Intersecting D-branes – A Path to the Standard Model?

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I) Introduction

Goal of superstring theory:

Embedding of the Standard Model into a unified description of gravitational and gauge forces.

Obstacles on the way:

- How to derive the precise SM spectrum?
- How to determine the precise SM couplings?
- How to break space-time supersymmetry?
- How to fix the values of the moduli?
- How to select the groundstate from an (apparent) huge vacuum degeneracy?
- How to describe the cosmological evolution of the universe?
- What is the structure of space and time at short distances?

I) Introduction

Further plan of the talk:

II) Intersecting brane world models:

- Construction
- The question of space-time supersymmetry

III) MSSM-like models and gauge coupling unification

IV) Type II Compactifications with D-branes and fluxes

V) Geometrical backreaction of D-branes/fluxes $\longrightarrow$ Non Calabi-Yau compactifications

VI) Heterotic Strings with fluxes

II) Intersecting Brane World Models

The progress in type II string physics was made possible due the discovery of D-branes.

(Polchinski)

D(p)-branes are higher(p)-dimensional topological defects, i.e. hypersurfaces, on which open strings are free to move.

They have led to several new insights:

- Non-Abelian gauge bosons as open strings on the world volumes π of the D-branes $\rightarrow$ Brane world models

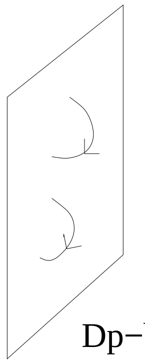
- Chiral fermions are open strings living on the intersections of two D-branes

$$N_F = I_{ab} \equiv \#(\pi_a \cap \pi_b) \equiv \pi_a \circ \pi_b$$

- Correspond to non-trivial gravitational backgrounds $\rightarrow$ AdS/CFT correspondence

II) Intersecting Brane World Models

Simplest D-brane configuration: 1 single Dp-brane:



Massless open string spectrum: $U(1)$ gauge boson $\longrightarrow$ supersymmetric $U(1)$ gauge theory in $p+1$ dimensions

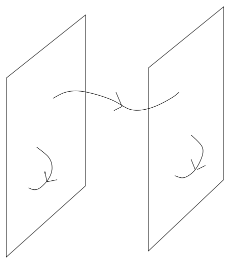
$$\mathcal{S}_{\text{eff}} = \int_\pi dx^{p+1} \; (\underbrace{\mathcal{L}_{\text{DBI}}(g,F,\phi)}_{\text{Tension}} + \underbrace{\mathcal{L}_{\text{CS}}(F,C_{p+1})}_{\text{Charge}})$$

Effective gauge interactions due to the exchange of open strings:

$$\begin{aligned} S_{DBI} &= \tau_p \int \mathrm{d}^{p+1}x \sqrt{\det(g_{\mu\nu} + \tau^{-1} F_{\mu\nu})} \\ &= \left(\frac{M^{p-3}_{\text{string}}}{g_{\text{string}}}\right) \int \mathrm{d}^{p+1}x \, F^2_{\mu\nu} + \ldots \end{aligned}$$

Other D-brane configurations (in flat space-time):

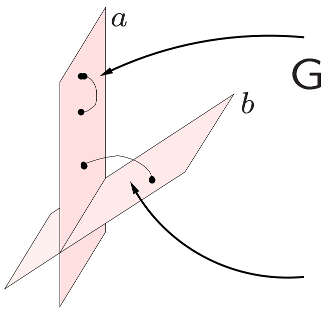
- N parallel Dp-branes



$\mathcal{N}=4$ supersymmetric $U(N)$ gauge theory in $p+1$ dimensions

- Intersecting D-branes

(Berkooz, Douglas, Leigh)



Gauge bosons in adj.

Chiral matter in $(N, \bar{M})$

Open string spectrum:

(i) $\mathcal{N}=4$ gauge bos. in adj. repr. of $U(N) \times U(M)$

(ii) Massless fermions in chiral $(N, \bar{M})$ repres.

(iii) Massive scalars in $(N, \bar{M})$ repes.

II) Intersecting Brane World Models

Intersecting D-branes break space-time supersymmetry!
This supersymmetry breaking manifests itself as the a massive/tachyonic scalar groundstate:

$$M_{ab}^2 = \frac{1}{2}\sum_I \Delta\Phi_{ab}^I - \max\{\Delta\Phi_{ab}^I\}$$

Massless scalars $\Leftrightarrow$ open string sector is supersymmetric.

Two flat supersymmetric D6-brane configurations:

- 2 intersecting D6-branes, intersect in 4-5 and 6-7 planes, parallel in 8-9 plane:

  $$1/4 \text{ BPS } (\mathcal{N} = 2\ SUSY): \quad \Phi^1 + \Phi^2 = 0$$

- 2 intersecting D6-branes, intersect in 4-5, 6-7 and 8-9 planes:

  $$1/8 \text{ BPS } (\mathcal{N} = 1\ SUSY): \quad \Phi^1 + \Phi^2 + \Phi^3 = \pi$$

In case the open string scalar is tachyonic ($M_{ab}^2 < 0$) $\longrightarrow$ the 2 different branes will recombine into a single brane.
Brane recombination $\longleftrightarrow$ Tachyonic Higgs effect *(Sen)*

Intersecting type IIA brane-world-models:

(Blumenhagen, Görlich, Körs, D.L., hep-th/0007024);

(Blumenhagen, Braun, Körs, D.L., hep-th/0206038)

(i) Choose compact orientifold background

$\mathcal{M}^{10} = (\mathbb{R}^{3,1} \times \mathcal{M}^6)/(\Omega\overline{\sigma})\,,\quad \Omega$: world sheet parity

$\overline{\sigma}$: $z_i \to \bar{z}_i$ anti-holomorphic involution. The orientifold 6-plane π_{O6} is the fixed locus, Fix($\overline{\sigma}$), which is a sLag 3-cycle, implying

$$\text{Vol}(\text{Fix}(\overline{\sigma})) = \int_{\text{Fix}(\overline{\sigma})} \Re(\Omega_3)\,.$$

(ii) Introduce D6-branes wrapped around the supersymmetric (sLag) 3-cycles π_a and their $\Omega\overline{\sigma}$ images π'_a of the internal Calabi-Yau space $\mathcal{M}^6$, which intersect in $\mathcal{M}^6$.

Massless spectrum:

- $\mathcal{N} = 1$ supergravity in the 10D bulk
- 7-dim. $\mathcal{N} = 1$ $U(N_a)$ gauge bosons localized on the D6-branes wrapped around 3-cycles π_a ($codim = 3$).
- 4-dim. chiral fermions localized on the intersections of the D6-branes ($codim = 6$).

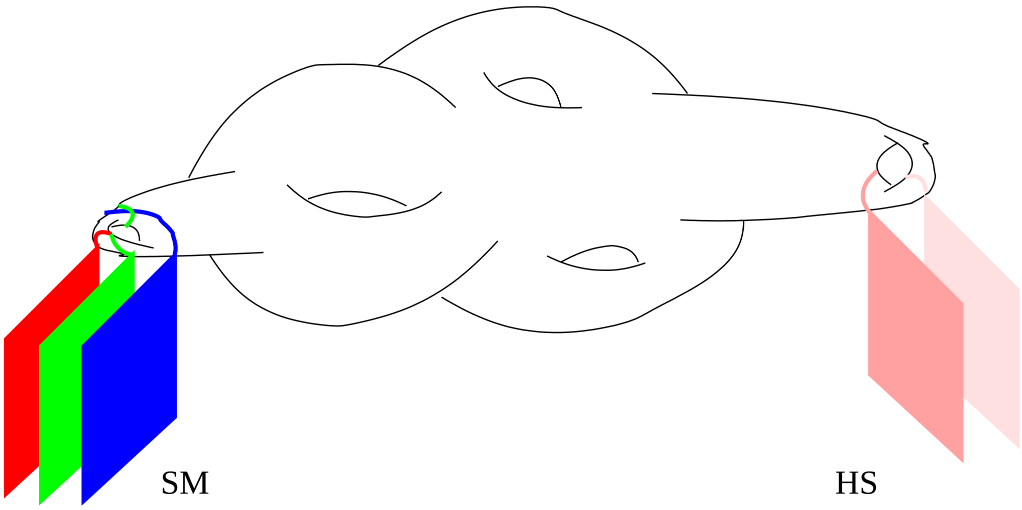
Since the chiral spectrum has to satisfy some anomaly constraints, we expect that it is given by purely topological data (Atiyah-Singer index theorem).

The chiral massless spectrum indeed is completely fixed by the topological intersection numbers of the 3-cycles of the configuration.

Sector	**Rep.**	**Number**
$a'\,a$	A_a	$\frac{1}{2}\,(\pi'_a \circ \pi_a + \pi_{O6} \circ \pi_a)$
$a'\,a$	S_a	$\frac{1}{2}\,(\pi'_a \circ \pi_a - \pi_{O6} \circ \pi_a)$
$a\,b$	$(\overline{N}_a, N_b)$	$\pi_a \circ \pi_b$
$a'\,b$	(N_a, N_b)	$\pi'_a \circ \pi_b$

The non-abelian gauge anomalies will cancel after satisfying the tadpole conditions and mixed $U(1)_a - SU(N)_b^2$ anomalies are canceled by a generalized Green-Schwarz mechanism involving dimensionally reduced RR-forms.

