SUPERSTRINGS IN CURVED SPACE-TIME

Costas KOUNNAS

INTRODUCTION

EINSTEIN GRAVITY ⊕ MATTER INTERACTIONS

is meaningful ONLY IN LOW ENERGIES.

"effective Theory"

$$S_0 = \int d^4x \sqrt{g} \left(\frac{M^2}{2} R + \mathcal{L}_{matter} \right)$$

After Quantum corrections $\boxed{S_0 + \Delta S \to S}$

$$\Delta S = \int d^4x \sqrt{g} \Big(\alpha R^2 + \beta R_{\mu\nu} R^{\mu\nu} + \gamma R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}$$
$$+ \frac{1}{M^2} \left(\delta R_\mu{}^\nu R_\nu{}^\sigma R_\sigma{}^\mu + \varepsilon R^3 + \dots \right)$$
$$+ \frac{1}{M^4} \left(\zeta R \cdot R \cdot R \cdot R + \dots \right)$$
$$+ \mathcal{O}\left(\frac{1}{M^6} \right)$$

ΔS contains infinitely many UNKNOWN parameters $(\alpha, \beta \dots) = (\alpha_i)$

S_0 is meaningful only when we can neglect higher curvature terms and terms which involve more than two derivatives on matter and gauge fields.

$$\mathcal{O}\left(\frac{R_{\mu\nu\rho\sigma}}{M^2} , \frac{\partial_\mu}{M} \right) \text{ expansion.}$$

$\Big\downarrow$ implies

- "There is no meaning to speak about a big-bang initial singularity or black-hole solutions since near singularities the $\mathcal{O}(\frac{1}{M^2})$ expansion breaks down with

$$\boxed{R \cdot R \gg M^4}$$ "

- "In cosmology it implies the non knowledge of the evolution of our universe at early times

$$t \leq M^{-1} \quad \text{or} \quad T(t) \geq M$$ "

- "All Quantum gravitational phenomena are not predicted for distances

$$\boxed{d \leq \mathcal{O}(\tfrac{1}{M}) \simeq 10^{-32}\ \text{cm}}$$

The only known theory in which all fundamental interactions, including gravity, are well defined is (super)-String theory.

$$\begin{Bmatrix} \text{STRING} \\ \text{theory} \end{Bmatrix} \xrightarrow[\alpha'\text{-expansion}]{} \begin{Bmatrix} \text{UNIFIED Effective} \\ \text{GRAVITY} \oplus \text{MATTER} \\ \oplus \text{GAUGE INTERACTIONS} \end{Bmatrix}$$

$$\oplus\ \alpha' \{\text{higher derivatives}\}$$

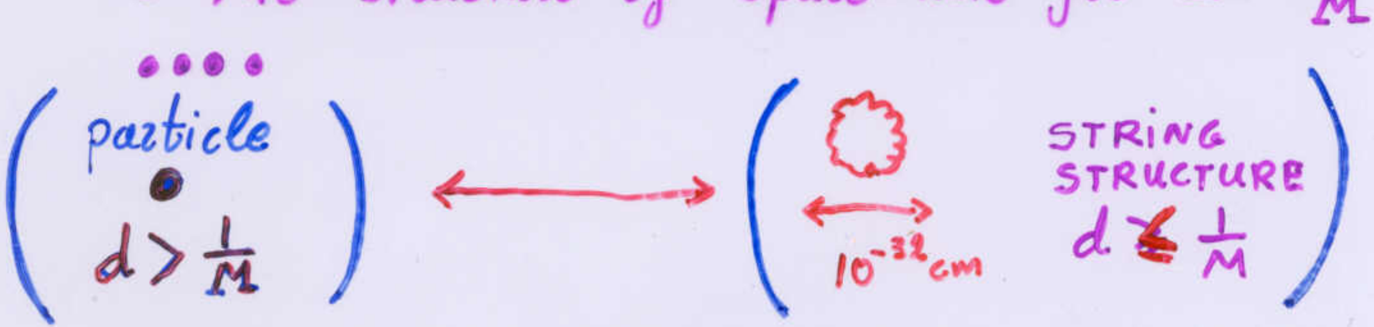
with $\left(\alpha' \equiv \frac{1}{M^2}\right)$

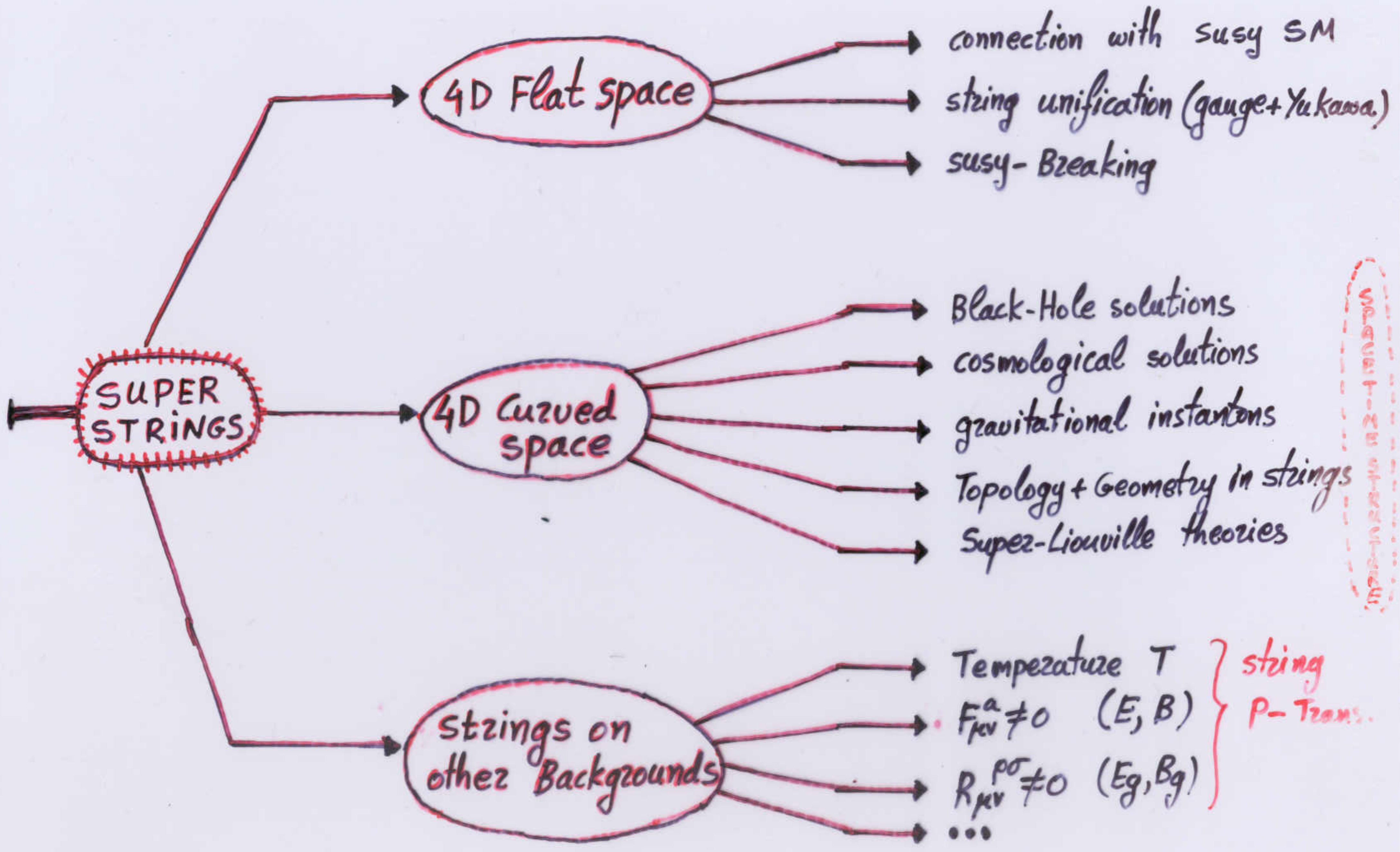
Fradkin + Tseytlin / Callan + Friedan + Martinec + Perry.

