Closed String Tachyons and Non-Commutative Instabilities

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OUTLINE

Introduction

Closed String versus Field Theory Spectrum

String Interpretation

INTRODUCTION

- Non-Commutative R^N:

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}$$

- Non-commutative field theory

$$S = \int d^N x \, \mathcal{L}(\phi, \partial\phi)_\star \qquad (\theta^{0i} = 0)$$

$\star$-product: $\quad e^{ikx} \star e^{iqx} = e^{\frac{i}{2} k_\mu \theta^{\mu\nu} q_\nu} \, e^{i(k+q)x}$

- UV/IR Mixing (Minwalla, Seiberg, Van Raamsdonk)

Ex.: non-planar tadpole

$$\lambda \int \frac{e^{i\tilde{p}\cdot k}}{p^2 + m^2} d^d k \quad , \qquad \tilde{p}^\mu = \theta^{\mu\nu} p_\nu$$

$1/\tilde{p}$ acts as UV cut-off in non-planar graphs

Consequence

UV divergences translate into IR divergences

$$\Lambda^2 \to \frac{1}{\tilde{p}^2} \quad , \qquad m^2 \log\frac{\Lambda}{m} \to m^2 \log\frac{1}{\tilde{p}m}$$

IMPLICATIONS

Modified dispersion relations

$$\omega^2 = p^2 - c\frac{\lambda}{\tilde{p}^2}$$

c: model dependent constant

$c < 0$: - minimum of ω at $p > 0$

- $\omega \to \infty$ as $p \to 0$

$c > 0$: low momentum modes are unstable!

- Non-commutative gauge theories

$$S = -\frac{1}{4}\int \mathrm{Tr}\ (F_{\mu\nu} \star F^{\mu\nu} + D_\mu X \star D^\mu X)$$

$c = N_B - N_F$, $\quad N_{B,F}$: # dof in the adjoint

$N_B > N_F$: unstable (pure YM!!) (Ruiz; E.L., Landsteiner, Tytgat)

- Motivation: String theory

The worldvolume of D-branes on a constant B-field becames NC (Connes, Douglas, Schwarz; Douglas, Hull; Cheung, Krogh; Chu, Ho; Schomerus)

$$[x^\mu, x^\nu] = i\frac{1}{B_{\mu\nu}}$$

A constant B-field does not affect closed strings

Thanks to D-branes it has been uncovered a deep relation between gravity and gauge theory

Can we obtain additional information by using NC D-branes?

CLOSED STRING VERSUS FIELD THEORY SPECTRUM

- The models: C^3/Z_N orbifolds

Action of the orbifold on the spinors

$$\psi \to L\psi \quad , \ \psi \in 4 \text{ of } SU(4) \sim SO(6)$$

$$L = \text{diag}(e^{\frac{2\pi i a_1}{N}}, e^{\frac{2\pi i a_2}{N}}, e^{\frac{2\pi i a_3}{N}}, e^{\frac{2\pi i a_4}{N}}) \,, \sum a_\alpha = 0 \bmod N$$

and on the complex coordinates of C^3

$$Z^\beta \to e^{\frac{2\pi i b_\beta}{N}} Z^\beta$$

where $b_1 = a_2 + a_3$, $b_2 = a_3 + a_1$, $b_3 = a_1 + a_2$.

These models include C/Z_N and C^2/Z_N, type 0, and type 0 orbifolds.

When some $a_\alpha = 0 \Rightarrow$ SUSY orbifold

Otherwise non-supersymetric $\Rightarrow$ contain tachyons in the twisted sectors

- D3-branes at the orbifold fix point

Gauge theory on n
regular D3-branes

$$\Downarrow$$

Gauge group:	$U(n)^N \,, \quad g_i = g$
Weyl fermions:	$(n_i, \bar{n}_{i+a_\alpha})$, $\alpha=1,..,4$
Complex scalars:	$(n_i, \bar{n}_{i+b_\beta})$, $\beta=1,2,3$

The theory is SUSY when at least one $a_\alpha = 0$

Switching a constant B-field along two (spatial) worldvolume directions the field theory becomes NC