View on the internal Calabi-Yau space:



Many non-supersymmetric as well as $\mathcal{N} = 1$ supersymmetric intersecting brane world models on tori, orbifolds, or the quintic Calabi-Yau manifold with gauge group

$$G = SU(3)_c \times SU(2)_L \times U(1)_Y$$

and 3 families of quark and leptons can be explicitly constructed.

(Blumenhagen, Braun, Görlich, Ott, Körs, D.L. (2000/01/02); Aldazabal, Cremades, Franco, Ibanez, Marchesano, Rabadan, Uranga; Cvetic, Shiu, Uranga; Bailin, Kraniotis, Love; Kokorelis; Förste, Honecker, Schreyer; Ellis, Kanti, Nanopoulos)

3 possibilities for supersymmetry breaking:

- SM-branes are non-supersymmetric:

  $$M_{\text{susy}} \simeq M_{\text{string}} \sim \mathcal{O}(1 \text{ TeV})$$

  Need for large transversal dimensions $R_\perp$ on the CY!

- SM-branes are supersymmetric ("local" supersymmetry), but are non-supersymmetric with respect to hidden sector branes $\longrightarrow$
  Gravity mediated supersymmetry breaking:

  $$M_{\text{susy}} \simeq \frac{M_{\text{string}}^2}{M_{\text{Planck}}} \simeq \mathcal{O}(1\text{TeV}) \;\Rightarrow\; M_{\text{string}} \simeq \mathcal{O}(10^{11}\text{GeV})$$

  Here the transversal dimensions on the CY are only moderately enlarged, $R_\perp \simeq \mathcal{O}(10^9)$GeV).

- All branes are supersymmetric ("global" supersymmetry) $\longrightarrow$
  Dynamical supersymmetry breaking in hidden sector:

  $$M_{\text{susy}} \simeq \frac{M_{\text{hidden}}^3}{M_{\text{Planck}}^2} \simeq \mathcal{O}(1\text{TeV}) \;\Rightarrow\; M_{\text{hidden}} \simeq \mathcal{O}(10^{13}\text{GeV})$$

Consistency requirements for intersecting branes:

(i) RR-charge cancellation:

This implies absence of anomalies in the effective field theory!

Chern-Simons actions:

$$\begin{aligned}\mathcal{S}_{\mathrm{CS}}^{(\mathrm{D}p)} &= \mu_p \int_{\mathrm{D}p} \mathrm{ch}(\mathcal{F}) \wedge \sqrt{\frac{\hat{A}(\mathcal{R}_T)}{\hat{A}(\mathcal{R}_N)}} \wedge \sum_q C_q, \\ \mathcal{S}_{\mathrm{CS}}^{(\mathrm{O}p)} &= -2^{p-4} \mu_p \int_{\mathrm{O}p} \sqrt{\frac{\hat{\mathcal{L}}(\mathcal{R}_T/4)}{\hat{\mathcal{L}}(\mathcal{R}_N/4)}} \wedge \sum_q C_q.\end{aligned}$$

For the case of D6-branes → equation of motion of C_7:

$$\frac{1}{\kappa^2} d \star dC_7 = \mu_6 \sum_a N_a \, \delta(\pi_a) + \mu_6 \sum_a N_a \, \delta(\pi'_a) + \mu_6 Q_6 \, \delta(\pi_{\mathrm{O6}}),$$

Integrate over $\mathcal{M}^6$ → RR-tadpole cancellation as equation in homology:

$$\sum_a N_a \, (\pi_a + \pi'_a) - 4\pi_{\mathrm{O6}} = 0.$$

(ii) Stability of the scalar potential: NS tadpole cancellation

Due to the tension of the D-branes a vacuum energy $\mathcal{V}(\phi_4, U_i)$ is induced which depends on the NS background fields: dilaton ϕ_4 , complex structure moduli U_i.

For flat 4-dim. Minkowski space-time we need a stable minimum of $\mathcal{V}(\phi_4, U_i)$ with $\mathcal{V}_{\text{min}} = 0 \longleftrightarrow$ Vanishing of NS tadpoles!

Scalar (D-term) potential:

$$\begin{aligned}
\mathcal{V} &= \mathcal{T}_6 \frac{e^{-\phi_4}}{\sqrt{\text{Vol}(\mathcal{M}^6)}} \left(\sum_a N_a \ \text{Vol}(\text{D6}_a) - 4\text{Vol}(\text{O6}) \right) \\
&= \mathcal{T}_6\, e^{-\phi_4} \left(\sum_a N_a \int_{\pi_a + \pi'_a} \Re(e^{i\phi_a} \widehat{\Omega}_3) - 4 \int_{\pi_{\text{O6}}} \Re(\widehat{\Omega}_3) \right)
\end{aligned}$$

Minimization of $\mathcal{V}$ will fix (part of) the complex structure moduli U_i.

3 possible scenarios:

- "Global" $\mathcal{N} = 1$ supersymmetry:
  Minima are such that all angles are supersymmetric $\longleftrightarrow$ all D6-branes conserve the same supersymmetries as orientifold plane, i.e. all D6-branes be calibrated with respect to $\Re(\widehat{\Omega}_3)$ $\Rightarrow \mathcal{V}_{\text{min}} = 0$

- "Local" $\mathcal{N} = 1$ supersymmetry:
  Minima are such that only SM angles are supersymmetric $\longleftrightarrow$ only SM D6-branes conserve the same supersymmetries as orientifold plane, i.e. only SM D6-branes be calibrated with respect to $\Re(\widehat{\Omega}_3)$. (Here hidden sector is in general necessary for RR tadpole cancellation.)

- No supersymmetry:
  Minima are such that SM angles are non-supersymmetric $\longleftrightarrow$ SM D6-branes do not conserve the same supersymmetries as orientifold plane. (Stability is very difficult to achieve!)

III) MSSM-like models and gauge coupling unification

(Blumenhagen, Stieberger, D.L., hep-th/0305146;
cfr. Antoniadis, Kiritsis, Tomaras, hep-th/0004214)

The three Standard Model gauge couplings g_s, g_w and g_y have different values at the weak scale.

Extrapolating these couplings due to the one-loop running

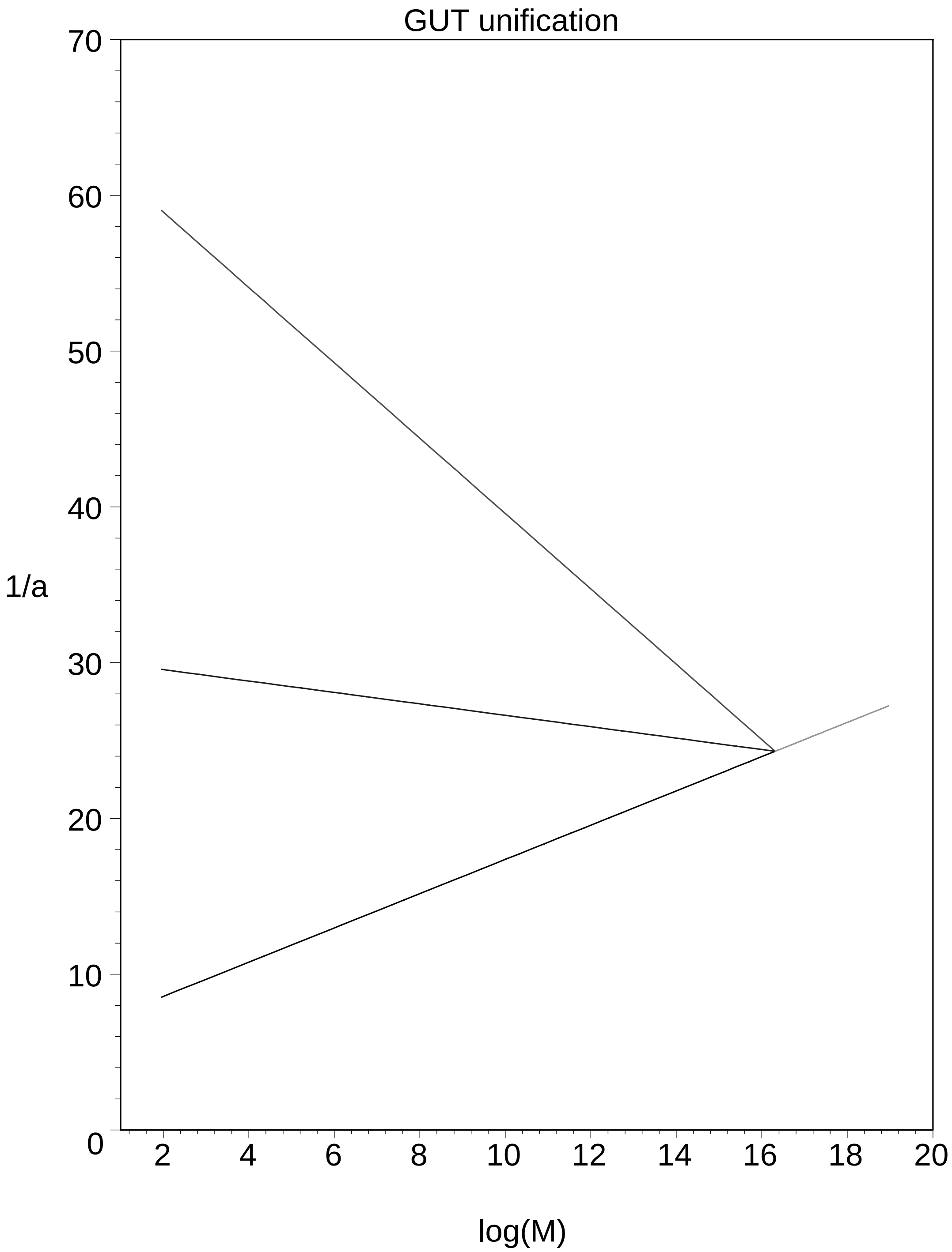
$$\frac{4\pi}{g_a^2(\mu)} = k_a \frac{4\pi}{g_X^2} + \frac{b_a}{2\pi} \log\left(\frac{\mu}{M_X}\right) + \Delta_a$$

to higher scales, one finds that they all meet at

$$M_X \simeq 2 \cdot 10^{16}\ \text{GeV}, \qquad \alpha_s = \alpha_w = \frac{3}{5}\alpha_Y = \alpha_X \simeq \frac{1}{24},$$

if the light spectrum contains just the MSSM particles.

