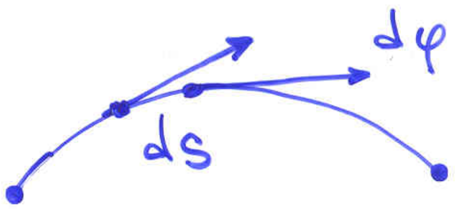


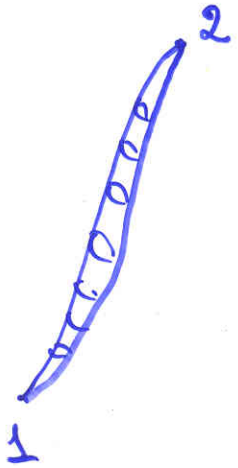
$$ds = \sqrt{\vec{x}'^2}\, d\sigma$$

Tensionless Strings

$$S = m \cdot \int \sqrt{1 - \vec{V}_\perp^2(s,t)}\, \underline{K(s,t)\, ds\, dt}$$

$$K = \frac{d\varphi}{ds}$$

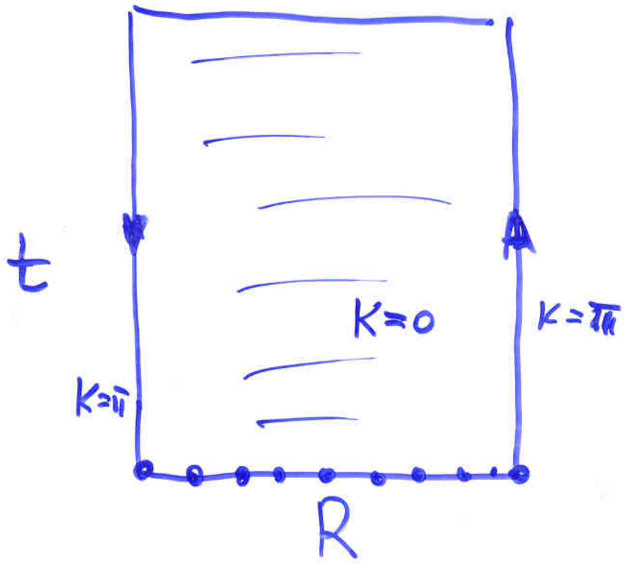