In this respect, string theory extends the validity of Field Theory at very small distances $\Delta x \sim \mathcal{O}(\frac{1}{M})$ and in the regions where the space-time is highly-curved.

- Initial singularity in Cosmological solutions
- Black-holes solutions
- Wormholes
- The structure of space-time for $\Delta x < \frac{1}{M}$

....




SUPER STRINGS
4D Flat space
connection with susy SM
string unification (gauge+Yukawa)
susy-Breaking
4D Curved space
Black-Hole solutions
cosmological solutions
gravitational instantons
Topology + Geometry in strings
Super-Liouville theories
SPACE-TIME STRUCTURE
Strings on other Backgrounds
Temperature T
$F^a_{\mu\nu} \neq 0$ (E, B)
$R_{\mu\nu}^{\ \ \rho\sigma} \neq 0$ (E_g, B_g)
...
string P-Trans.

4D – SUPERSTRINGS

- Effective Supergravities in low energies $E_t < M_{string}$
- Connections with the $SU(3) \times SU(2) \times U(1)$ with $E_t \simeq M_Z$
- Super Higgs ⊕ Higgs mechanism.

$\left(\frac{E_t}{M_{string}}\right)$ expansion

⊕ string loop expansion

"Infrared behaviour in string theories"

- Initial Cosmological singularity. "Big-Bang" $\|R_{\mu\nu\rho\sigma}\| \sim M_{string}$.

- Black-Holes
- Wormholes
- Gravitational Instantons
- ...

- The structure of our universe in $\Delta x \sim M_{string}$

- Strings in finite Temperature
- Strings in finite Electric or Magnetic fields
- Strings in non-trivial Backgrounds

(STRINGY PHENOMENA) → $\|R_{\mu\nu\rho\sigma}\| \sim M_{string}$, $T \sim M_{string}$, $\|\vec{B}\|, \|\vec{E}\| \sim M_{string}$

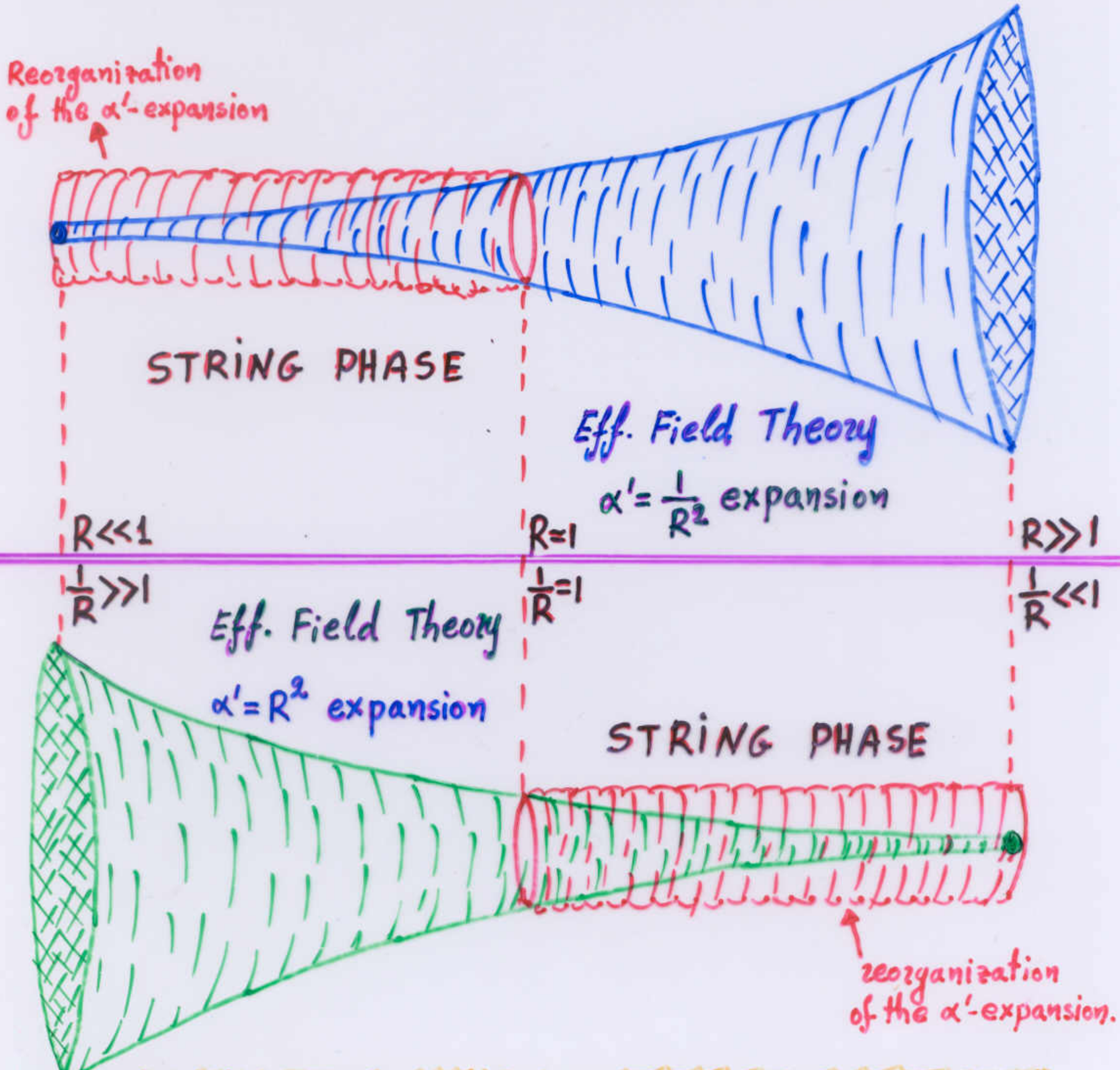
?

(PARADOXES IN EFF. FIELD THEORY)

2. STRING PHENOMENA

PARADOXES in Effective Field Theory.

- $$\boxed{ds^2 = -dt^2 + R^2(t)\left[d\theta_1^2 + d\theta_2^2 + d\theta_3^2\right]}$$