This is in accord with for instance an $SU(5)$ Grand Unified gauge group at the GUT scale.


GUT unification
70
60
50
40
30
20
10
0
1/a
2
4
6
8
10
12
14
16
18
20
log(M)

In string theory one has a new scale M_s, so that it is natural to relate M_X to M_s. In the heterotic string one finds

$$k_a = \text{level of SU(N}_\text{a}\text{) Kac} - \text{Moody algebra.}$$

At one loop level the relation between the string and the Planck scale was found to be

$$M_s \simeq g_{st} \cdot 0.058 \cdot M_{pl},$$

which, using $g_{st} \simeq 0.7$, led to $M_s \simeq 5 \cdot 10^{17}$ GeV.
(Kaplunovsky (1988); Derendinger, Ferrara, Kounnas, Zwirner (1992))

The discrepancy between M_X and M_s needs to be explained by moduli-dependent string threshold corrections Δ_a (or alternatively by heterotic M-theory).
(Ibanez, Ross, D.L. (1991/92); Nilles, Stieberger; Witten (1996) ...)

III) MSSM-like models and gauge coupling unification

In contrast to the heterotic string, in D-brane models each gauge factor comes with its own gauge coupling, which at string tree-level can be deduced from the Dirac-Born-Infeld action

$$\frac{4\pi}{g_a^2} = \frac{M_s^3\, V_a}{(2\pi)^3\, g_{st}\, \kappa_a}, \qquad V_a = (2\pi)^3 R_a^3\,.$$

with $\kappa_a = 1$ for $U(N_a)$ and $\kappa_a = 2$ for $SP(2N_a)/SO(2N_a)$.

By dimensionally reducing the type IIA gravitational action one can similarly express the Planck mass in terms of stringy parameters ($M_{pl} = (G_N)^{-\frac{1}{2}}$)

$$M_{pl}^2 = \frac{8\, M_s^8\, V_6}{(2\pi)^6\, g_{st}^2}, \qquad V_6 = (2\pi)^6 R^6\,.$$

Eliminating the unknown string coupling g_{st} gives

$$\frac{1}{\alpha_a} = \frac{M_{pl}}{2\sqrt{2}\,\kappa_a\, M_s} \frac{V_a}{\sqrt{V_6}}.$$

Due to

$$\frac{V_a}{\sqrt{V_6}} = \int_{\pi_a} \Re(e^{i\phi_a} \widehat{\Omega}_3)$$

the gauge coupling only depends on the complex structure moduli.

III) MSSM-like models and gauge coupling unification

Consider the gauge coupling unification in a model in-depedent bottom up approach.

3 phenomenological requirements:

- The SM branes mutually preserve $\mathcal{N} = 1$ supersymmetry.
- The intersecting numbers realize a 3 generation MSSM
- The $U(1)_Y$ gauge boson is massless

We will show that using these 3 reasonable assumptions gauge coupling unification is achieved in a natural way!

Two simple ways to embed the SM!

(Blumenhagen, Körs, D.L., hep-th/0012156; Cremades, Ibanez, Marchesano, Rabadan, hep-th/0105155, hep-th/0302105)

Both of them use four stacks of $D6$-branes:

$$\begin{aligned} A &: \quad U(3)_a \times SP(2)_b \times U(1)_c \times U(1)_d \\ B &: \quad U(3)_a \times U(2)_b \times U(1)_c \times U(1)_d. \end{aligned}$$

The chiral spectrum of the intersecting brane world model should be identical to the chiral spectrum of the standard model particles.

This fixes uniquely the intersection numbers of the four 3-cycles, $(\pi_a, \pi_b, \pi_c, \pi_d)$.

field	**sector**	**I**	$SU(3) \times SU(2) \times U(1)^3$
q_L	(ab)	3	$(3, 2; 1, 0, 0)$
u_R	(ac)	3	$(\overline{3}, 1; -1, 1, 0)$
d_R	(ac')	3	$(\overline{3}, 1; -1, -1, 0)$
e_L	(db)	3	$(1, 2; 0, 0, 1)$
e_R	(dc')	3	$(1, 1; 0, -1, -1)$
ν_R	(dc)	3	$(1, 1; 0, 1, -1)$

The hypercharge Q_Y is given as the following linear combination of the three $U(1)$s

$$Q_Y = \frac{1}{3} Q_a - Q_c - Q_d.$$

For intersecting brane worlds it can happen that some of the stringy $U(1)$s are anomalous and get a mass via some generalized Green-Schwarz mechanism or that via axionic couplings some anomaly-free abelian gauge groups become massive.

The condition that a linear combination $U(1)_Y = \sum_i c_i U(1)_i$ remains massless reads

$$\sum_i c_i \, N_i \, \left(\pi_i - \pi'_i\right) = 0.$$

In general, if the hypercharge is such a linear combination of $U(1)$s, $Q_Y = \sum_i c_i Q_i$, then the gauge coupling is given by

$$\frac{1}{\alpha_Y} = \sum_i \frac{N_i \, c_i^2}{2} \frac{1}{\alpha_i}.$$

In our case

$$\frac{1}{\alpha_Y} = \frac{1}{6} \frac{1}{\alpha_a} + \frac{1}{2} \frac{1}{\alpha_c} + \frac{1}{2} \frac{1}{\alpha_d}.$$

Realization of the MSSM:

Assume that the 3-cycles π_a and π_b have intersection number $\pi_a \circ \pi_b = 3$, then homologically choosing $\pi_d = \pi_a$ gives the right intersection numbers for π_d.

Therefore

$$V_a = V_d.$$

The condition that $U(1)_Y$ remains massless simply implies $\pi_c' = \pi_c$.

Therefore, at the bottom of this simple realization there lies an extended Pati-Salam like model

$$U(4)_{a+d} \times SU(2)_b \times SU(2)_c.$$

From the field theory point of view it is very natural to assume, that the two gauge couplings of the two $SU(2)$ factors are the same, i.e.

$$V_c = V_b.$$

III) MSSM-like models and gauge coupling unification

From the stringy point of view, even though we cannot rigorously prove it in the general case, the constraints from supersymmetry and the intersection numbers $\pi_a \circ \pi_b = -\pi_a \circ \pi_c = 3$ do not seem to leave very much room to evade that $V_b = V_c$.

Therefore

$$\alpha_d = \alpha_a = \alpha_s, \quad \alpha_c = \frac{1}{2}\alpha_b = \frac{1}{2}\alpha_w,$$

which implies the Pati-Salam like tree-level relation

$$\frac{1}{\alpha_Y} = \frac{2}{3}\frac{1}{\alpha_s} + \frac{1}{\alpha_w}.$$

This relation will allow for natural gauge coupling unification!

III) MSSM-like models and gauge coupling unification

In the absence of threshold corrections, the one-loop running of the three gauge couplings is described by the well known formulas

$$\begin{aligned}\frac{1}{\alpha_s(\mu)} &= \frac{1}{\alpha_s} + \frac{b_3}{2\pi}\ln\left(\frac{\mu}{M_s}\right)\\ \frac{\sin^2\theta_w(\mu)}{\alpha(\mu)} &= \frac{1}{\alpha_w} + \frac{b_2}{2\pi}\ln\left(\frac{\mu}{M_s}\right)\\ \frac{\cos^2\theta_w(\mu)}{\alpha(\mu)} &= \frac{1}{\alpha_Y} + \frac{b_1}{2\pi}\ln\left(\frac{\mu}{M_s}\right),\end{aligned}$$

where (b_3, b_2, b_1) are the one-loop beta-function coefficients for $SU(3)_c$, $SU(2)_L$ and $U(1)_Y$.

Using the tree level relation at the string scale yields

$$\frac{2}{3}\frac{1}{\alpha_s(\mu)} + \frac{2\sin^2\theta_w(\mu) - 1}{\alpha(\mu)} = \frac{B}{2\pi}\ln\left(\frac{\mu}{M_s}\right)$$

with

$$B = \frac{2}{3}b_3 + b_2 - b_1.$$

III) MSSM-like models and gauge coupling unification

Employing the measured Standard Model parameters

$$\begin{aligned} M_Z &= 91.1876 \text{ GeV}, \quad \alpha_s(M_Z) = 0.1172, \\ \alpha(M_Z) &= \frac{1}{127.934}, \quad \sin^2\theta_w(M_Z) = 0.23113 \end{aligned}$$

the resulting value of the unification scale only depends on the combination B of the beta-function coefficients.