$$\longrightarrow \quad S = m \int \sqrt{1 - \vec{V}^2} \cdot 2\pi \cdot dt$$

point like particle

Tensionless Strings



flat Wilson loop

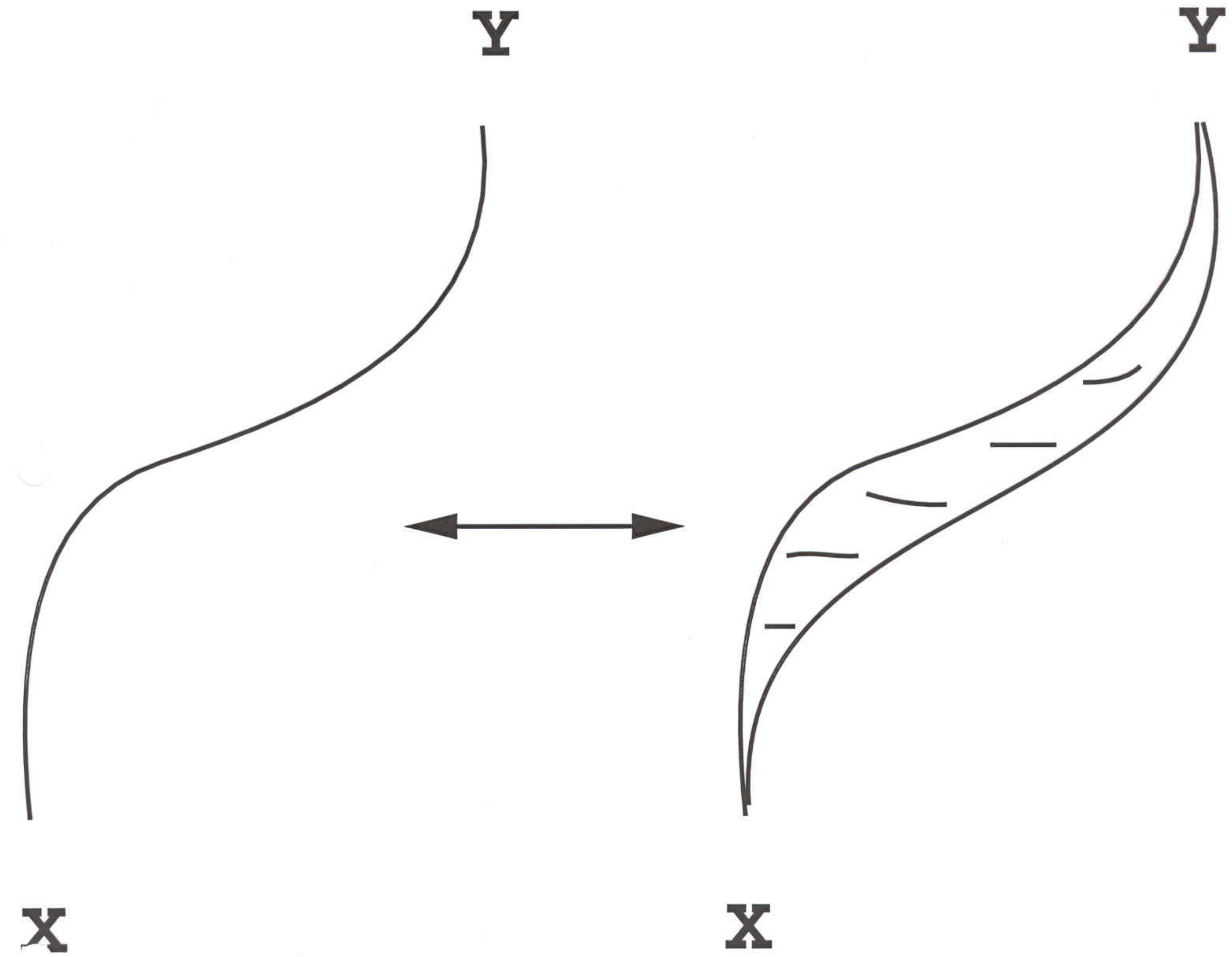
$$S \sim m(R+t)$$

$$e^{-m(R+t)} \sim e^{-V(R)\cdot t}$$

$$t \to \infty$$

$$V(R) = m$$

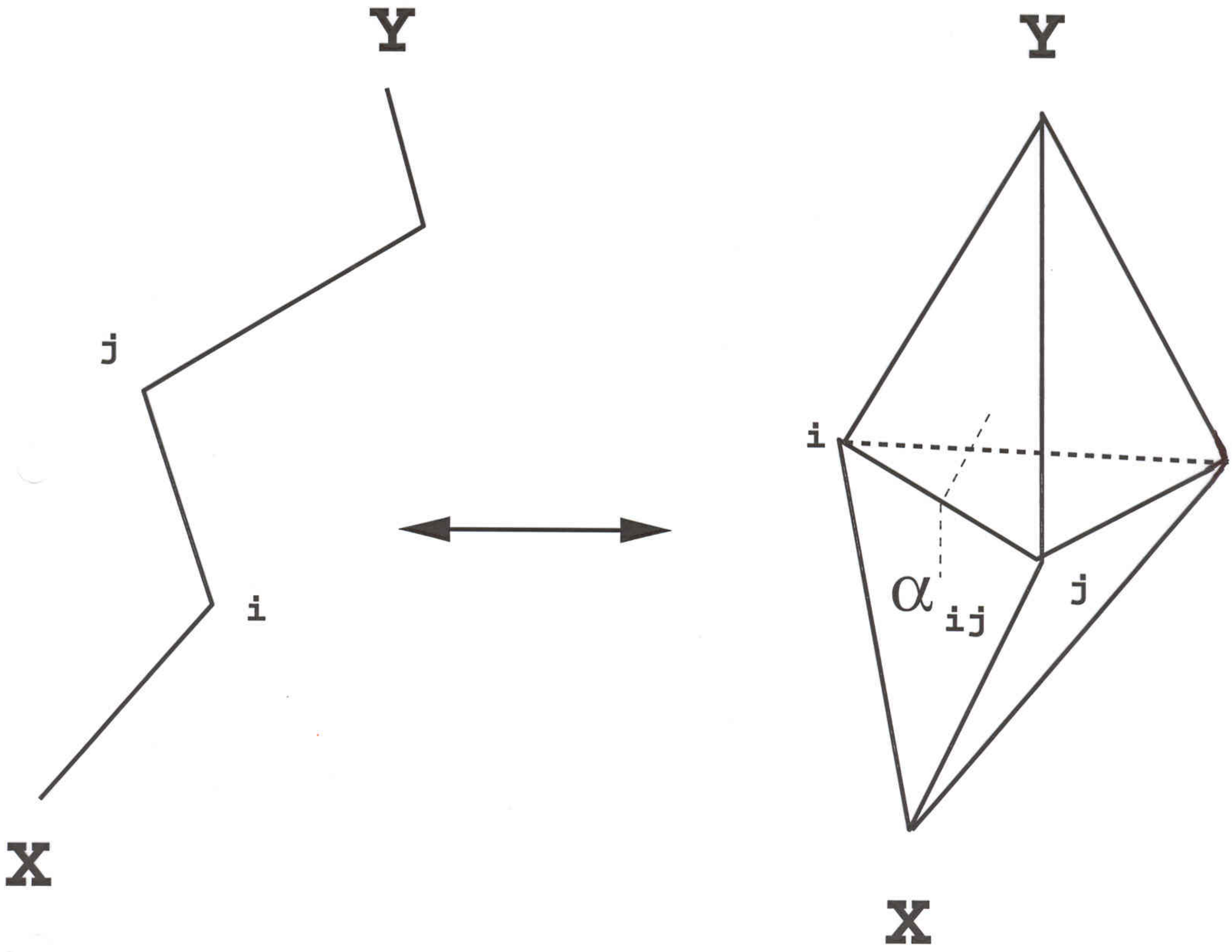
$$F = \frac{\partial V}{\partial R} = 0$$



