Cones & Cosmological Singularities

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based on: BK 02 06 220
BK & Wiseman
BK & H. NICOLAI – IN PROGRESS

OUTLINE

- BACKGROUND
- IDEA IN BRIEF
- BKL
- CONES

GIVEN IN WONDERFUL CRETE

- GR carries the seeds to its own destruction : SINGULARITIES

- Especially interesting : Cosmological Big Bang/Crunch
  Inside black holes

- Recent interest in String Theory (non-representative list! see Elitzur's talk)

  Maldacena AdS-Schw
  Liu-Moore-Seiberg
  Jerusalem group Elitzur, Giveon Rabinovici
  Stanford (very recent)

HERE: VENEZIANO, BANKS

AND MANY MANY OTHERS ...

People say: "DESPITE ALL THIS WORK IT IS FAIR TO SAY THAT WE DO NOT COMPLETELY UNDERSTAND..."

Actually I'd say

WE KNOW CLOSE TO NOTHING..

WHAT DO WE KNOW?

Belinskii Khalatnikov Lifshitz

Classical GR approach to (space-like) singularity

Actually most of the problems found is String Theory computations originate (in hindsight) in Classical GR.

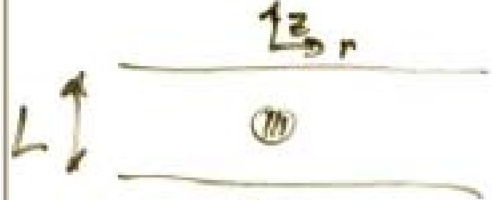
Much recent activity summarized in "Cosmological Billiards" - Damour Henneaux Nicolai hep-th/.....

A CENTRAL RESULT: EXTENDED THEORIES OF GRAVITY CAN BE DIVIDED INTO 2 "BKL TYPES" or critical dimension (I'll review shortly).

BLACK HOLE BLACK STRING TRANSITION

– Original <u>motivation</u>

2 phases in presence of compact dimension



BH

Black String

- We know a lot about 4d class. GR. Surprisingly new qualitative issues arise in $d > 4$:
- Phase TRANSITION
- GENUINE TOPOLOGY CHANGE
- NON-UNIQUENESS
- EXPLOSION / THUNDERBOLT $E \approx 10^{38}\, J/\mu \sim 10^{28}\, J/(eV)^{-1}$
- CONTROVERSY W. HOROWITZ-MAEDA
- NUMERICAL RESULTS BY WISEMAN; CHOPTUIK, LEHNER ET. AL.
- Importance of cones ...
- Likely to violate Cosmic Censorship.

CONES exhibit a CRITICAL DIMENSION for stability

BK 0206220

Related: Klebanov, Rangamani-Wills, C. Böhm

Relation to BKL? Gibbons.

Maybe NEW CRITERION for BKL types.

IDEA IN BRIEF

	BKL	CONE
PHENOM.	CHAOS $d \leq 10$ (AND STR. SUGRA: IIA, IIB, hd) (osc. soln)	tachyon $d < 10$
	KASNER STEADY STATE $d \geq 11$	stable $d > 10$
MECHANISM	• finite/infinite Billiard table • Hyperbolic/non algebras U duality West	• Effectively particle in 1d: $V(x) = -\frac{c}{x^2}$, $-\partial_x^2 + V(x) = H$ $c = \frac{2}{d-2}$ $c_{crit.} = \frac{1}{4}$

STRING TH. INTERPRETATION?

WHY CHAOTIC?

