Area Spectrum & Quasinormal Modes of Black Holes

hep-th/0304135

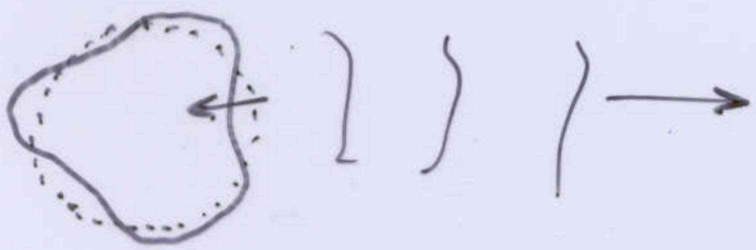
Schwarzschild Black Hole:

$$A = 16\pi M^2 \qquad (c = G = 1)$$

$$S_{BH} = \frac{1}{4} A \qquad (k_B = \hbar = 1)$$

$$\left(\frac{1}{T_{BH}} = \frac{\partial S_{BH}}{\partial M} = 8\pi M \right)$$

Quasinormal Modes:



(Nollert, Kokkotas, Schmidt)

$$\omega_n = \frac{0.04371\ldots}{M} - i\,\frac{2\pi}{8\pi M}\left(n + \frac{1}{2}\right)$$

$$(\text{Hod}) = \frac{\ln 3}{8\pi M} - i\,\frac{2\pi}{8\pi M}\left(n + \frac{1}{2}\right), \quad n \gg 1$$

(Analytic proof by Motl; M+Neitzke)

(Hod) $\hbar \omega_{real} = \Delta E = \Delta M$

$$\Delta A = 32\pi M \Delta M = 4 \ln 3$$

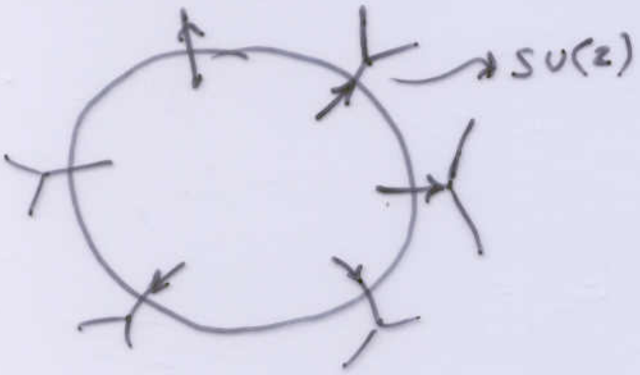
Suggest "it from bit" picture with 3-state bits:

$$\left.\begin{aligned} A &= N \cdot 4 \ln 3 \\ S &= N \cdot \ln 3 \end{aligned}\right\} \longrightarrow S = \frac{1}{4} A$$

Is there a microscopic explanation? . . .

LOOP QUANTUM GRAVITY (SPIN FOAM)

(Ashtekar, Smolin, Rovelli, Corichi, Krasnov, Lewandowski...)



$$A = \sum_{spins} a_j$$

$$S = \ln \Omega_{spins}$$

$$a_j = \lambda f(C_{2,j}) = \lambda \sqrt{j(j+1)} \quad *$$

($\hookrightarrow$ Immirzi parameter)

Gauss Law constraint (SU(2)-singlet)

For large black holes ($A \gg 1$)

$$\Omega = \prod_{j=0}^{\infty} (2j+1)^{N_j} \quad *$$

(N_j = # of spin-j links)

Introduce a temperature parameter dual to area:

$$Z = \sum_{\text{states}} \Omega(A)\, e^{-\beta A}$$

$$= \sum_{\{N_j\}} \prod_j (2j+1)^{N_j}\, e^{-\beta \sum_j N_j a_j}$$

$$= \prod_j \frac{1}{1-(2j+1)\, e^{-\beta a_j}}$$

We have a Bose condensation at $\beta = \beta_c$:

$$\beta_c = \max_j \left(\frac{\ln(2j+1)}{a_j} \right)$$

With standard spectrum ($a_j \sim \sqrt{j(j+1)}$) condensate forms at $j = \frac{1}{2}$ (2-state "bits")

$\Rightarrow$ incompatible with Hod's result.