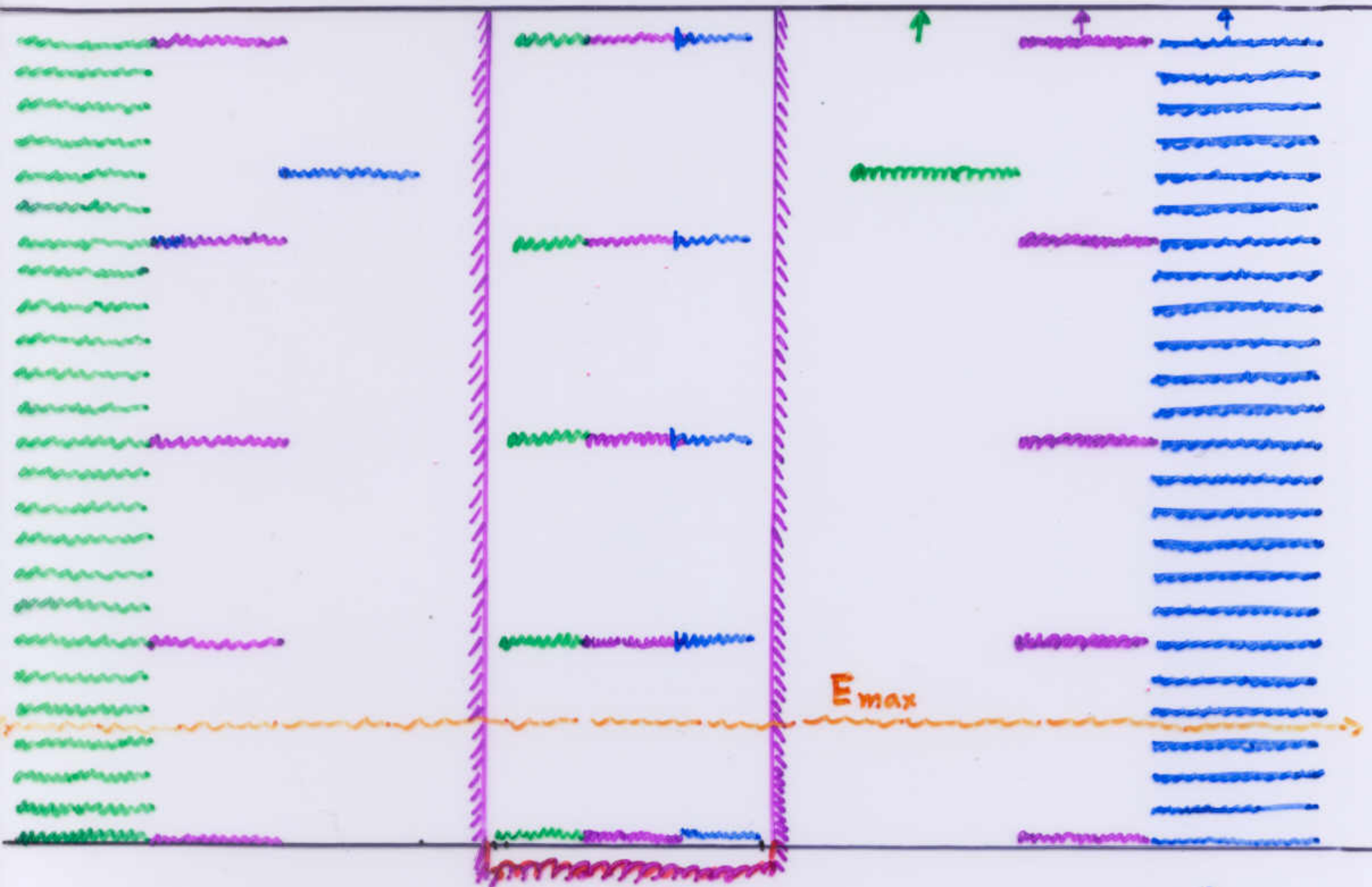
- $$\boxed{ds^2 = -dt^2 + \frac{1}{R^2(t)}\left[d\theta_1^2 + d\theta_2^2 + d\theta_3^2\right]}$$

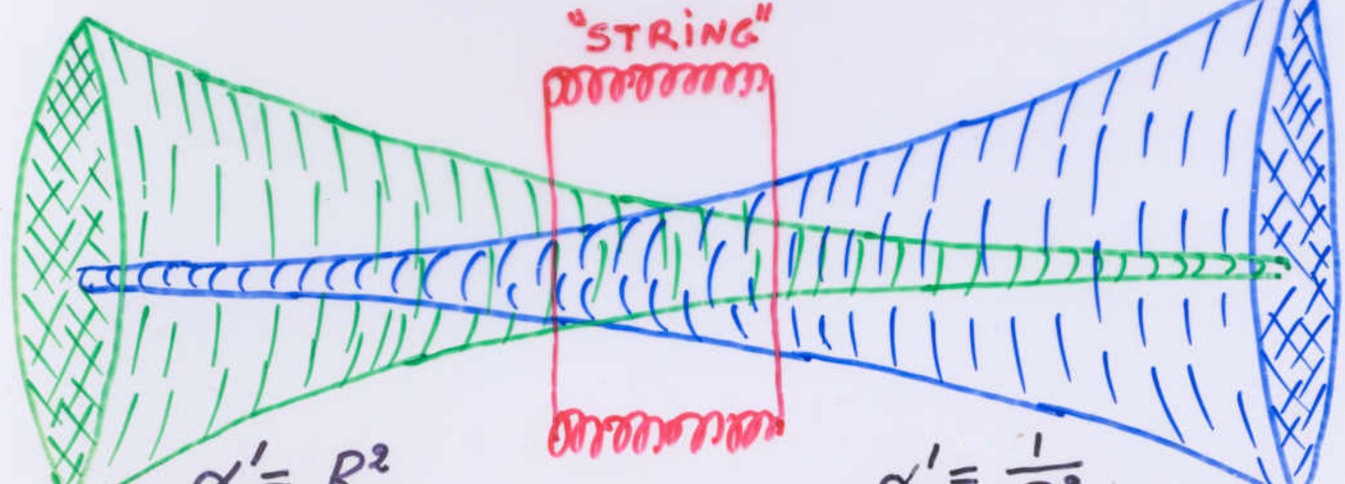
DUALITY "PARADOXE"

$$ds^2 = -dt^2 + R^2(t)\left[d\theta_i^2\right]$$ ← Eff. Field Theory "momenta"

Equiv.

$$ds^2 = -dt^2 + \frac{1}{R^2(t)}\left[d\theta_i^2\right]$$ ← Eff. Field. Theory "windings"





$\alpha' = R^2$ $\qquad\qquad$ $\alpha' = \frac{1}{R^2}$

- For $t \to -\infty$ the lower mass-excitations are the "WINDING"-STATES
- For $t \to +\infty$ the lower mass-excitations are the "MOMENTUM" STATES

$$\boxed{\underset{(t\to-\infty)}{\text{WINDING}} \rightsquigarrow \underset{(t\to+\infty)}{\text{MOMENTUM}}}$$

- It is then necessary, either to go beyond the field theory picture and
- work directly on the String level, (using the powerful techniques of the underlying two-dimensional (super) conformal theory)

"OR"

- to find a way to extend the field theory picture. (e.g. reorganization of the α'-expansion or else?)

Kiritsis Kounnas.

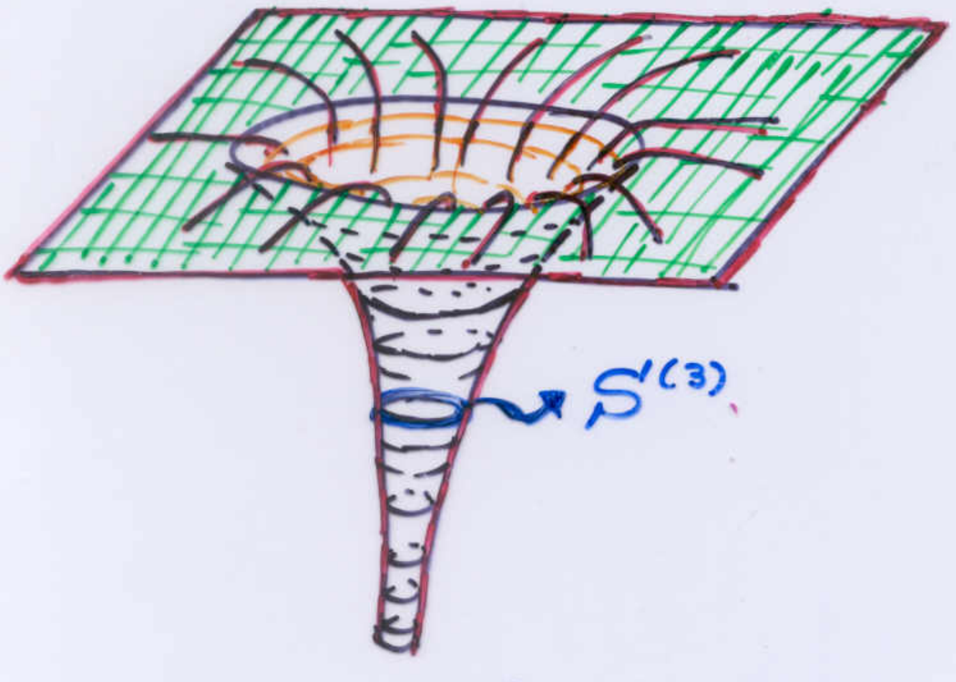
Special, non-trivial, $(N=4, \hat{c}=4)$ realizations.