- Anomalies

- $B = 0$

$$\begin{cases} U(n)_i^3 \text{ anomalies cancel} \\ U(1)_i U(n)_j^2 \text{ do not cancel} \end{cases}$$

$N - 1$ non-trivial $U(1)$'s are anomalous

$\Downarrow$ Green-Schwarz mechanism (Ibañez, Rabadan, Uranga)

$$U(1) \otimes SU(n)^N$$

- $B \neq 0$

The diagrams which could produce mixed anomalies are non-planar

$$\begin{cases} \tilde{p} \neq 0 : \text{ anomalies are absent (Martin)} \\ \tilde{p} = 0 : \text{ mixed anomalies survive (Ardalan, Sadooghi)} \end{cases}$$

NC Green-Schwarz mechanism gives masses to $\tilde{p} = 0$ modes of non-trivial $U(1)$'s (Armoni, E. L., Theisen) $\Rightarrow$

$$\Rightarrow \tilde{p} \neq 0: \quad U(n)^N$$

- Polarization tensor

$$(\Pi^{NPl})^{\mu\nu}_{ij} = M_{ij}\frac{g^2 n}{\pi^2}\frac{\tilde{p}^\mu \tilde{p}^\nu}{\tilde{p}^4}$$

- Π^{NPl} has a quadratic divergence $1/\tilde{p}^2$, but fulfills the Ward identity

$$p_\mu(\Pi^{NPl})^{\mu\nu} \sim p_\mu \tilde{p}^\mu = p_\mu \theta^{\mu\nu} p_\nu = 0$$

- Π^{NPl} affects only $U(1)_i \subset U(n)_i$

- M_{ij} can be read from the matter content

$$\left.\begin{array}{lll} M_{ii}: & \text{adjoint} & \text{matter} \\ M_{ij}: & \text{bifundamental} & \text{matter} \end{array}\right\} \begin{array}{l} \text{bosons}: +1 \\ \text{fermions}: -1 \end{array}$$

$$M_{ij} = 2\delta_{ij} - \sum_\alpha (\delta_{i,j-a_\alpha} + \delta_{i,j+a_\alpha}) + \sum_\beta (\delta_{i,j-b_\beta} + \delta_{i,j+b_\beta})$$

M can be diagonalized by $e^{(k)}$, $e_j^{(k)} = e^{\frac{2\pi i j k}{N}}$, with eigenvalues $(k=0,..,N-1)$

$$\epsilon^{(k)} = 16 \prod_\alpha \sin \frac{\pi k a_\alpha}{N} \qquad (a_4=-(a_1+a_2+a_3))$$

$$\Delta S \sim \int \frac{d^4p}{(2\pi)^4} \frac{\tilde{p}^\mu \tilde{p}^\nu}{\tilde{p}^4} \sum_{i,j=1}^{N} M_{ij} \, \mathrm{Tr}\, A_\mu^{(i)}(p) \mathrm{Tr}\, A_\nu^{(j)}(-p)$$

$$\Downarrow \quad B_\mu^{(k)} = \tfrac{1}{\sqrt{N}} \textstyle\sum_{j=1}^{N} e^{2\pi i \, \frac{jk}{N}} \, \mathrm{Tr}\, A_\mu^{(j)}$$

$$\Delta S \sim \int \frac{d^4p}{(2\pi)^4} \frac{\tilde{p}^\mu \tilde{p}^\nu}{\tilde{p}^4} \sum_{k=0}^{N-1} \epsilon^{(k)} \, B_\mu^{(N-k)}(p) \, B_\nu^{(k)}(-p)$$

Dispersion relation for $B^{(k)}$ modes polarized along the NC direction

$$E^2 = \vec{p}^2 - g^2 n \frac{\epsilon^{(k)}}{\pi^2} \frac{1}{\tilde{p}^2}$$

Adjoint scalars get similar IR divergent contributions to the self-energy; not bifundamentals or fermions