Dreyer: relevant group is $SO(3)$ rather than $SU(2)$ (? ...)

$\rightarrow$ only integer spins allowed

Criticality now at $j=1$ (3-state bits)

$\hookrightarrow$ recovery of $4\ln 3$ result.

However...

(Alekseev, A.P., Smedbäck)

Formula for $c_{e,j}$ receives additive quantum correction: $c_{e,j} + \frac{1}{4}$

$$\Rightarrow * \; a_j = \lambda \left(j + \tfrac{1}{2}\right)$$ equidistant spectrum (Bekenstein....)

Now critical value for β shifts to $j = 1$!

↪ reproduces $\Delta A = 4 \ln 3$ without restriction to even spins...

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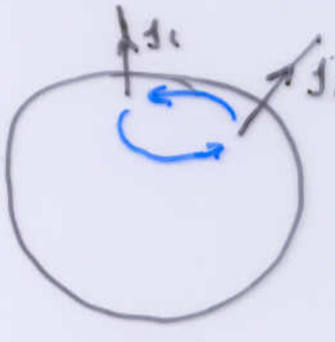
But...

Standard formula for Ω implies a peculiar partial distinguishability for links:



If $j_1 \neq j_2$:

$$|j_1, m_1; j_2, m_2\rangle = |j_2, m_2; j_1, m_1\rangle$$

However, if $j_1 = j_2$:

$$|j, m_1; j, m_2\rangle \neq |j, m_2; j, m_1\rangle$$

If fully indistinguishable: NO area-entropy law

We argue for fully distinguishable links.

Now Ω, thermodynamics change:

$$Z = \sum_N Z_1^N = \frac{1}{1 - Z_1}$$

where $Z_1 = \sum_j (2j+1)\, e^{-\beta a_j}$

Macroscopic black hole at $\beta = \beta_c$:

$$Z_1(\beta_c) = 1$$

No Bose condensate, Boltzmann distribution of j's

$$S = \beta A + \ln Z$$

$(\beta \to \beta_c,\ Z \sim A)$

$$S = \beta_c A$$

- Non-equidistant spectrum $(\sim \sqrt{j(j+1)})$ is OUT
- For $a_j = \lambda (j + \frac{1}{2}) \Rightarrow \Delta a = \lambda \cdot \frac{1}{2} = 4 \ln 3$

we get:

$$\beta_c = \frac{1}{4} \ln \frac{3+\sqrt{5}}{6} = 0.876\, \frac{1}{4}$$ (rats...!)

What can be done?

Keep equidistant spectrum and distinguishable statistics, but modify $SU(2) \rightarrow SO(3)$ as in Dreyer.

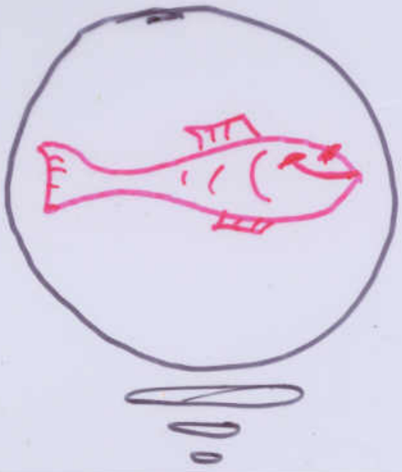
Then:

$$Z_1 = \sum_{n=0}^{\infty} (2n+1)\, e^{-\beta\gamma(n+1)}$$

$$= \frac{e^{\beta\gamma}+1}{(e^{\beta\gamma}-1)^2} \quad \text{and we get:}$$

$$\left.\begin{aligned} \beta_c &= \frac{\ln 3}{\gamma} \\ \gamma &= 4 \ln 3 \end{aligned}\right\} \Rightarrow \beta_c = \frac{1}{4}$$

- Reproduces quasinormal mode picture
- NO 3-state domination.

- Which picture is true (if any)?
- Charge, angular momentum ... ?
- Physics of decay process...?
- Is all this just a ...  ?