For the MSSM one has $(b_3, b_2, b_1) = (3, -1, -11)$, i.e $B = 12$ and the unification scale is the usual GUT scale

$$M_s = M_X = 2.04 \cdot 10^{16} \text{ GeV}.$$

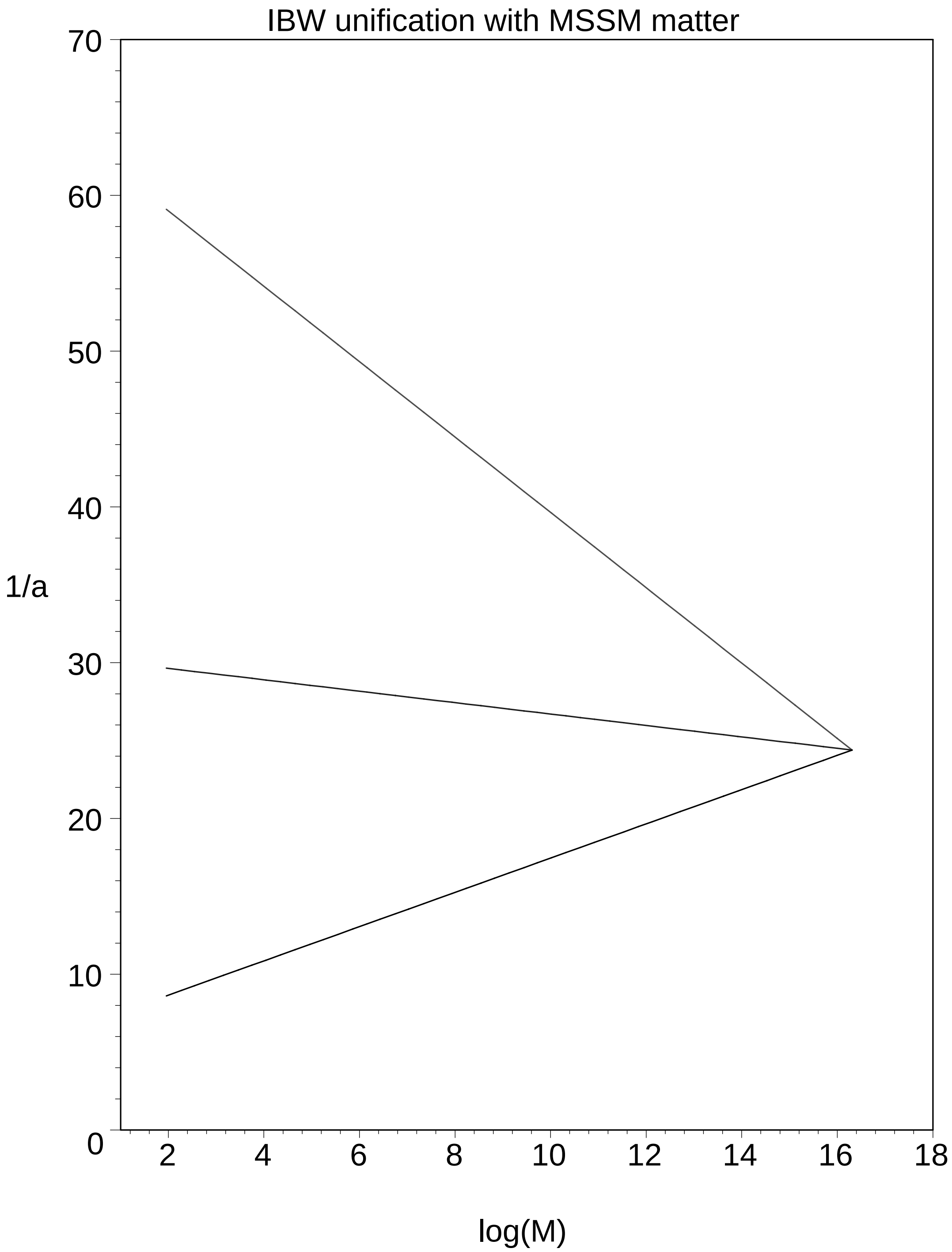
Of course, for the individual gauge couplings at the string scale we get

$$\alpha_s(M_s) = \alpha_w(M_s) = \frac{5}{3}\alpha_Y(M_s) = 0.041,$$

which are just the supersymmetric GUT scale values with the Weinberg angle being $\sin^2\theta_w(M_s) = 3/8$.

Assuming $g_{st} = g_X$, for the internal radii one obtains

$$M_s R = 5.32, \quad M_s R_s = 2.6, \quad M_s R_w = 3.3.$$


IBW unification with MSSM matter
70
60
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log(M)

In general besides the chiral matter string theory contains also additional vector-like matter.

This is also localized on the intersection loci of the $D6$ branes and also comes with multiplicity n_{ij} with $i, j \in \{a, b, c, d\}$.

One finds the following contribution to B

$$\begin{aligned} B &= 12 - 2\, n_{aa} - 4\, n_{ab} + 2\, n_{a'c} + 2\, n_{a'd} - 2\, n_{bb} + 2\, n_{c'c} \\ &\quad + 2\, n_{c'd} + 2\, n_{d'd}. \end{aligned}$$

B does not depend on the number of weak Higgs multiplets n_{bc}.

Example A:

If we have a model with a second weak Higgs field, i.e. $n_{bc} = 1$, we still get $B = 12$ but with

$$(b_3, b_2, b_1) = (3, -2, -12).$$

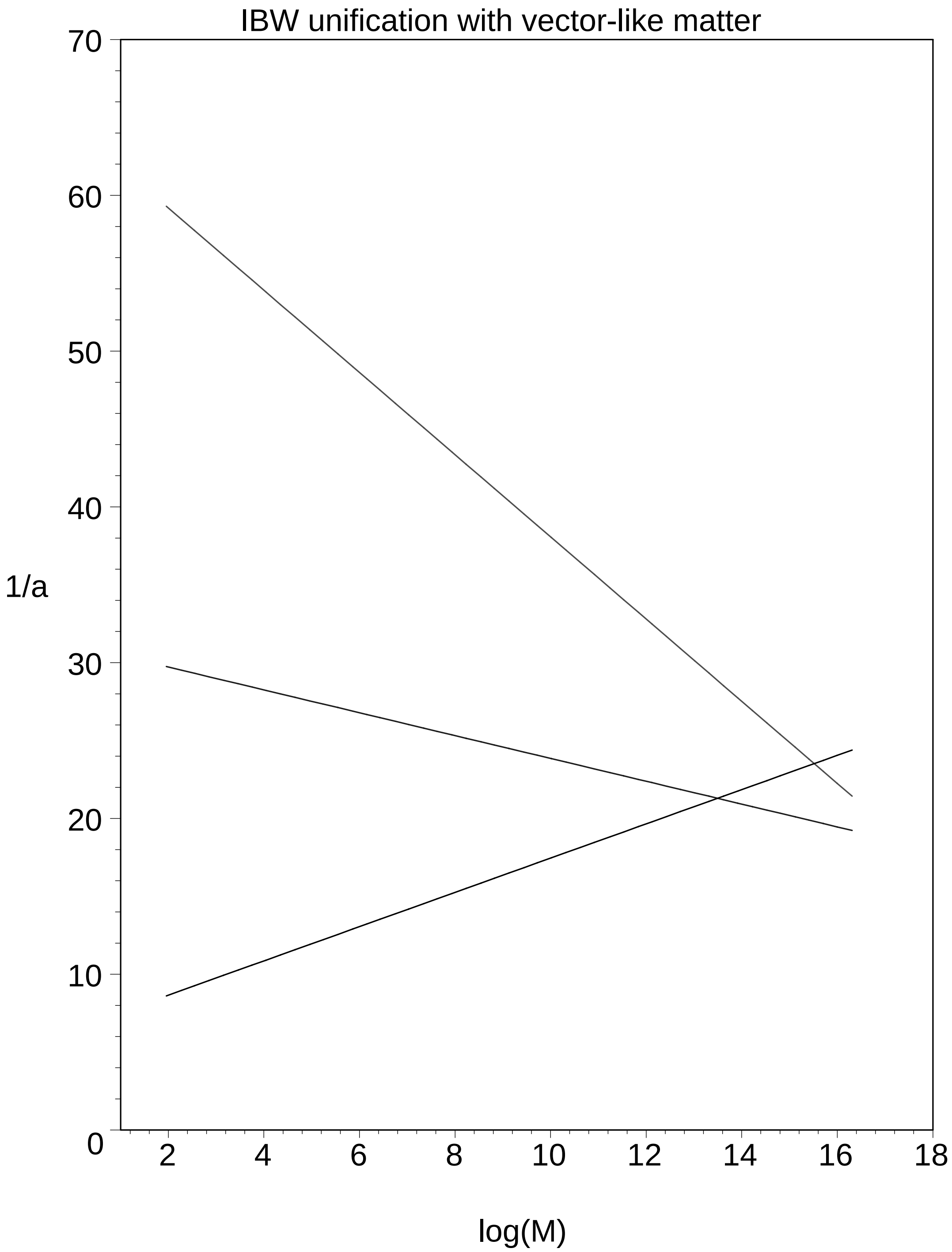
The gauge couplings "unify" at the scale

$$M_s = 2.02 \cdot 10^{16}\text{GeV}.$$

However they are not all equal at that scale

$$\alpha_s(M_s) = 0.041, \quad \alpha_w(M_s) = 0.052, \quad \alpha_Y(M_s) = 0.028.$$

III) MSSM-like models and gauge coupling unification



Example B: intermediate scale model

For models with gravity mediated supersymmetry breaking (hidden anti-branes) the string scale is naturally in the intermediate regime $M_s \simeq 10^{11}$GeV.