- Closed string theory spectrum

$N-1$ twisted sectors + untwisted sector

k^{th} twisted sector:

worlsheet bosons: modings $n \pm \theta_i$

$$\text{worlsheet fermions:} \left\{ \begin{array}{ll} n + 1/2 \pm \theta_i & \text{in NS} \\ n \pm \theta_i & \text{in R} \end{array} \right.$$

where

$$\theta_i = \frac{k b_i}{N} \qquad \left(b_1 = a_2 + a_3 \; , b_2 = a_3 + a_1 \; , b_3 = a_1 + a_2\right)$$

The spectrum is unchanged under

$$\theta_i \to \theta_i + k_i \quad , \qquad \sum k_i = \text{even}$$

This allows to set

$$\theta_i \in (-1, 1] \;\; + \;\; \theta_i \geq 0 \;\; \text{or} \;\; \theta_i < 0$$

We search for tachyons in NSNS, since

No tachyons on NSR, RNS and RR:

R zero point energy=0 + level matching

Lowest states on the NS k^{th} twisted sector

$$\begin{aligned}
\alpha' m_1^2 &= |\theta_2| + |\theta_3| - |\theta_1| &&= 2\epsilon\,\frac{ka_1}{N} + r_1 \\
\alpha' m_2^2 &= |\theta_3| + |\theta_1| - |\theta_2| &&= 2\epsilon\,\frac{ka_2}{N} + r_2 \\
\alpha' m_3^2 &= |\theta_1| + |\theta_2| - |\theta_3| &&= 2\epsilon\,\frac{ka_3}{N} + r_3 \\
\alpha' m_4^2 &= 2 - \textstyle\sum_{i=1}^{3} |\theta_i| &&= 2 + 2\epsilon\,\frac{ka_4}{N} - \sum r_i
\end{aligned}$$

r_i are even intergers

$\epsilon = 1$ if $\theta_i \geq 0$, $\epsilon = -1$ if $\theta_i < 0$

Then

$$\epsilon^{(k)} = -16 \prod_{\alpha} \sin \frac{\pi \alpha' m_\alpha^2}{2}$$

- Since $-1 \leq \alpha' m_\alpha^2 \leq 2$

$$\mathrm{sign}(\alpha' m_\alpha^2) = \mathrm{sign}\left(\sin \frac{\pi \alpha' m_\alpha^2}{2}\right)$$

- At most one $\alpha' m_\alpha^2$ can be negative

  Let us chose $|\theta_1| \leq |\theta_2| \leq |\theta_3|$

$$\Downarrow$$

  only m_3^2 and m_4^2 can be negative

$$m_3^2 < 0 \quad \rightarrow \quad |\theta_1| + |\theta_2| < |\theta_3|$$
$$m_4^2 < 0 \quad \rightarrow \quad |\theta_1| + |\theta_2| > 2 - |\theta_3|$$

Thus $m_3^2, m_4^2 < 0$ implies

$$|\theta_3| > 1$$

This contradicts $\theta_i \in (-1, 1]$

We have then shown

$$\epsilon^{(k)} = 16 \prod_\alpha \sin \frac{\pi k a_\alpha}{N} = -16 \prod_\alpha \sin \frac{\pi \alpha' m_\alpha^2}{2}$$

$$\Downarrow$$

$\epsilon^{(k)}$ positive	$\leftrightarrow$	tachyons in the k^{th} sector
$\epsilon^{(k)}$ negative	$\leftrightarrow$	non-SUSY without tachyons
$\epsilon^{(k)} = 0$	$\leftrightarrow$	SUSY sector

- Gauge inv. effective action

$$\Delta S \sim \int \frac{d^4p}{(2\pi)^4} \, \frac{\tilde{p}^\mu \tilde{p}^\nu}{\tilde{p}^4} \, A_\mu(p) \, A_\nu(-p)$$

is not gauge invariant

$$\delta_\lambda A_\mu(p) = ip_\mu \lambda(p) - 2 \int \frac{d^4q}{(2\pi)^4} \lambda(p-q) A_\mu(q) \sin\frac{\tilde{p}.q}{2}$$