(i) Wormhole 4-d space.

$$W_k^{(4)} \equiv \left\{ SU(2)_k \otimes U(1)_Q \right\}_{SUSY}$$

- $ds^2[W] = k \dfrac{dz\,d\bar{z} + dw\,d\bar{w}}{(z\bar{z} + w\bar{w})}$
- $2\Phi = \log(z\bar{z} + w\bar{w}) + \text{const.}$
- $H_{ijk} = e^{2\Phi} \varepsilon_{ijk}{}^{\ell} \partial_\ell \Phi = Q \underbrace{\varepsilon_{ijk}}_{SU(2)\text{ S.C.}}{}^{4}$

$$Q = \sqrt{\frac{2}{k+2}}$$



$$\rho = e^{-2\phi}$$
$$t = \log \rho$$

$$ds^2 = dt^2 + d\Omega^{(3)}$$

σ-model / background interpretation {Callan + Harvey + Strominger

5-brane interpretation : Duff + Lu.

Exact Conformal Construction : Kounnas + Porrati + Rostand.

*** Both categories of Liouville representations give rise to

UNITARY REPRESENTATIONS OF $N=4$, $\hat{c}=4$

once they are combined with the $SU(2)_k$ and $(SU(2)^+, SU(2)^-)$ degrees of freedom.

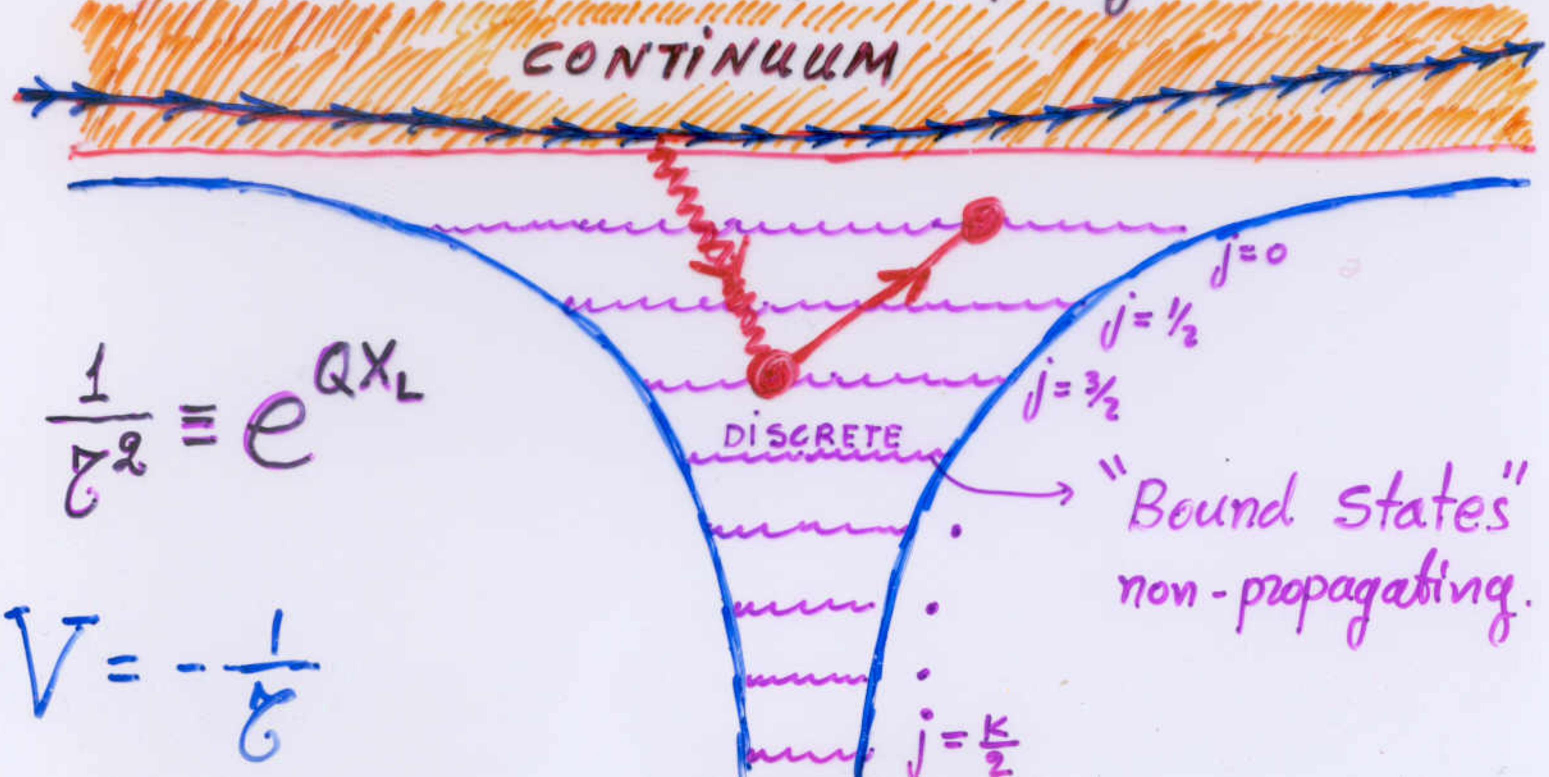
- Continuous Repr. $\longrightarrow$ long (massive) $N=4$ Rep.

$$h > s^+$$

- discrete Repr. $\longrightarrow$ short ("chiral") $N=4$ Repr.

The locality respect to the $N=4$ operators implies

$$\begin{cases} s^+_{1,2} = s^-_1 = \frac{1}{2} & \tilde{\beta} = -(j+1) & h = s^+_{1,2} \\ s^+_{1,2} = s^-_2 = \frac{1}{2} & \tilde{\beta} = j & h = s^+_{1,2} \end{cases}$$



$$\frac{1}{r^2} \equiv e^{QX_L}$$

$$V = -\frac{1}{r}$$

- Kounnas

(ii) (2d - Bell) ⊗ (2d - Cylinder)

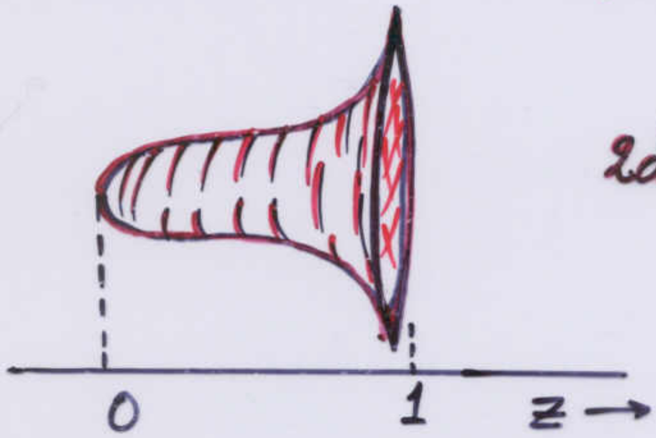
$$G_K^{(4)} \equiv \left\{ \left(\frac{SU(2)}{U(1)}\right)_K \otimes U(1)_R \otimes U(1)_Q \right\}_{SUSY}$$

- $ds^2[C] = K\left[\frac{dz\,d\bar{z}}{1-z\bar{z}} + dw\,d\bar{w}\right]$

- $2\Phi = \log(1-z\bar{z}) + w + \bar{w} + \text{const.}$

- $N=4$ relations : $Q = \sqrt{\frac{2}{K+2}}$, $R = \sqrt{2K}$

z



2d - BELL

$|z| < 1$

w

2d - CYLINDER

- $G_K^{(4)}$ and $W_K^{(4)}$ are DUAL to each other.