$$K(x,y)$$

$$\sum_{\text{paths}} e^{-A_{xy}} \longleftrightarrow \sum_{\text{surfaces}} e^{-A_{xy}}$$

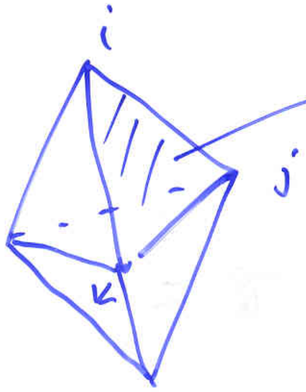
$$[\,A_{xy}\,] = \text{cm}$$



A_{xy}

$$m\sum_{<ij>}\lambda_{ij} \longleftrightarrow m\sum_{<ij>}\lambda_{ij}\,(\pi-\alpha_{ij})^{\zeta}$$

Regge gravity



area s_{ijk}

$$S = \int_{M_4} R\sqrt{g}\, d^4x \longleftrightarrow$$

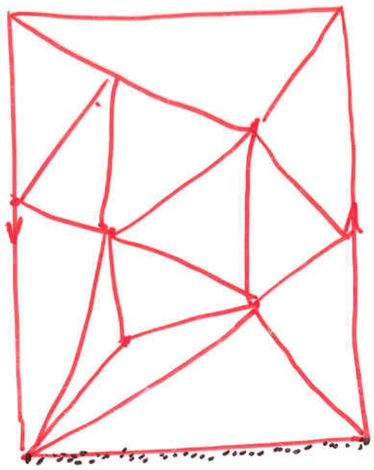
$$\to \sum_{\langle ijk\rangle} s_{ijk} \cdot \omega_{ijk}$$