$\Downarrow$ cancels

$\Downarrow$ needs higher-point functions to cancel

Summing all N-point functions

$$\Delta S \sim \int \frac{d^4p}{(2\pi)^4} \, \frac{1}{\tilde{p}^4} \, W(p) \, W(-p)$$

(Van Raamsdonk; E.L., Armoni)

with the straigh Wilson line operator

$$W(p) \; = \; \mathrm{Tr} \int d^4x \; P_* \left(e^{i\, g \int_0^1 d\sigma \, \tilde{p}^\mu \, A_\mu(x+\tilde{p}\,\sigma)} \right) * \, e^{ipx}$$

(Ishibashi, Ito, Kawai, Kitazawa)

$W(p)$ is gauge invariant $\Rightarrow$ ΔS is gauge invariant

STRING INTERPRETATION

- <u>Orbifold models</u>

$$\Delta S \sim \sum_{k=0}^{N-1} \int \frac{d^4 p}{(2\pi)^4} \, \frac{\epsilon^{(k)}}{\tilde{p}^4} \, W^{(N-k)}(p) \, W^{(k)}(-p)$$

with

$$W^{(k)} = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} e^{2\pi i \frac{jk}{N}} \widetilde{W}^{(j)}$$

and

$$\widetilde{W}^{(j)}(p) \; = \; \mathrm{Tr} \int d^4 x \; P_* \left(e^{i\,g \int_0^1 d\sigma \, \tilde{p}^\mu \, A_\mu^{(j)}(x+\tilde{p}\,\sigma)} \right) * e^{ipx}$$

- Closed strings couple to straight open Wilson line operators on NC D-branes (Das, Trivedi; Okawa, Ooguri; Liu, Michelson)

- Modes in the k^{th} twisted sector couple to field theory operators as (Douglas, Moore)

$$\mathrm{Tr}(\gamma_k \lambda_j) \, \phi_k \, \mathcal{O}_j = e^{2\pi i \frac{jk}{N}} \, \phi_k \, \mathcal{O}_j$$

$$\Downarrow$$

ΔS is suggestive of a closed string exchange

Can $1/\tilde{p}^4$ be related to a closed string propagator?

- Closed string exchange between D-branes

$d+4=4,6,8,10$: # dim. where ϕ_k lives

Contribution of ϕ_k to the D-brane eff. action

$$\Delta S \sim \int \frac{d^4p}{(2\pi)^4}\, W^{(N-k)}(p)\, W^{(k)}(-p) f(\tilde{p},u)$$

where f is the closed string propagator

$$f(\tilde{p},u) = \alpha'^{-\frac{d+2}{2}} \int \frac{d^dy}{(2\pi)^d} \frac{e^{iyu}}{y^2+\tilde{p}^2+(2\pi\alpha' m)^2}$$

$$\left(y=2\pi\alpha' p_\perp \right)$$

$$\Downarrow$$

$f(\tilde{p},u)$ diverges as $\alpha' \to 0$, but

$$f(\tilde{p},u)|_{\alpha'^0} \sim \int \frac{d^dy}{(2\pi)^d} \frac{e^{iyu}}{(y^2+\tilde{p}^2)^{\frac{d}{2}+2}} \to_{u\to 0} \frac{1}{\tilde{p}^4}$$

$$\Delta S \sim \sum_{k=0}^{N-1} \epsilon^{(k)} \int \frac{d^4p}{(2\pi)^4} \frac{1}{\tilde{p}^4} W^{(N-k)}(p)\, W^{(k)}(-p)$$

$\Downarrow$ $\Downarrow$

$\Downarrow$ $\sim$ closed string propagator

all scalar modes contribute

$\epsilon^{(k)} =$ NSNS $-$ RR

$\epsilon^{(k)} = 0$ if SUSY

} measure of misalignment between NSNS and RR

Consistent with

- $\epsilon^{(k)}$ determined by lowest NS states
- R zero-point energy $= 0$

- Open question

Do NC instabilities and closed string tachyons have a related condensation process?