{Kiritsis
Kounnas
Lust

{Rocek
E. Verlinde

(iii) <u>(2d - BELL) ⊗ (2d - CIGAR)</u> (Axial).

(iv) <u>(2d - BELL) ⊗ (2d - TRUMPET)</u> (Vector).

$$\Delta^{(4)}_{K} = \left\{ \left(\frac{SU(2)}{U(1)}\right)_{K} \otimes \left(\frac{SL(2,R)}{U(1)}\right)_{K+4} \right\}_{SUSY}$$

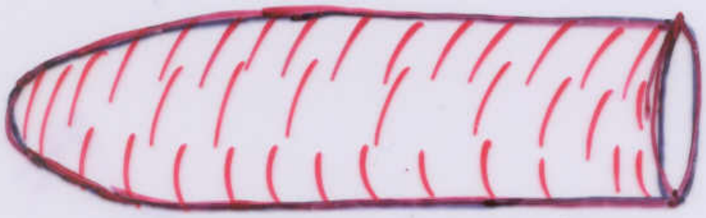
↑ (Axial) or (Vector)

- $ds^2(\Delta_{A,V}) = K\,\frac{dz\,d\bar{z}}{(1-z\bar{z})} + K'\,\frac{dw\,d\bar{w}}{(w\bar{w}\pm 1)}$
- $2\Phi = \log(1-z\bar{z}) + \log(1\pm w\bar{w})$ + const

z

2d - BELL

w^A



2d - CIGAR

w^V

2d - TRUMPET

<u>Construction</u>: C.K.

<u>σ-model</u>: Kiritsis, Kounnas, Lust

<u>Cosmological solutions</u>: - Kounnas + Lust
- Witten + Nappi
- Gasperini + Veneziano. ...

DUAL

Δ (Axial) ↭ Δ (Vector)

- Kiritsis
- Giveon
- (Verlinde)² + Dijkgraaf

- The full spectrum of excitation is derived in terms of well-known characters ($SU(2)_n$ in W; toroidal, parafermionic, $U(1)$, in G, Δ and F). A.F.K.

- For small κ (large curvatures),
The
  - field theory states
  - string-states
  - "Winding"-like states

  } are degenerate

$$\text{F.T.S.} \sim \mathcal{O}\left(\frac{1}{\kappa+2}\right) \qquad \kappa \sim 0, 1, 2, 3. < 10$$

$$\text{S.S.} \sim \mathcal{O}(1)$$

$$\text{W.L.S} \sim \mathcal{O}\left(\frac{\kappa-2}{16}\right)$$

A.F.K

- The constructions based on W or G are connected to the non-critical strings, and define SUPER-LIOUVILLE theories in the STRONG-COUPLING regime, coupled to UNITARY MATTER SYSTEMS.

$$\boxed{\begin{aligned} \hat{C}_L &= 1 + 4\left(\frac{1}{\kappa_1+2} + \frac{1}{\kappa_2+2}\right) \\ \hat{C}_M &= 1 - 4\left(\frac{1}{\kappa_1+2} + \frac{1}{\kappa_2+2}\right) \end{aligned}}$$

$$\boxed{5 \leq C_M < 9}$$

!!! K+A.F.K.

(iii) $U(1)_Q$ - Liouville-like characters. X_L

They can classified in two categories:

(A) Continous representations.

they are generated by the lowest weight operators

$$e^{\beta X_L} \;:\; \beta = -\frac{1}{2}Q + ip$$

with p - real and $h_p = \frac{Q^2}{8} + \frac{p^2}{2}$

The fixed imaginary part in the momentum $(i\frac{Q}{2})$ of the plane-waves is due to the non-trivial dilaton motion.

$$\boxed{Z^L_{cont.} = \frac{\mathrm{Im}\,\tau^{-1/2}}{\eta(\tau)\,\bar{\eta}(\bar{\tau})}}$$

(B) Discrete representations.

$$e^{\beta X_L} \;:\; \beta = \tilde{\beta} Q \;,\; \tilde{\beta}\text{.real}$$

$$h_\beta = -\frac{\tilde{\beta}(\tilde{\beta}+1)}{K+2} \qquad \boxed{\left(\frac{j(j+1)}{K+2}\right)_{SU(2)_K}}$$

A Resolution of the Cosmological Singularity

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Lorenzo Cornalba and Miguel Costa, hep-th/0204261

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String gravity and cosmology: Some new ideas, gr-qc/9509017.

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JHEP 0203 (2002) 052, hep-th/0111025.

Introduction

In the *standard cosmological models*, the existence of an initial *singularity in the past*, (Big–Bang), *is inevitable*.

The standard cosmological models *far below* the Planck scale are *extremely successful* in explaining the experimental data.

The existence of a *minimum distance in String Theory* $\longrightarrow$ *Pre–Big–Bang Scenario Conjecture.*

However, either:

• the exact string backgrounds *are not realistic*
Kiritsis–Kounnas '95
or
• They show *Time–Like singularities* whose nature has remained unclear.
Kiritsis–Kounnas '95,
Burgess, Martineau, Quevedo, Rajesh and Zhang, '01 ,
Cornalba–Costa, '02

More recently, in the framework of *brane cosmology:*

• • • The big–bang singularity $\longleftrightarrow$ collision of branes • • •
Ekpyrotic Universe
Khoury, Ovrut, Steinhardt and Turok '01 $\oplus$ *Seiberg '01* $\oplus$ *'02,*
Elitzur, Giveon, Kutasov and Rabinovici, '02

In all these scenarios the *cosmological singularity* is *always Space–Like* and *does not have* a clear *brane interpretation.*

The simplest *singularity free cosmology* is that of *full de Sitter Space–Time* with positive cosmological constant in the bulk.

de Sitter cosmology is problematic:

- In string theory *it has not been possible*, up to now, to construct a *meta–stable string background* with a de Sitter geometry.

- From the cosmological point of view it is difficult to *connect a de Sitter phase* to the usual matter dominated universes and at *the same time* to retain a *singularity–free space–time*.

- de Sitter space *avoids the cosmological singularity* by introducing an effectively *repulsive* part to the gravitational interaction which we can interpret either as a *positive cosmological constant*, or as a *space–filling brane.*

A New Cosmological Scenario

Cornalba–Costa–Kounnas '02

We propose a new cosmological scenario which *resolves* the *conventional* initial singularity problem.