ω_{ijk} – deficit angle on $\langle ijk\rangle$

$$= (2\pi - \alpha_1 - \alpha_2 \dots - \alpha_n)_{ijk}$$

s_{ijk} – area of $\langle ijk\rangle$

Tensionless String



$$A = (R + T) \cdot m$$

$$W(C) \underset{T\to\infty}{\simeq} e^{-V(R)\cdot T}$$

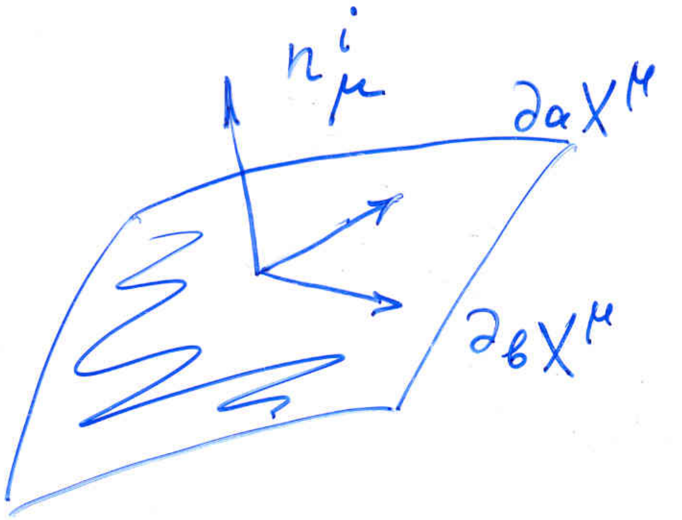
$$V(R) = m = \text{const}$$

$$\sigma_{\text{classical}} = 0$$

String tension is zero

partons are free

The Action



$\mu = 0, 1, \cdots D-1$

$i = 1, \ldots, D-2$

$a, b = 1, 2$

$$g_{ab} = \partial_a X^\mu \partial_b X^\mu \qquad (\mathrm{I})$$

$$K^i_{ab} = \partial_a \partial_b X^\mu \cdot n^i_\mu \qquad (\mathrm{II})$$

$$S = m \cdot \int \sqrt{K^{ia}_{a} K^{ib}_{b}} \sqrt{g}\, d^2 \xi$$

$$\sim m \cdot \int dl$$

$$[K^i] = \frac{1}{cm}$$

Willmore - Kleinert - Polyakov

$$\frac{1}{e^2}\int K^{ia}_{a} K^{ib}_{b} \sqrt{g}\, d^2\xi$$

$$[e^2] = m^0$$

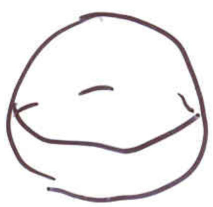
$$= \frac{1}{e^2}\int \left(\Delta(g) X^\mu\right)^2 \sqrt{g}\, d^2\xi$$

in conformal gauge $g_{ab} = \rho\,\delta_{ab}$

$$= \frac{1}{e^2}\int \underline{\frac{1}{\rho}} \left(\partial^2 X^\mu\right)^2 d^2\xi$$

Geometry:

- $\frac{1}{e^2}\int \left(\frac{2}{R}\right)^2 R^2 \cdot 4\pi = \frac{1}{e^2}\cdot \underline{16\pi}$



- $m\int \sqrt{\left(\frac{2}{R}\right)^2} R^2 4\pi = \underline{m}\cdot 8\pi\, \underline{R}$

Weyl Invariance

$$\text{Action} = m \cdot \int d^2\xi \sqrt{g} \sqrt{K^2}$$

$$= m \cdot \int d^2\xi \sqrt{g} \sqrt{(\Delta(g) X^\mu)^2}$$

$$\Delta(g) = \frac{1}{\sqrt{g}} \partial_a \sqrt{g}\, g^{ab} \partial_b$$

in conformal gauge $\quad g_{ab} = \rho \cdot \eta_{ab}$

$$\text{Action} \sim m \int d^2\xi \cdot \sqrt{\rho^2} \cdot \sqrt{\left(\frac{1}{\rho}\right)^2} \sim 1$$

The action is ρ · independent.

Model A

$$g_{ab} = \partial_a X^\mu \partial_b X^\mu$$

Model B

fields $\underline{g_{\alpha\beta}}$ and $\underline{X^\mu}$

are independent variables,

two-dimensional quantum gravity interacting with scalar fields X^μ.

Energy-momentum tensor (B)

$$S = \frac{m}{\pi} \cdot \int d^2\xi \sqrt{g} \cdot \sqrt{(\Delta(g) X^\mu)^2}$$

$$\Delta(g) = \frac{1}{\sqrt{g}} \partial_a \sqrt{g}\, g^{ab} \partial_b$$

$$\delta S = -\frac{1}{2\pi} \int \sqrt{g} \cdot T_{ab} \cdot \delta g^{ab} \cdot d^2\xi$$

$$T_{ab} = \partial_{\{a} \left(m \frac{\Delta(g) X^\mu}{\sqrt{(\Delta(g) X)^2}} \right) \cdot \partial_{b\}} X^\mu - g_{ab} g^{cd} \partial_c \left(m \frac{\Delta(g) X^\mu}{\sqrt{(\Delta(g) X)^2}} \right) \partial_d X^\mu$$

- it is traceless:

$$g^{ab} T_{ab} = 0 \quad , \quad \nabla^a T_{ab} = 0$$

Equation for X^μ:

$$\frac{\pi}{\sqrt{g}} \frac{\delta S}{\delta X^\mu} = \Delta(g) \cdot \left(m \frac{\Delta(g) X^\mu}{\sqrt{(\Delta(g) X)^2}} \right) = 0$$