Choosing vector-like matter

$$n_{a'a} = n_{a'd} = n_{d'd} = 2, \quad n_{bb} = 1$$

leads to $B = 18$.

The string scale turns out to be

$$M_s = 3.36 \cdot 10^{11} \text{GeV}.$$

The running of the couplings with

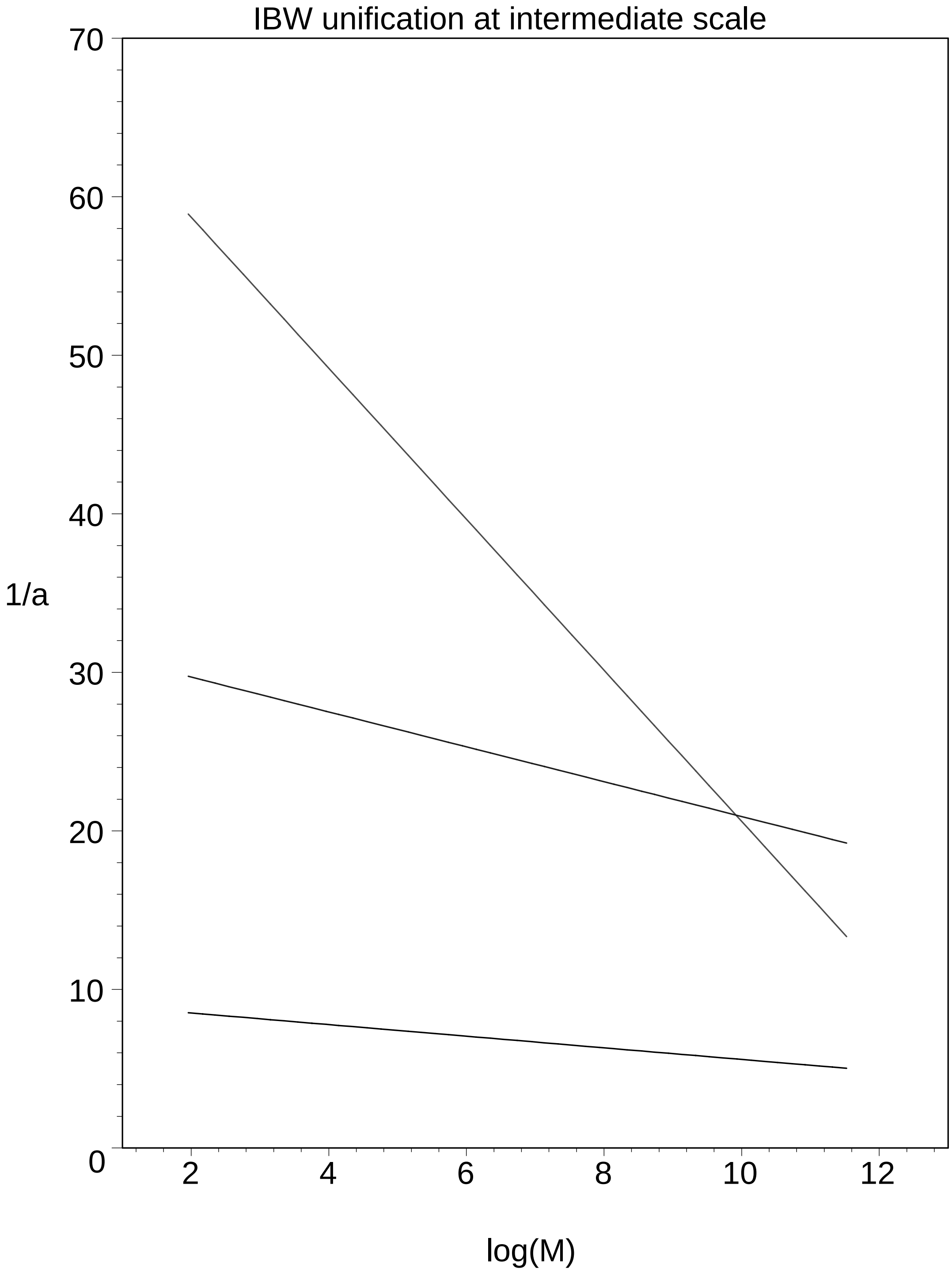
$$(b_3, b_2, b_1) = (-1, -3, -65/3)$$

leads to the values of the gauge couplings at the string scale

$$\alpha_s(M_s) = 0.199, \quad \alpha_w(M_s) = 0.052, \quad \alpha_Y(M_s) = 0.045.$$

Assuming $g_{st} \simeq 1$, for the internal radii one obtains

$$M_s R = 230, \quad M_s R_s = 1.7, \quad M_s R_w = 3.3.$$


IBW unification at intermediate scale
70
60
50
40
30
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0
1/a
2
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8
10
12
log(M)

Example C: Planck scale model

Interestingly for $B = 10$ one gets

$$\frac{M_s}{M_{pl}} = 1.24 \sim \sqrt{\frac{\pi}{2}}.$$

Choosing vector-like matter

$$n_{aa} = 1,$$

the beta-function coefficients read

$$(b_3, b_2, b_1) = (0, -1, -11).$$

The couplings at the string scale turn out to be

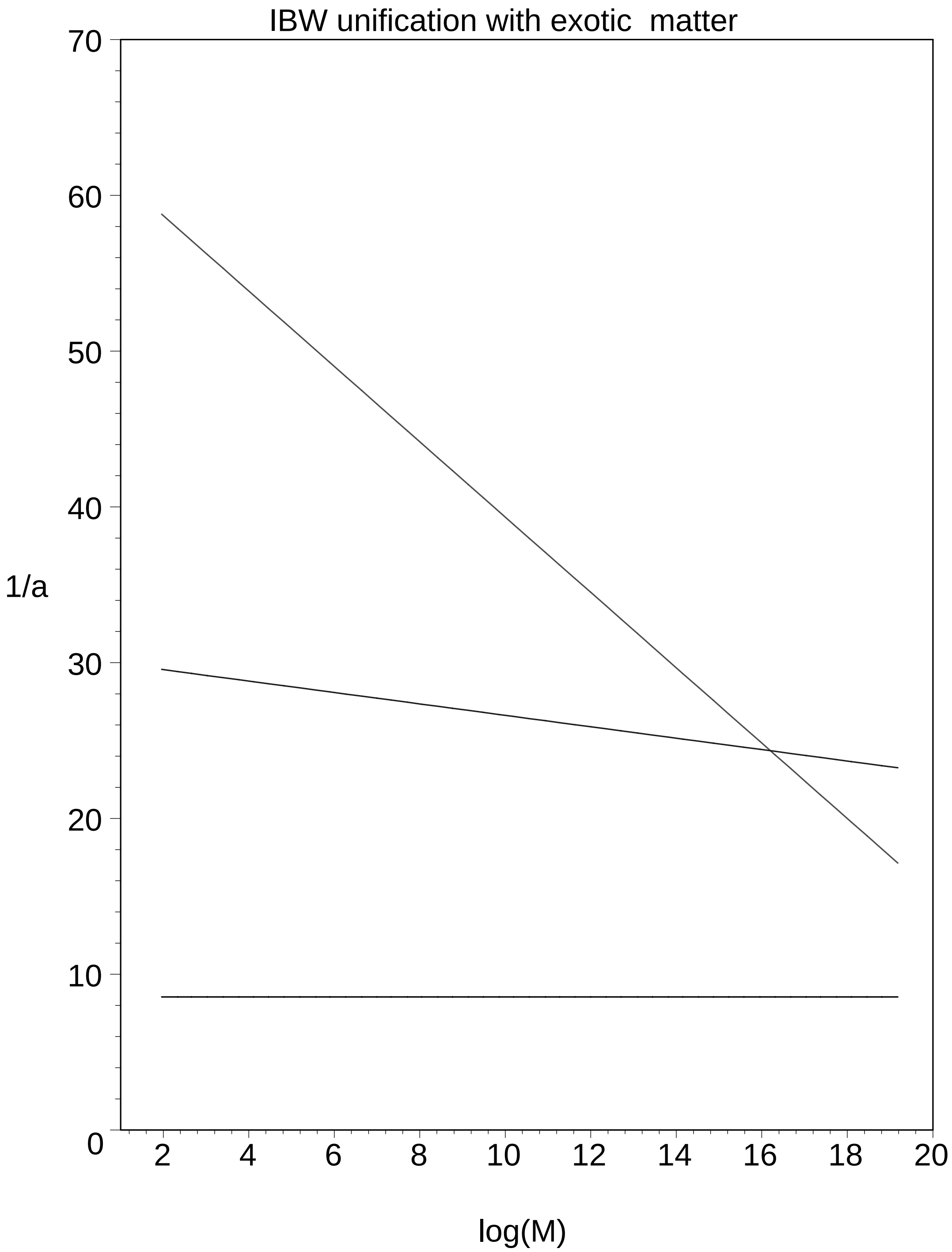
$$\alpha_s(M_s) = 0.117, \quad \alpha_w(M_s) = 0.043, \quad \alpha_Y(M_s) = 0.035$$

leading to $\sin^2\theta_w(M_s) = 0.445$.