The space–time geometry has an *unconventional Time–Like singularity on a lower dimensional hypersurface,* with *localized negative energy density.*

The natural interpretation in string theory is that of *negative tension branes*, e.g. the *orientifolds* of type II string theory. *The Space-time ends* at the *orientifolds*, and it is divided in *three regions:*

- *A Contracting Region* with a future cosmological *horizon.*

- *An Intermediate Region* which *ends at the orientifols.*

- *An expanding Cosmological Region.* It is separated from the intermediate region by a *past cosmological horizon.*

*** The Orientifold are hypersurfaces of arbitrary dimension, *charged under the Ramond–Ramond fields*, with *negative tension* and *no localized dynamical degrees of freedom*

O–planes act as source in the super–gravity action

$$S_O = T \int_\Gamma d^d x \, e^{-\phi} \sqrt{-\det G} \ \pm Q \int_\Gamma A_d$$

$T \longrightarrow$ tension
$Q \longrightarrow$ charge
$\Gamma \longrightarrow$ d–dimensional hypersurface.

S_O acts as a *cosmological term localized* to the hypersurface.

The basic idea behind the new cosmological models is the *replacement of the big-bang singularity at $t = 0$* by a

- *cosmological horizon*

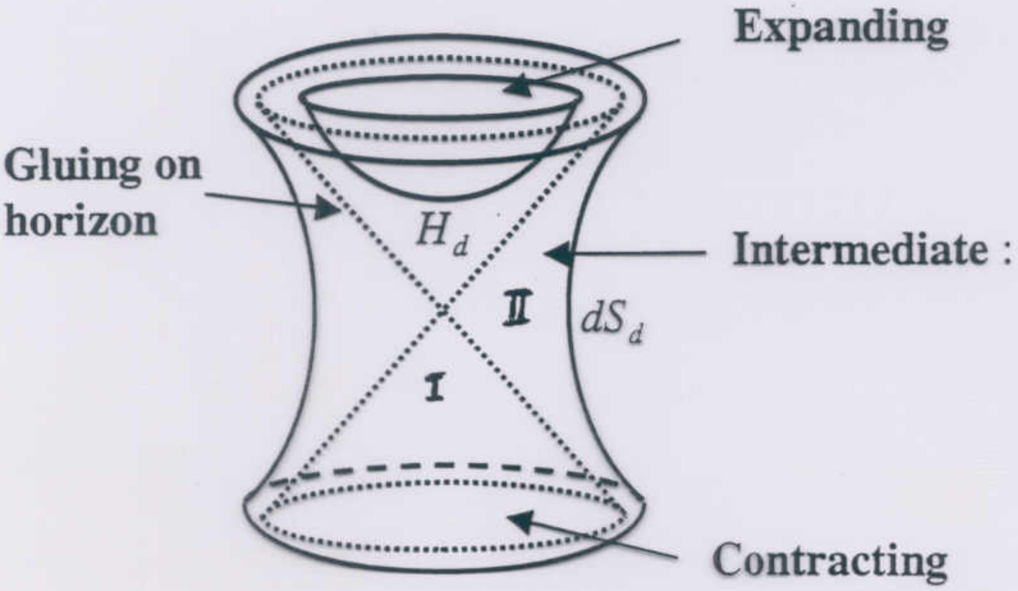
and to

- *continue space-time across the horizon.*

- Gluing conditions on the horizon and analytic continuation

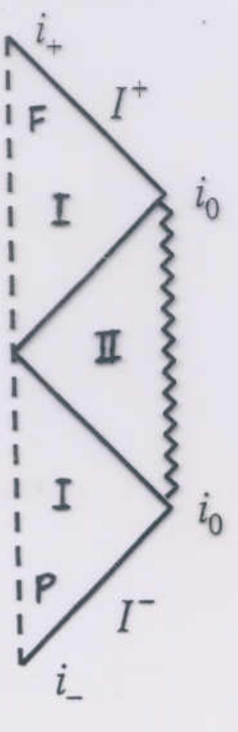
$$a_I(t) = t + o(t^3) \qquad \psi_I(t) = \psi_0 + o(t^3)$$

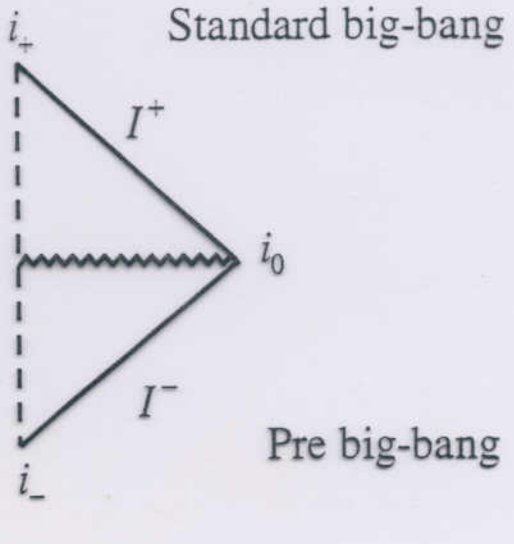
$$a_{II}(x) = -i a_I(ix) \qquad \psi_{II}(x) = \psi_I(ix)$$



Any singularity will have a *branelike interpretation*

- Carter-Penrose diagram





The simplest model with this causal structure is the

KL-string 4D model

$$\frac{SL(2,R)_{(-k)}}{O(1,1)} \otimes U(1) \otimes U(1) \otimes K\left(\delta c = \frac{-6}{k+2}\right)$$

$$ds_E^2 = -dt^2 + t^2\ dz^2 + (t^2 + R^2)\ dx^2 + (t^2 + R^2)\ dy^2$$

For large positive $t^2 >> R^2$ it describes:

• • • A Contracting Isotropic Universe for $t << 0$

• • • An Expanding Isotropic Universe for $t >> 0$

• • • The would be $t = 0$ singularity is replaced by

• • • *a non singular horizon:*

$$ds_E^2(t \sim 0) = -d\tau^2 + d\rho^2 + R^2\ dx^2 + R^2\ dy^2$$

Beyond the horizon ($t^2 < 0$), the extended metric develops a

Time-Like singularity at $t^2 + R^2 = 0$

$$ds_E^2(t^2 < 0) = -\bar{\rho}^2\ d\bar{\tau}^2 + d\bar{\rho}^2 + (R^2 - \bar{\rho}^2)\ dx^2 + (R^2 - \bar{\rho}^2)\ dy^2$$

setting $\bar{\rho}^2 = R^2 sin^2\sigma$

$$ds_E{}^2(t^2<0) = R^2(-sin^2\sigma\ d\bar{\tau}^2 + cos^2\sigma\ (d\sigma^2 + dx^2 + dy^2)$$

$B_0 - \bar{B}_0$ *Negative Tension zero-Branes at* $\bar{\rho} = \pm R$

Other *isotropic examples* with the same space–time causal structure can be constructed:

• Klein compactification of flat space–time, by a string theory orbifold construction.
Grojean, Quevedo, Tasinato and Zavala, '01

• Imposing the existence of a Cosmological Horizon in String solutions with a Non-Trivial Electric Background.
Cornalba and Costa '01, $\oplus$ *Kounnas '02*

• Performing Double Analytic Continuation in Non-Extremal Black-Holes or Brane Solutions.
Israel and Kounnas, '02 (in print)

For the description of the Cosmological Horizon in the isotropic models, it is convenient to use the Milne and Rindler 'polar' coordinates system.

$$ds_M^2 = -dt^2 + a(t)^2\ dH_d^2, \qquad t = 0 \rightarrow \text{flat},$$

$$ds_R^2 = dy^2 + a(y)^2\ dS_d^2, \qquad y = 0 \rightarrow \text{flat}$$

H_d is the d–dimensional hyperbolic space.

$$dH_d^2 = \frac{dx^2 + dx_i^2}{x^2}$$

S_d is the d–dimensional De Sitter space.

$$dS_d^2 = \frac{-d\tau^2 + dx_i^2}{\tau^2}$$

Crossing the horizon $t \to iy$ and $x \to i\tau$

The 'cosmological' time t in Milne metric

becomes

A space–like coordinate y, in Rindler metric.