Let us introduce:

$$\Pi^{\mu} = m \frac{\Delta(g) X^{\mu}}{\sqrt{(\Delta(g) X)^2}}$$

then

$$\Pi^{\mu} \Pi_{\mu} = m^2$$

and our equations are:

$$\Delta(g) \cdot \Pi^{\mu} = 0$$

$$T_{ab} = \nabla_{\{a} \Pi^{\mu} \nabla_{b\}} X^{\mu} - g_{ab}\, g^{cd} \nabla_c \Pi^{\mu} \nabla_d X^{\mu} = 0$$

<u>Conformal gauge</u>

$$g_{ab} = \rho \cdot \eta_{ab} \qquad \eta_{ab} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\partial^2 \left(m \frac{\partial^2 X^{\mu}}{\sqrt{(\partial^2 X)^2}} \right) = 0$$

$$T_{ab} = \partial_a \left(m \frac{\partial^2 X^{\mu}}{\sqrt{(\partial^2 X)^2}} \right) \cdot \partial_b X^{\mu} - \eta_{ab} \cdot \partial^c \left(m \frac{\partial^2 X^{\mu}}{\sqrt{(\partial^2 X)^2}} \right) \partial_c X_{\mu} = 0$$

Light cone coordinates $\tau^{\pm} = \tau^0 \pm \tau^1$

$$g_{ab} = \frac{1}{2}\begin{pmatrix} 0 & \rho \\ \rho & 0 \end{pmatrix}$$

The components of energy-momentum tensor are:

$$T_{++} = \frac{1}{2}(T_{00} + T_{01}) = 2\,\partial_+ \Pi^\mu \cdot \partial_+ X^\mu$$

$$T_{--} = \frac{1}{2}(T_{00} - T_{01}) = 2\,\partial_- \Pi^\mu \cdot \partial_- X^\mu$$

$$T_{+-} = \frac{1}{2}(T_{00} - T_{11}) = 0 \quad \leftarrow \text{trace}$$

The conservation of the T_{ab} take the form:

$$\begin{aligned} \partial_- T_{++} &= 0 \\ \partial_+ T_{--} &= 0 \end{aligned} \quad \Longrightarrow \quad \begin{aligned} T_{++} &= T_{++}(\tau^+) \\ T_{--} &= T_{--}(\tau^-) \end{aligned}$$

$$S' = \frac{4m}{\pi} \int \sqrt{(\partial_+ \partial_- X^\mu)^2}\, d\tau^+ d\tau^-$$

$$\tau^+ = f(\tilde{\tau}^+)$$

$$\tau^- = g(\tilde{\tau}^-)$$

Solution of classical equations.

$$\partial^2\left(m\,\frac{\partial^2 X^\mu}{\sqrt{(\partial^2 X^\mu)^2}}\right)=0 \qquad \Pi^\mu = m\,\frac{\partial^2 X^\mu}{\sqrt{(\partial^2 X)^2}}$$

$$X^\mu = \frac{1}{2}\left[X_L^\mu(\xi^+) + X_R^\mu(\xi^-)\right] +$$

$$+\frac{1}{2}\int_0^{\xi^+}\!\!\int_0^{\xi^-}\left[\Pi_L^\mu(\tilde\xi^+) + \Pi_R^\mu(\tilde\xi^-)\right]\cdot\underline{\Omega}(\tilde\xi^+,\tilde\xi^-)\,d\tilde\xi^+\,d\tilde\xi^-$$

$$T_{++} = \frac{1}{2}\,\dot\Pi_L^\mu(\xi^+)\cdot\dot X_L^\mu(\xi^+)$$

$$T_{--} = \frac{1}{2}\,\dot\Pi_R^\mu(\xi^-)\cdot\dot X_R^\mu(\xi^-)$$

are indeed functions of only one light cone variable.