For the scales of the overall Calabi-Yau volume and the 3-cycles we obtain

$$M_s R = 0.6, \quad M_s R_s = 1.9, \quad M_s R_w = 3.3.$$

III) MSSM-like models and gauge coupling unification



Summary: brane world models

Under a few natural assumptions supersymmetric Intersecting Brane World Models can make interesting predictions about gauge coupling unification.

The challenge remains to construct realistic supersymmetric IBW models with the chiral spectrum of the MSSM and only a mild amount of vector-like matter.

Other interesting topics:

- Computation of Yukawa couplings
  (Cremades, Ibanez, Marchesano; Cvetic, Papadimitriou)

- Dynamical supersymmetry breaking
  (Cvetic, Langacker, Wang)

- Proton decay: $\tau_{\text{proton}} \sim 10^{36}$ years
  (Klebanov, Witten; Friedmann, Witten)

- Flavor changing neutral currents
  (Abel, Masip, Santiago; Abel, Owen)

- 1-loop gauge threshold corrections Δ
  (S. Stieberger, D.L., hep-th/0302221.)

Explicit computation of Δ_a in toroidal and orbifold models:

(i) $\mathcal{N}=4$ sectors: $\Delta_a=0$.

(ii) $\mathcal{N}=2$ sectors:

$$\Delta_{ab}^{\mathrm{N}=2}=b_{ab}^{N=2}\ \ln(T_2^i V_a^i|\eta(T^i)|^4)+\mathrm{const.}\ ,$$

with the wrapped brane volume

$$V_a^i=\frac{1}{U_2^i}|n_a^i+U^i m_a^i|^2\,.$$

(iii) $\mathcal{N}=1$ sectors:

$$\Delta_{ab}^{\mathrm{N}=1}=b_{ab}^{N=1}\ \ln\frac{\Gamma(1-\frac{1}{\pi}\phi_{ab}^1)\ \Gamma(1-\frac{1}{\pi}\phi_{ab}^2)\ \Gamma(1+\frac{1}{\pi}\phi_{ab}^1+\frac{1}{\pi}\phi_{ab}^2)}{\Gamma(1+\frac{1}{\pi}\phi_{ab}^1)\ \Gamma(1+\frac{1}{\pi}\phi_{ab}^2)\ \Gamma(1-\frac{1}{\pi}\phi_{ab}^1-\frac{1}{\pi}\phi_{ab}^2)}\,,$$

$$\cot(\phi_{ab}^j)=\frac{n_a^j n_b^j\frac{R_1^j}{R_2^j}+m_a^j m_b^j\frac{R_2^j}{R_1^j}}{n_a^j m_b^j-n_b^j m_a^j}\ .$$

(Δ_a still depend on moduli! (Cfr. heterotic $\mathcal{N}=1$ sectors: DKL))

(iv) $\mathcal{N}=0$ sectors: UV divergent 1-loop threshold corrections. Note: in "local" supersymmetric models all SM thresholds can be made finite!

(S. Stieberger, D.L.: work in progress)

IV) Type II Compactifications with D-branes and H-fluxes

Problem of moduli stabilization:

Type IIA: D6-branes wrapped around 3-cycles $\mathcal{C}_3 \subset \mathcal{M}^6$, Potential $\mathcal{V}_{brane} \sim$ Vol($\mathcal{C}_3$) $\longrightarrow$ fixes complex structure moduli U_i!

Q: How to fix the Kähler moduli $T_i \sim$ Vol($\mathcal{C}_2$) of $\mathcal{M}^6$?

A: Turn on H-fluxes, i.e. background expectation values for H-field strength fields!

E.g. RR 2-form field strength:

$$\langle H_R^{(2)} \rangle = \oint_{\mathcal{C}_2} H_R^{(2)} \Rightarrow \mathcal{V}_{flux} \sim \langle H_R^{(2)} \rangle^2$$

Aim: Construct compactifications with D-branes and fluxes:

(Blumenhagen, Taylor, D.L., hep-th/0303016; Cascales, Uranga, hep-th/0303024)

D6-branes: Non-Abelian gauge bosons, chiral fermions $\rightarrow$ SM, non-trivial $\mathcal{V}_{brane}(U_i)$.

H-fluxes: No chirality, non-trivial $\mathcal{V}_{flux}(T_i)$.

IV) Type II Compactifications with D-branes and H-fluxes

T-dual type IIB (mirror) picture:

D-branes: Stacks of $D9_a$ branes which wrap mirror $\tilde{\mathcal{M}}^6$ CY plus open string magnetic fields F_{ab} through 2-cycles of $\tilde{\mathcal{M}}^6 \longrightarrow$ Non-Abelian gauge bosons, chiral fermions $\rightarrow \mathcal{V}_{brane}(T_i)$, fixes Kähler moduli of $\tilde{\mathcal{M}}^6$.

H-fluxes: RR and NS 3-form flux $\langle H_R^{(3)}\rangle, \langle H_{NS}^{(3)}\rangle \neq 0$ through 3-cycles of $\tilde{\mathcal{M}}^6 \longrightarrow \mathcal{V}_{flux}(U_i)$, fixes complex structure moduli of $\tilde{\mathcal{M}}^6$.

Effective flux induced action:

(Taylor, Vafa; Kachru, Schulz, Trivedi; ...)

(i) Kinetic energy of 3-forms $\Longrightarrow$ scalar potential $\mathcal{V}_{flux}$

$$\begin{aligned}
\mathcal{S}_{eff} &= -\frac{1}{4\kappa_{10}^2 \Im(\tau)} \int_{\tilde{\mathcal{M}}^6} G \wedge \star G\,, \\
G &= \tau H_{NS}^{(3)} + H_R^{(3)}\,, \qquad \tau = C_0 + ie^{-\phi}\,.
\end{aligned}$$

Expand G in terms of a basis of $H^3(\tilde{\mathcal{M}}^6, Z)$:

$$\begin{aligned}
G &= e_\Lambda X^\Lambda + m^\Lambda F_\Lambda\,, \\
e_\Lambda &= \tau e_\Lambda^1 + e_\Lambda^2\,, \qquad m^\Lambda = \tau m_1^\Lambda + m_2^\Lambda\,.
\end{aligned}$$

IV) Type II Compactifications with D-branes and H-fluxes

Scalar potential:

$$\begin{aligned}\mathcal{V}_{flux} &= -\frac{\mu_3}{2\Im\tau}[(e+m\bar{\mathcal{N}})(\Im\mathcal{N})^{-1}(\bar{e}+\bar{m}\mathcal{N})] \\ &+ \mu_3(m\times e) = \mathcal{V}_F + \mathcal{V}_D\end{aligned}$$

$\mathcal{N}$ denotes the period matrix:

$$\begin{aligned}\mathcal{N}_{\Lambda\Sigma} &= \bar{F}_{\Lambda\Sigma} + 2i\frac{\Im(F_{\Lambda\Gamma})\Im(F_{\Sigma\Delta})X^\Gamma X^\Delta}{\Im(F_{\Gamma\Delta})X^\Gamma X^\Delta}, \\ X^\Lambda &= \int_{A^\Lambda}\Omega_3\,, \quad F_\Lambda = \int_{B_\Lambda}\Omega_3\end{aligned}$$

$\mathcal{V}_{flux}$ depends on the compex structure moduli U_i and τ.

$\mathcal{V}_F$ can be derived from a superpotential:

$$W = \frac{1}{\sqrt{2}\kappa_{10}}\int_{\tilde{\mathcal{M}}^6}\Omega_3\wedge G = \sqrt{\mu}_3(e_\Lambda X^\Lambda + m^\Lambda F_\Lambda)$$

For certain choices of fluxes with $N_{flux} = m\times e \neq 0$ supersymmetric minimima of W with $W_{U_i} = W_\tau = W = 0$, i.e. $\mathcal{V}_F = 0$ can be found.

Note: Since at the minimum $V_{flux} = \mu_3(m\times e) > 0$ one needs orientifold planes to cancel the vacuum energy of the 3-form fluxes.

(ii) Topological action of 3-forms $\Longrightarrow$ RR-tadpoles

$$\mathcal{S}_{CS} = \frac{1}{2\kappa_{10}^2} \int \frac{C_4 \wedge G \wedge \bar{G}}{4i\Im\tau}$$

This induces a RR tadpole for C_4 given by

$$N_{flux} = \frac{1}{2\kappa_{10}^2 \mu_3} \int H_R^{(3)} \wedge H_{NS}^{(3)} = m \times e$$

So we need D-branes and orientifold planes in order to cancel the flux RR-tadpole and the unbalanced flux vacuum energy!

$\Longrightarrow$ D9-branes with magnetic fluxes plus orientifold planes.