Generic model in Type II string theory

In the Rindler patch the metric develops a *time-like naked singularity where the universe ends.*

The metric has the correct form to be interpreted as an *orientifold plane with a de Sitter world-volume.*

The geometry describes *an open universe* with:

- *contracting and expanding regions,*

together with

• *an intermediate region bounded by an orientifold plane.*

The intermediate region *smoothly connects* the would be *'big–crunch'* and *'big–bang'* of conventional cosmologies.

The *resolution of the cosmological singularity* allows the calculation of *transition amplitudes* from the *collapsing* to the *expanding phases.*

In particular, *the past/future vacuum to vacuum amplitude* for a free field propagating in the cosmological background.

• Assuming *a trivial vacuum at the asymptotic past,*

$\longrightarrow$ *a thermal spectrum in the far future.*

This analysis is similar to that of Hawking in the derivation of *black hole radiation, applied to our cosmology.*

A dynamical description of the universe:

•• Consider a *large spherical charged brane* as the boundary of the universe.

•• The brane starts to contract due to the brane's tension.

••• During the collapse, *the brane will back-react* on the geometry

$\longrightarrow$

•• *It creates, in the center of the sphere, a positive energy density.*

•• If the brane has *positive tension*, it will *collapse* due to its own *gravitational and electric forces.*

•• If the brane has *negative tension*, it will interact with its own *gravitational and electric fields* *with opposite sign*

•• This will *invert the contraction to an expanding phase.*

$\longrightarrow$ *Localized negative* tension objects can *avoid the big-bang singularity.*

???? Why Orientifolds ????

This choice *is natural in string theory*:

• In order to create a *non-trivial gravitational* background with *broken supersymmetry*

It is necessary to start with *non-BPS configuration* of branes.

• *If not*, the vacuum solution will be *stable and stationary* due to the supersymmetry.

One possibility is to construct type II models where the RR charge is canceled *only with orientifold planes* $O - \overline{O}$ localized in different points, *without any D or* $\overline{D}$ *branes.*

• Models of this kind are constructed in the literature:
Antoniadis, Dudas and Sagnotti, '99
Kachru, Kumar and Silverstein, '00

• $O - \overline{O}$ configuration *breaks supersymmetry*, and it is *incompatible* with a *flat space-time* and *constant dilaton.*

• • The *induced geometry* in the presence of orientifolds is our *cosmological solution*

The $O - \overline{O}$ configuration is stable!!!
Cornalba, Costa and Kounnas '02

• • The *attractive electric* force is balanced by an effective *repulsive* force created by the gravitational back-reaction of the orientifolds.

• • Another initial conficuration, is *to dress the orientifolds* with *branes*, $OD - \overline{OD}$.

The $OD - \overline{OD}$ configuration is *consistent classically* with a flat space-time and constant dilaton.

At the *quantum level* $OD - \overline{OD}$ *is unstable* due to the lack of supersymmetry. *An effective potential is created* bringing the *branes and anti-branes together* which *annihilate* each other.

$\longrightarrow$ $D - \overline{D}$ *Annihilation*

After this annihilation, *we are left with the* $O - \overline{O}$ orientifolds,

• The *background fields are no longer flat* and give rise to the cosmological scenario described above.

••• The structure of *the effective potential* among *branes* and *orientifolds* is determined *by probing* with *D–branes* the geometry.

A Specific Type II String Model

We consider backgrounds with a non-trivial RR field.

The 10d Type II effective action has the form:

$$S = \frac{2\pi}{l_s{}^8}\left[\int d^{10}x\sqrt{g}e^{-2\phi}\left(R+4\left(\nabla\phi\right)^2\right) - \frac{1}{2}\int F\wedge\widetilde{F}\right] ,$$

- $l_s = 2\pi\sqrt{\alpha'}$

- F is the RR $(d+1)$–form field strength

- $\widetilde{F} = \star F$ is the dual $\tilde{d}$–form, $\tilde{d} = 9 - d$.

A family of solutions is parameterized by g_s

$$\mathcal{E}^2\, ds^2 = \Lambda^{-\frac{1}{2}\,\frac{d+1}{d-1}}\; ds^{\,2}_{d+1} + \Lambda^{\frac{1}{2}}\; ds^2(E^{\tilde{d}})$$

$$e^{\phi} = g_s\;\Lambda^{\frac{4-d}{4}}\;, \qquad\qquad \widetilde{F} = \frac{1}{g_s\;\mathcal{E}^{\tilde{d}-1}}\,\epsilon(E^{\tilde{d}})$$

In the intermediate region II, the geometry develops a time-like curvature singularity.

$$a_{II}\left(x\right) = a_s\left(x_s - x\right)^{\gamma} \qquad\qquad \phi_{II}\left(x\right) = -\log\left[\eta\left(x_s - x\right)\right]^2$$

The dimensionless constant a_s *only depends* on the boundary conditions imposed *at the horizon.*

•• The form of the metric closed to the singularity can be determined from:

• The behavior of the scalar field at *the singularity* ($\phi \to +\infty$), and *the horizon* ($\phi = \phi_0$)

• Its *asymptotic behavior in the future* ($\phi \to -\infty$)

• The *growing behavior* of $(-\phi)$ as one moves from the *singularity to the horizon*

• The *growing behavior* $(-\phi)$ from the horizon to the *asymptotic future*

$$\mathcal{E}^2 ds^2 = \Lambda^{-\frac{1}{2}} \mu \, ds^2(dS_d) + \Lambda^{\frac{1}{2}} \left[d\Lambda^2 + ds^2(E^{\tilde{d}}) \right] ,$$

$$e^{\phi} = g_s \Lambda^{\frac{4-d}{4}} , \qquad F = -\frac{1}{g_s \mathcal{E}^d} \, d\Lambda^{-1} \wedge \left[\sqrt{\mu^d} \, \epsilon(dS_d) \right] ,$$

μ is a constant determined from a_s.

Substituting the *de Sitter space* dS_d by the *Minkowski space* M^d we obtain *naively* the solution for *Dp–branes* ($p = d - 1$) at $\Lambda(x) = 0$, *uniformly smeared* along $E^{\tilde{d}}$.

The *harmonic function* H is simply $H = \Lambda$.

• The *tension is negative.* $\longrightarrow$ The singularity is interpreted as *non-dynamical* *$(d-1)$-orientifold* smeared along the $E^{\tilde{d}}$.

When $\Lambda \to 0$ the *radius of the de Sitter slice* $\sqrt{\mu}\,\Lambda^{-1/4}$ *diverges.* The orientifold looks flat approaching the usual supersymmetric solution.

Away from the orientifold the solution becomes

$$\mathcal{E}^2 ds^2 = \frac{G}{\Lambda^{\frac{1}{2}}}\,\mu\, ds^2\,(dS_d) + \frac{\Lambda^{\frac{1}{2}}}{H}\, d\Lambda^2 + \Lambda^{\frac{1}{2}}\, ds^2(E^{\tilde{d}})$$

$$e^{\phi} = g_s\,\Lambda^{\frac{4-d}{4}}\ , \qquad F = \frac{1}{g_s\,\mathcal{E}^d}\,\frac{1}{\Lambda^2}\sqrt{\frac{G^d}{H}}\,d\Lambda \wedge \left[\sqrt{\mu^d}\,\epsilon(dS_d)\right]$$

$G(\Lambda)$ and $H(\Lambda)$ are dimensionless functions defined by

$$\mu G = a_{II}^2(\Lambda)\ \Lambda^{-\frac{1}{d-1}}, \qquad H = \Lambda^{\frac{d}{d-1}}\ \left(\frac{d\Lambda}{dx}\right)^2$$

$g_s = e^{\phi}$ is the *string coupling at the horizon.*

$\mathcal{E} = g_s\sqrt{-F^2}$ is the *electric field at the horizon.*

The metric is known analytically for $d=1$. The functions G and H become linear in Λ.