Quantization

global symmetries $\delta X^\mu = \Lambda^{\mu\nu} X_\nu + a^\mu$

$$P^\mu_a = \partial_a \Pi^\mu = \partial_a \left(m \frac{\partial^2 X^\mu}{\sqrt{(\partial^2 X)^2}} \right)$$

$P^\mu(\sigma^0;\sigma^1) \equiv P^\mu_0 = \partial_0 \Pi^\mu$ is conjugate to X^μ

$$[X^\mu(\sigma^0;\sigma^1); P^\nu(\sigma^0;\tilde{\sigma}^1)] = i\eta^{\mu\nu}\delta(\sigma^1-\tilde{\sigma}^1)$$

One can deduce that the following relations should hold:

$$[\partial_+ X^\mu_L(\sigma); \partial_+ \Pi^\nu_L(\sigma')] = i\pi\,\delta'(\sigma-\sigma')$$

$$[\partial_- X^\mu_R(\sigma); \partial_- \Pi^\nu_R(\sigma')] = i\pi\,\delta'(\sigma-\sigma')$$

$$[\partial_\pm X^\mu_{L,R}(\sigma); \partial_\pm X^\nu_{L,R}(\sigma')] = 0$$

$$[\partial_\pm \Pi^\mu_{L,R}(\sigma); \partial_\pm \Pi^\nu_{L,R}(\sigma')] = 0.$$

The equivalent form of the action

$$S = \frac{m}{\pi}\int d^2\xi \sqrt{(\partial^2 X)^2} = \frac{1}{\pi}\int d^2\xi \cdot \Pi^{\mu}\partial^2 X^{\mu}$$

we can deduce the propagator:

$$\langle \Pi^{\mu}(\xi)\, X^{\nu}(\tilde{\xi})\rangle = -\frac{\eta^{\mu\nu}}{2}\ln\left(|\xi-\tilde{\xi}|\mu\right)$$

and then

$$\langle \Pi^{\mu}_{R}(\xi^{-})\, X^{\nu}_{R}(\tilde{\xi}^{-})\rangle = -\eta^{\mu\nu}\ln\left(|\xi^{-}-\tilde{\xi}^{-}|\mu\right)$$

$$\langle \Pi^{\mu}_{L}(\xi^{+})\, X^{\nu}_{L}(\tilde{\xi}^{+})\rangle = -\eta^{\mu\nu}\ln\left(|\xi^{+}-\tilde{\xi}^{+}|\mu\right)$$

Two point correlation function of the energy-momentum operator is:

$$\langle T\, T_{++}(\xi^{+})\, T_{++}(\tilde{\xi}^{+})\rangle =$$

$$= \frac{1}{4}\langle T : \dot{\Pi}^{\mu}_{L}(\xi^{+})\dot{X}^{\mu}_{L}(\xi^{+}) :: \dot{\Pi}^{\nu}_{L}(\tilde{\xi}^{+})\dot{X}^{\nu}_{L}(\tilde{\xi}^{+}) : \rangle$$

$$= \frac{1}{4}\langle \dot{\Pi}^{\mu}_{L}(\xi^{+})\dot{X}^{\nu}_{L}(\tilde{\xi}^{+})\rangle\langle \dot{X}^{\mu}_{L}(\xi^{+})\dot{\Pi}^{\nu}_{L}(\tilde{\xi}^{+})\rangle$$

$$= \frac{1}{4}\frac{\eta^{\mu\nu}}{(\xi^{+}-\tilde{\xi}^{+})^{2}}\cdot\frac{\eta^{\mu\nu}}{(\xi^{+}-\tilde{\xi}^{+})^{2}} = \frac{1}{4}\frac{D}{(\xi^{+}-\tilde{\xi}^{+})^{4}}$$

The ghost contribution is

$$-\frac{13}{4}\frac{1}{(\xi^{+}-\tilde{\xi}^{+})^{4}}$$

The absence of conformal anomaly requires that the total central charge should be zero

$$\frac{D}{4} - \frac{13}{4} = 0 \qquad D_c = 13$$

Mode expansion

for closed strings: $X^{\mu}(\tau,\sigma) = X^{\mu}(\tau,\sigma+2\pi)$

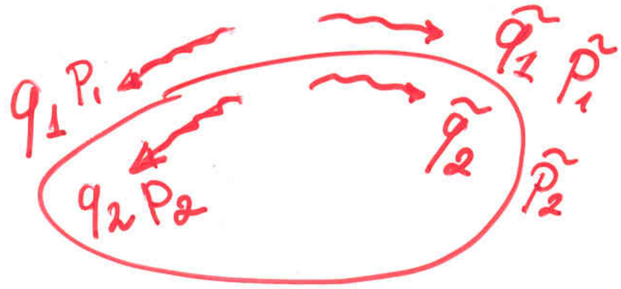
$$X_L^{\mu} = x^{\mu} + \frac{1}{m}\pi^{\mu}\xi^{+} + \sum_{n=1}^{\infty}\sqrt{\frac{2}{nm^2}}\left\{q_{1n}^{\mu}\sin n\xi^{+} + q_{2n}^{\mu}\cos n\xi^{+}\right\}$$