Example $Z_2 \times Z_2$ orientifold: 4 tadpole conditions

$$8\sum_a \prod_I n_a^I + N_{flux} = 32\,, \qquad 8\sum_a N_a n_a^1 m_a^2 m_a^3 = \pm 32\,,$$
$$8\sum_a N_a m_a^1 n_a^2 m_a^3 = -32\,, \qquad 8\sum_a N_a m_a^1 m_a^2 n_a^3 = -32$$

Total scalar potential:

$$\mathcal{V}_{total}(T_i, U_i, \tau) = \mathcal{V}_{flux}(U_i, \tau) + \mathcal{V}_{D9}(T_i, \tau) - \mathcal{V}_{O3,O7}(T_i, \tau)$$

IV) Type II Compactifications with D-branes and H-fluxes

Concrete example:

One can construct a $\mathcal{N} = 1$ supersymmetric $Z_2 \times Z_2$ orientifold model with supersymmetric D9-branes and supersymmetric 3-form fluxes:

(i) 3-form fluxes $\leftrightarrow$ complex structure moduli:

$$U^1 U^2 = -1 \,, \quad \tau = -U^3$$

(ii) 2 stacks of D9-branes:

$$\begin{aligned} 1^{st} \ \ \text{stack}: \ (n^I, m^I) &= \{(0,1),(1,-1),(1,-1)\} \\ 2^{nd} \ \ \text{stack}: \ (n^I, m^I) &= \{(1,0),(0,-1),(0,-1)\} \end{aligned}$$

Kähler moduli:

$$T^2 T^3 = (4\pi^2 \alpha')^2$$

Gauge group: $G = U(4) \times U(4)$

Chiral fermions: $(4,4) + (4,\bar{4})$-representations (anomalous, canceled by inflow mechanism).

V) Non Calabi-Yau compactifications

So far we assumed that our internal space $\mathcal{M}^6$ is a Ricci-flat CY space. However in general, the H-fluxes/wrapped D-branes induce a strong backreaction on the underlying space-time geometry! $\Longrightarrow$ $\mathcal{M}$ non-Ricci flat!

Supersymmetry transformations:

$$\delta\Psi_M = \nabla_M \epsilon - \frac{1}{4} H_M \epsilon = 0$$

β-functions (equations of motion):

$$R_{MN} - \frac{1}{4}(H)^2_{MN} = 0$$

Questions:

- What is the mathematical structure of $\mathcal{M}^6$? – Classification of possible spaces $\mathcal{M}^6$?
- Explicit examples for $\mathcal{M}^6$?
- What is the low-energy, 4-dim. effective action?

V) Non Calabi-Yau compactifications

Main results on the mathematical structure:

$\mathcal{N} = 1$ space time supersymmetry $\Longrightarrow$ $\mathcal{M}^6$ is equipped with a $SU(3)$ connection with torsion:

$$T_{mnp} \propto H_{mnp}$$

Spaces with $SU(3)$ structure and torsion are well known in the mathematical literature: the allowed torsion tensors fall into five different classes.

(Friedrich; Chiossi, Salomon)

	Flux	Background metric of $\mathcal{M}$
M-theory	–	$R_{mn} = 0$ (7-dim.) G_2 manifold
$\Updownarrow$ S^1-fibration		
Type IIA	$H_R^{(2)} \neq 0$ D6-branes	$R_{mn} \neq 0$, $d\Omega_3 \neq 0$ almost Kähler manifold
$\Updownarrow$ T-duality		
Type IIB	$H_R^{(3)} \neq 0$ $H_{NS}^{(3)} \neq 0$	$R_{mn} = 0$ Calabi-Yau manifold
$\Updownarrow$ string duality		
Heterotic	$H_{NS}^{(3)} \neq 0$	$R_{mn} \neq 0$, $dJ \neq 0$ complex Hermitian manifold

Take a 6-dim. manifold $\mathcal{M}^6$ with Riemannian metric g, almost complex structure J,

$$J = e^1 \wedge e^2 + e^3 \wedge e^4 + e^5 \wedge e^6,$$

with $J \cdot J = -1$ ($J \cdot e^a = J^a_b e^b$). This defines a $U(3)$ structure.

A $SU(3)$ structure is determined by the (3,0)-form

$$\Psi = (e^1 + ie^2) \wedge (e^3 + ie^4) \wedge (e^5 + ie^6).$$

Ψ has norm 1 and is subject to the compatibility relations

$$J \wedge \psi_\pm = 0\,, \quad \psi_+ \wedge \psi_- = \frac{2}{3} J \wedge J \wedge J\,,$$

where

$$\Psi = \psi_+ + i\psi_-\,, \quad \psi_- = J \cdot \psi_+\,.$$

V) Non Calabi-Yau compactifications

The failure of the holonomy group of g to reduce to $SU(3)$ can be measured by the intrinsic torsion τ. The space to which the torsion belongs can be decomposed into five classes:

$$\tau \in \mathcal{W}_1 \oplus \mathcal{W}_2 \oplus \mathcal{W}_3 \oplus \mathcal{W}_4 \oplus \mathcal{W}_5 \,,$$

described by the decomposition of τ into $SU(3)$ irreducible representations:

$$(1+1)+(8+8)+(6+\bar{6})+(3+\bar{3})+(3+\bar{3})\,.$$

The five $\mathcal{W}_i$ are fixed by dJ and $d\Psi$:

$$\begin{array}{lcll} \mathcal{W}_1 & \leftrightarrow & [dJ]^{(3,0)}\,, & \mathcal{W}_2 \leftrightarrow [d\Psi]^{(2,2)}\,, \\ \mathcal{W}_3 & \leftrightarrow & [dJ]^{(2,1)}\,, & \mathcal{W}_4 \leftrightarrow J \wedge dJ\,, \\ \mathcal{W}_5 & \leftrightarrow & [d\Psi]^{(3,1)}\,. & \end{array}$$

Depending on the class of torsion we deal with the following manifolds:

i) Complex, Hermitian manifolds ($\mathcal{W}_1 = \mathcal{W}_2 = 0$):

$$\begin{array}{rcl} \tau \in \mathcal{W}_3 & \Leftrightarrow & \text{special Hermitian manifolds,} \\ \tau \in \mathcal{W}_5 & \Leftrightarrow & \text{Kähler manifolds,} \\ \tau = 0 & \Leftrightarrow & \text{Calabi–Yau manifolds.} \end{array}$$

ii) Non-complex manifolds

$$
\begin{aligned}
\tau \in \mathcal{W}_1 &\Leftrightarrow \text{nearly–Kähler manifolds,} \\
\tau \in \mathcal{W}_2 &\Leftrightarrow \text{almost–Kähler manifolds,} \\
\tau \in \mathcal{W}_1^- \oplus \mathcal{W}_2^- \oplus \mathcal{W}_3 &\Leftrightarrow \text{half-flat manifolds.}
\end{aligned}
$$

A half-flat manifold can be lifted to a 7-dim. G_2-space:

M-theory on a G_2-space $X_7 \longleftrightarrow$ Type IIA with Ramond 2-form flux (or intersecting D6-branes):

(Acharya, Maldacena, Vafa, Atiyah, Witten, Brandhuber, Gubser, Gukov, Bilal, Derendinger, Sfetsos, Cvetic, Gibbons, Pope, Kaste, Kehagias, Minasian, Petrini, Tomasiello, Behrndt, Dall'Agata, Mahapatra, D.L. ...)

Circle fibration

$$
\pi : \quad X_7 \to \mathcal{M}^6 \,, \quad g = A \otimes A + \pi^* \hat{g} \,,
$$

$\hat{g}$ metric of $\mathcal{M}^6$, and $dA = \pi^* \rho$ with ρ some 2-form on $\mathcal{M}^6$ ($\rho \sim H_R^{(2)}$).

G_2 structure of X_7: $\phi = J \wedge A + \psi_-$

$$
\begin{aligned}
d\phi &= dJ \wedge A + d\psi_- + J \wedge \rho \,, \\
d \star \phi &= d\psi_+ \wedge A + J \wedge dJ - \psi_+ \wedge \rho \,.
\end{aligned}
$$

G_2 holonomy implies that $d\phi = d \star \phi = 0$. Therefore

$$dJ = 0\,,\;\; d\psi_+ = 0\,,\;\; d\psi_- = -J \wedge \rho\,,\;\; J \wedge dJ = 0\,.$$

So we finally obtain

$$\tau \in \mathcal{W}_2^- \,,$$

i.e. the type IIA space $\mathcal{M}^6$ with Ramond 2-form flux is an *almost-Kähler manifold*.

This result agrees with the construction of the IIA background $\mathcal{M}^6$ by applying the mirror (T-duality) transformation on type IIB on a CY^6 with 3-form fluxes.

(Gurrieri, Louis, Micu. Waldram; Kachru, Schulz, Tripathy, Trivedi)

VI) Heterotic Strings with 3-form flux

(Strominger; Cardoso, Curio, Dall'Agata, Zoupanos, Manousselis, D.L., hep-th/0211118; Gauntlett, Martelli, Waldram, hep-th/0302158)

Consider a warped compactification of the 10-dimensional heterotic string on $\mathbb{R}^{1,3} \otimes \mathcal{M}^6$ with metric:

$$ds^2 = e^{2\Delta(y)}(dx^\mu \otimes dx^\nu \eta_{\mu\nu} + dy^m \otimes dy^n g_{mn}(y)) .$$

The additional background fields are:

Dilaton $\phi(y)$, NS 3-form $H_{mnp}(y)$, Yang-Mills field F_{mn}.