• For *large cosmological times* the geometry in cosmological *region I* is that of a *curvature dominated* universe.

Probing the Geometry with Branes

From the *motion of D–branes* in the orientifolds background we can derive the *static effective potential* as seen by an observer on the brane

For *flat branes,* there is a *brane/orientifold repulsion.*

There is *no force between parallel branes and orientifolds* of opposite charge, due to the BPS nature of the configuration.

In our case, both the source and the probe brane induces a *repulsive force* and tends to move the probe brane away from the singularity.

The *energy density generated* by the orientifolds acts as an *anti-gravitational force* on the orientifolds preventing them from collapse.

The *static potential* can be deduced from the metric. The Born-Infeld and the Wess–Zumino pieces give the total action

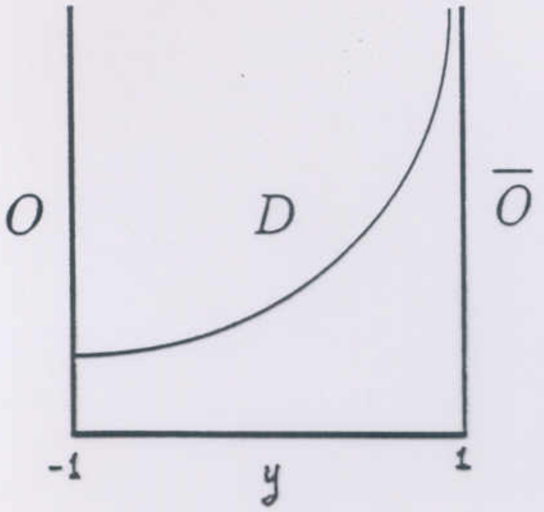
The effective potential for $d = 1$ becomes:

$$\mathcal{V}_D(y) = \frac{y^2}{1 - y^2\,\mathrm{sign}(y)}$$

The coordinate y is defined as $\Lambda = 1 - y^4$

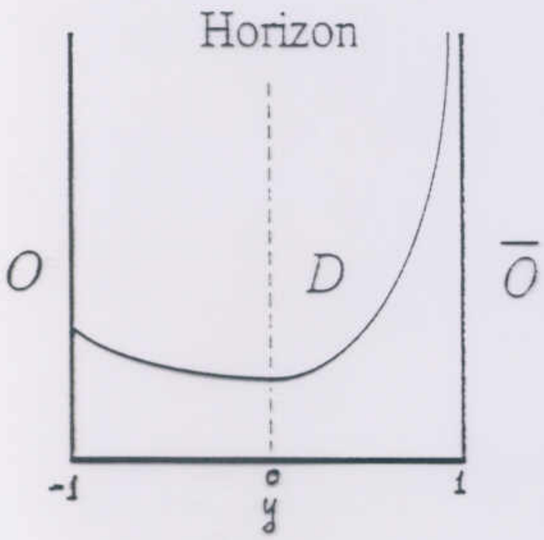
The horizon is at $y = 0$ and $y^2 = 1$ gives the position of the orientifold.

Consider a D-brane probe. It sees the following potential $V(y)$



Naïve **potential without backreaction**

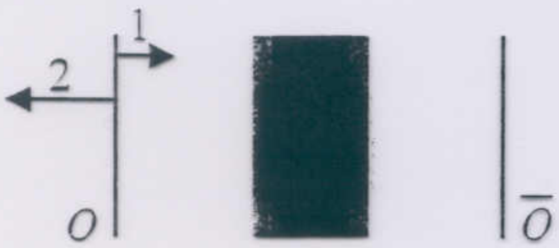
- OD is SUSY
- $\overline{O}D$ repel



Complete potential

D-brane attracted to core of geometry
It has + mass and + charge. Attracted by positive energy density in the core.

- **<u>Orientifold Repulsion</u>**



Energy	-	+	-
Charge	-	0	+

Two competing forces:

1. $O\overline{O}$ attraction
2. Repulsion from energy density in the core

Second force wins !

Cosmological Thermal Radiation

In the absence of initial singularity in the early universe, the propagation of matter can be studied for all times.

The $d = 1$ case is particularly simple, especially for scalar fields.

Let us consider a massless scalar field Ξ satisfying the Klein–Gordon equation in:

The Collapsing Region I

$$\Xi_{in} \sim \frac{1}{\sqrt{|p\ t|}} \left(a(p)_{in}\ e^{ip(t+x)} + a(p)^*_{in}\ e^{-ip(t+x)} \right)$$

The Intermediate Region II

- **Continuity at the horizon**

$$\Xi_{II}(y, \tau) = \Xi_I(it, ix)$$

- **Orientifold boundary conditions**

$$\Xi_{II} = 0, \qquad \partial_y \Xi_{II} = 0$$

The Expanding Region I

$$\Xi_{out} \sim \frac{1}{\sqrt{|p\ t|}} \left(ia(p)_{out}\ e^{ip(t+x)} - ia(p)^*_{out}\ e^{-ip(t+x)} \right)$$

With Bogolubov relations among "in" and "out" states:

$$a(p)_{out} = +\alpha a_{in}(p) - \beta a(-p)^*_{in}$$
$$a(p)^*_{out} = -\beta a(-p)_{in} + \alpha a(p)^*_{in}$$

The natural choice of the cosmological vacuum in the far past is the trivial vacuum $|0\rangle$

$$a_{in}(p)\,|0\rangle = 0$$

The *observer in the expanding universe* will detect an *average number*$N(p)$ of particles of momentum p:

$$N(p) = |\beta|^2 = \frac{1}{e^{p/\mathcal{T}(p)} - 1}$$

These particles *arise from the reflection* on the orientifold. The non-trivial coefficient β in the Bogolubov transformation *relates states with equal and opposite momentum.*

The effective dimensionless temperature approaches for large $|p|$

$$\mathcal{T}(p) \sim \frac{1}{2\pi}$$

At large cosmological times, the frequency ω is shifted $\longrightarrow$ thermal spectrum with *physical temperature*

$$T = \frac{\mathcal{E}}{2\pi\, a(t)}$$

$a(t)$ is the scale factor and t the proper cosmological time.

This is expected for radiation in a $(d+1)$–dimensional cosmology From Boltzmann law and the radiation equation of state $\rho = d\,p$

$$\rho \sim T^{d+1}, \qquad \rho \sim 1/a^{d+1}(t), \longrightarrow T \sim \frac{c}{2\pi\, a(t)}$$

Conclusion

• The presence of a localized cosmological constant on a time-like lower dimensional hypersurface avoids the conventional space-like singularity

• The localized energy density arises naturally in string theory as branes with negative tension like orientifolds.

• The resulting non-trivial cosmological background has an horizon and broken supersymmetry.

• Space-time is divided in three regions:

$* * *$ A collapsing region with a future cosmological horizon.
$* * *$ A 'static' intermediate region were the orientifolds are situated.
$* * *$ An expanding phase.

• The interpretation of the singularity is justified by probing the geometry with D–branes.

• The energy density at the core of the geometry prevents the orientifolds to collapse due to the gravitational repulsion.

• The orientifold provides well defined boundary conditions, necessary for the determination of past→future transition amplitudes.

• Assuming the trivial vacuum in the past, we derived a thermal spectrum in the far future with temperature

$$T = \frac{\mathcal{E}}{2\pi\, a(t)}$$