$$X_R^{\mu} = x^{\mu} + \frac{1}{m}\pi^{\mu}\xi^{-} + \sum_{n=1}^{\infty}\sqrt{\frac{2}{nm^2}}\left\{\tilde{q}_{1n}^{\mu}\sin n\xi^{-} + \tilde{q}_{2n}^{\mu}\cos n\xi^{-}\right\}$$

q_{1n} q_{2n} $\tilde{q}_{1n}$ $\tilde{q}_{2n}$ are internal degrees of freedom

$$\Pi_L^{\mu} = me^{\mu} + \kappa^{\mu}\xi^{+} + \sum_{n=1}^{\infty}\sqrt{\frac{2m^2}{n}}\left\{-p_{1n}^{\mu}\cos n\xi^{+} + p_{2n}^{\mu}\sin n\xi^{+}\right\}$$

$$\Pi_R^{\mu} = me^{\mu} + \kappa^{\mu}\xi^{-} + \sum_{n=1}^{\infty}\sqrt{\frac{2m^2}{n}}\left\{-\tilde{p}_{1n}^{\mu}\cos n\xi^{-} + \tilde{p}_{2n}^{\mu}\sin n\xi^{-}\right\}$$



zero modes are x^{μ}, p^{μ} e^{μ}, π^{μ}

From basic commutators

$$[X^{\mu}_{L,R}(s), P^{\nu}_{L,R}(s')] = 2\pi i \eta^{\mu\nu} \delta(s-s')$$

we can get:

$$[e^{\mu}, \pi^{\nu}] = i\eta^{\mu\nu}$$

$$[x^{\mu}, k^{\nu}] = i\eta^{\mu\nu}$$

$$[q^{\mu}_{in}, p^{\nu}_{jm}] = i\eta^{\mu\nu}\delta_{ij}\delta_{nm}$$

Introducing operators

$$a^{\mu}_{n} = \frac{p^{\mu}_{2n} + i p^{\mu}_{1n}}{\sqrt{2}} \qquad a^{+\mu}_{n} = \frac{p^{\mu}_{2n} - i p^{\mu}_{1n}}{\sqrt{2}}$$

$$b^{\mu}_{n} = \frac{q^{\mu}_{1n} - i q^{\mu}_{2n}}{\sqrt{2}} \qquad b^{+\mu}_{n} = \frac{q^{\mu}_{1n} + i q^{\mu}_{2n}}{\sqrt{2}}$$

and then

$$\alpha^{\mu}_{n} = m\sqrt{n}\, a^{\mu}_{n} \quad n>0 \qquad \alpha^{\mu}_{0} = k^{\mu}$$

$$\alpha^{\mu}_{-n} = m\sqrt{n}\, a^{+\mu}_{n} \quad n>0$$

$$\beta^{\mu}_{n} = \frac{1}{m}\sqrt{n}\, b^{\mu}_{n} \quad n>0 \qquad \beta^{\mu}_{0} = \frac{1}{m}\pi^{\mu}$$

$$\beta^{\mu}_{-n} = \frac{1}{m}\sqrt{n}\, b^{+\mu}_{n} \quad n>0$$

$$[\alpha^\mu_n, \beta^\nu_m] = n\,\eta^{\mu\nu}\,\delta_{n+k,0}$$

$$P^\mu_L = \sum \alpha^\mu_n e^{-in\xi^+}$$

$$\partial_+ X^\mu_L = \sum \beta^\mu_n e^{-in\xi^+}$$

We can find now Virasoro generators:

$$L_n = \langle e^{in\xi^+} : P^\mu_L \partial_+ X^\mu_L \rangle$$

$$= \langle e^{in\xi^+} T_{++} \rangle$$

$$= \sum_k : \alpha_{n-k} \beta_k :$$

Now we can compute the central charge

$$[L_n, L_m] = (n-m)\cdot L_{n+m} + \frac{D}{6}(n^3 - n)\,\delta_{n+m,0}$$

We have two time more degrees of freedom than in the standard case

$$\frac{D}{12} \qquad\qquad 2\cdot\frac{D}{12} = \frac{D}{6}$$