Conditions for $\mathcal{N} = 1$ space-time supersymmetry in 4 dimensions:

gravitino : $$\delta\Psi_M = \mathcal{D}_M\epsilon = \nabla_M\epsilon - \frac{1}{4}H_M\epsilon = 0 ,$$

gaugino : $$\delta\chi = -\frac{1}{4}\Gamma^{MN}\epsilon F_{MN} = 0 ,$$

dilatino : $$\delta\lambda = \nabla\phi + \frac{1}{24}H\epsilon = 0 ,$$

VI) Heterotic String with 3-form flux

Conditions on the geometry of $\mathcal{M}^6$:

- $\mathcal{M}^6$ must be complex:

  $$\text{Nijenhuis tensor}: \quad N^p_{mn} \equiv J^q_m \partial_{[q} J^p_{n]} - J^q_n \partial_{[q} J^p_{m]} = 0\,.$$

  Spin connection with torsion has $SU(3)$ holonomy:

  $$\delta\Psi_m = \mathcal{D}_m \eta_+ = \partial_m \eta_+ + \frac{1}{4}(\omega^{np}_m - H^{np}_m)\Gamma_{np}\eta_+ = 0$$

  Therefore H^{np}_m denotes the torsion of $\mathcal{M}^6$.

  Integrability:

  $$[\mathcal{D}_m, \mathcal{D}_n]\eta_+ = \frac{1}{4}\tilde{R}^{pq}_{mn}\Gamma_{pq}\eta_+ = 0 \implies \tilde{R}^{pq}_{mn} J_{pq} = 0$$

- There exists one holomorphic (3,0)-form ω with:

  $$\star d \star J = i(\bar{\partial} - \partial) \log ||\omega||\,, \qquad \Psi = \frac{\omega}{||\omega||}\,.$$

VI) Heterotic Strings with 3-form flux

Conditions which link the matter fields to geometry:

- H-field:

$$H = \frac{i}{2}(\partial - \bar{\partial})J \,.$$

- Dilaton:

$$\phi(y) = \frac{1}{8}\log||\omega|| + \text{const}\,, \qquad \Delta(y) = \phi(y) + \text{const}\,.$$

- Yang-Mills fields:

$$F_{mn}J^{mn} = 0\,.$$

- Bianchi-identity:

$$dH = tr(\tilde{R}\wedge\tilde{R}) - tr(F\wedge F)\,.$$

V) Heterotic Strings with 3-form flux

Now reformulate the geometrical conditions in terms of the five torsion classes:

- $\mathcal{M}^6$ is complex:

$$\mathcal{W}_1 = \mathcal{W}_2 = 0 \iff dJ^{(3,0)} = d\Psi^{(2,2)} = 0 \, .$$

- Holomorphic (3,0)-form ω:

$$J \wedge dJ = -d \log ||\omega|| \, , \quad \mathcal{W}_4 = \frac{1}{2} J \wedge dJ \, , \quad \mathcal{W}_5 = d \log ||\omega|| \, .$$

$$\tau \in \mathcal{W}_3 \oplus \mathcal{W}_4 \oplus \mathcal{W}_5 \, , \quad 2\, \mathcal{W}_4 + \mathcal{W}_5 = 0$$

$$\mathcal{W}_4 \quad \text{and} \quad \mathcal{W}_5 \quad \text{exact} \, .$$

$$H = \frac{i}{2}(\partial - \bar{\partial}) J \, , \quad F_{mn} J^{mn} = 0 \, , \quad \phi(y) = \frac{1}{8} \log ||\omega|| \, ,$$

$$dH = tr(\tilde{R} \wedge \tilde{R}) - tr(F \wedge F) \, .$$

IV) Heterotic Strings with 3-form flux

Special case:

$$\mathcal{W}_4 = \mathcal{W}_5 = 0$$

So

$$\tau \in \mathcal{W}_3 \,,$$

i.e. $\mathcal{M}^6$ is a special-Hermitian manifold.

In this case the dilaton ϕ and the warp factor Δ are constants, and ω is a closed, holomorphic (3,0)-form of constant norm.

The only difference between $\mathcal{M}^6$ and a CY-space is given by a non-trivial 3-form H.

Examples:
Nilmanifolds *(Salamon)*

Moishezon manifolds
(Gutowski, Ivanov, Papadopoulos)

BPS action and superpotential for heteroti string compactification with fluxes:

(Lopes Cardoso, Curio, Dall'Agata, D.L., hep-th/0306088;

cfr: Becker, Becker, Dasgupta, Prokushkin, hep-th/0304001)

Bosonic part of the 10D effective action ($\mathcal{O}(\alpha'^2)$):

$$S = \int d^{10}x \sqrt{g}\, e^{8\phi} \left[\frac{1}{4} R - \frac{1}{12} H_{MNP} H^{MNP} + 16(\partial_M \phi)^2 - \frac{1}{4}\alpha' \left(F^I_{MN} F^{I\,MN} - R^+_{MNPQ} R^{+\,MNPQ} \right) \right] .$$

After compactification this yields

$$\begin{aligned} S &= -\int d^4x \sqrt{g_4}\, V \\ V &= -\Big\{ -\frac{1}{2} \int_{\mathcal{M}_6} \mathrm{e}^{8\phi}\, (8d\phi + \theta) \wedge \star\, (8d\phi + \theta) + \frac{1}{8} \int_{\mathcal{M}_6} \mathrm{e}^{8\phi}\, J \wedge J \wedge \hat{R}^{ab} J_{ab} \\ &- \frac{1}{4} \int d^6y \sqrt{g_6}\, \mathrm{e}^{8\phi}\, N_{mn}{}^p\, g^{mq} g^{nr} g_{ps}\, N_{qr}{}^s \\ &+ \frac{1}{2} \int_{\mathcal{M}_6} \mathrm{e}^{8\phi} \left(H + \frac{1}{2} \star \mathrm{e}^{-8\phi}\, d(\mathrm{e}^{8\phi} J) \right) \wedge \star \left(H + \frac{1}{2} \star \mathrm{e}^{-8\phi}\, d(\mathrm{e}^{8\phi} J) \right) \\ &- \frac{\alpha'}{2} \int d^6y \sqrt{g_6}\, \mathrm{e}^{8\phi} \left[\mathrm{tr}(F^{(2,0)})^2 + \mathrm{tr}(F^{(0,2)})^2 + \frac{1}{4} \mathrm{tr}(J^{mn} F_{mn})^2 \right] \\ &+ \frac{\alpha'}{2} \int d^6y \sqrt{g_6}\, \mathrm{e}^{8\phi} \left[\mathrm{tr}(R^{+\,(2,0)})^2 + \mathrm{tr}(R^{+\,(0,2)})^2 + \frac{1}{4} \mathrm{tr}(J^{mn} R^+_{mn})^2 \right] \Big\} . \end{aligned}$$

Sum of BPS squares $\Longrightarrow$ The supersymmetry conditions provide a solution of the equations of motion!

Use:

$$H = -\frac{1}{2} \star \mathrm{e}^{-8\phi} d(\mathrm{e}^{8\phi} J) = \frac{1}{2} i(\partial - \bar{\partial}) J \, .$$

Corresponding superpotential:

Show that V can be partially written in standard $\mathcal{N} = 1$ form:

$$V = \mathrm{e}^{\mathcal{K}} \left[g^{i\bar{\jmath}} D_i W \, \overline{D_{\bar{\jmath}} W} - 3|W|^2 \right] \, ,$$

Consider the term:

$$V = -\frac{1}{2} \int \mathrm{e}^{8\phi} \left(H + \frac{1}{2} \star \mathrm{e}^{-8\phi} d(\mathrm{e}^{8\phi} J) \right) \wedge \star \left(H + \frac{1}{2} \star \mathrm{e}^{-8\phi} d(\mathrm{e}^{8\phi} J) \right) \, .$$

Now introduce

$$\mathcal{H} \equiv H + \frac{i}{2} \mathrm{e}^{-8\phi} d(\mathrm{e}^{8\phi} J) \, ,$$

Then (under some assumptions) the superpotential is:

$$W = \int \mathcal{H} \wedge \Omega = \int \left(H + \frac{i}{2} dJ \right) \wedge \Omega \, .$$

The deviation from a Calabi-Yau space is measured by the geometrical term $dJ \wedge \Omega$!

This superpotential is in quite analogy with the type IIB superpotential with 3-form flux G on a CY-space:

(Mayr; Taylor, Vafa)

$$W = \int G \wedge \Omega = \int (H_R^{(3)} - \tau H_{NS}^{(3)}) \wedge \Omega \,.$$

It is also similar to the type IIA superpotential with 2-form flux:

(Vafa; Curio, Körs, D.L.; Gurrieri, Louis, Micu, Waldram)

$$W = \int (H_R^{(2)} \wedge J + d\Omega) \wedge J \,.$$

Here the deviation from a Calabi-Yau space is measured by the term $d\Omega \wedge J$!

VII) Conclusions

Important question: Does it make at all sense to construct 4-dim. string vacua without knowing the dynamical selection process which determines the unique string ground state (if it exists)?

(Preliminary) answer: Probably Yes!

Statistics of string/M theory vacua:

(M. Douglas, hep-th/0303194)

Assume that we can construct the SM spectrum from strings in several ways, where the SM couplings for each model are statistically, i.e. uniformly distributed.

SM fills the following volume in the space of coupling constants (measured in natural units):

$$\delta V_{SM} \sim 10^{-238}$$

Therefore we need at least $\mathcal{O}(10^{238})$ brane/flux string vacua with SM spectrum in order to make the statistical statement that string theory contains the SM.

This seems to be possible!