BKL (exposition)

Schw $$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 d\Omega^2$$

$$f = 1 - 1/r$$

near $r \to 0$ $\quad f \sim -1/r$

new coor. $\quad d\rho \sim \sqrt{r}\, dr$

$$ds^2 = -d\rho^2 + \rho^{-2/3} dt^2 + \rho^{4/3} d\Omega^2$$

COMPARE W. KASNER

$$ds^2 = -dt^2 + \sum_i t^{2p_i} dx_i^2$$

(in Schw $p_i = -\frac{1}{3}, \frac{2}{3}, \frac{2}{3}$)

soln iff:
$$\begin{cases} \sum p_i^2 = 1 \\ \sum p_i = 1 \end{cases}$$ KASNER SPHERE

$$\exists p_i > 0, \ \exists p_j < 0$$

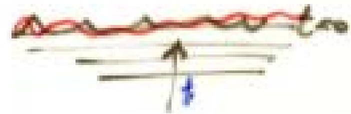
Is the singularity generic? (late 60's)

BKL - no (at first)

Penrose - yes

BKL REDEEMED... - We have a detailed description of approach to singularity.

THE PROCEDURE:



$$ds^2 = -dt^2 + ds^2_x(x,t)$$

$\downarrow$ $\partial_x \ll \partial_t$ assump. (ultra-locality)

$$ds^2 = -dt^2 + ds^2_x(t)$$

$$ds^2_x = N^T \cdot \exp(\mathrm{diag}(\beta_1 \ldots \beta_{D-1})) \cdot N$$

N - upper-diag $= \mathbb{1} + \begin{bmatrix} 0 & * \\ & 0 \end{bmatrix}$ IWASAWA decomposition GRAHAM-SCHMIDT like

$I \sim \int dt \sqrt{g}\,(\partial\beta/dt)^2_{t\to\infty}$: canonical Kinetic $\Rightarrow$

Define $d\tau = dt/\sqrt{g}$

$$I \sim \int d\tau\, \dot{\beta}^2 + \ldots$$

$$\mathcal{L} = \sum_{i=1}^{D-1} \dot{\beta}_i^2 - \Big(\sum_{i=1}^{D-1} \dot{\beta}_i\Big)^2 - \mathcal{V}_S - \mathcal{V}_G$$

$$\mathcal{V}_G = -g R_x$$

$$R \sim -\frac{1}{4}\sum_{abc} e^{2(\beta_a - \beta_b - \beta_c)} \left(C^a{}_{bc}\right)^2 + e^{-2\beta_a} F_a(\partial^2\beta, \partial\beta, \partial c, c)$$

$d\Theta^a = -\frac{1}{2} C^a{}_{bc}\, \Theta^b \wedge \Theta^c$ (defn of $C^a{}_{bc}$ - structure constants.)

BKL limit $\quad |\beta| \to \infty$

$\exp(w_i \beta^i) \to \Theta(w_i \beta^i)$

$$\Theta(x) = \begin{cases} 0 & x < 0 \\ \infty & x > 0 \end{cases}$$ INFINITE WALL POTENTIAL

BILLIARD IN β-SPACE

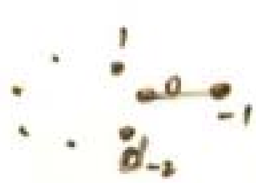
projectivized:

free motion (KASNER epoch) $\quad$ chaotic $\qquad$ non-chaotic

$\beta \sim \tau$, $\tau \sim \log t$

- U-duality, group theory: wall reflections are Weyl reflections.

e.g. $AE_d = A_{d-2}^{\wedge\wedge}$
for pure gravity in d dimensions.



For concreteness:

In 4d, explicit BKL eq's:

$$\begin{cases} 2\,\alpha_{\tau\tau} = (\mu b^2 - \nu c^2)^2 - (\lambda a^2)^2 \\ 2\,\beta_{\tau\tau} = (\lambda a^2 - \nu c^2)^2 - (\mu b^2)^2 \\ 2\,\gamma_{\tau\tau} = (\lambda a^2 - \mu b^2)^2 - (\nu c^2)^2 \end{cases}$$

λ, μ, ν const

$$\begin{cases} a = e^{\alpha} \\ b = e^{\beta} \\ c = e^{\gamma} \end{cases}$$

+ constraint

$$\frac{1}{2}(\alpha+\beta+\gamma)_{\tau\tau} = \alpha_\tau \beta_\tau + \alpha_\tau \gamma_\tau + \beta_\tau \gamma_\tau$$

& substituting the second derivatives from eq. of motion.

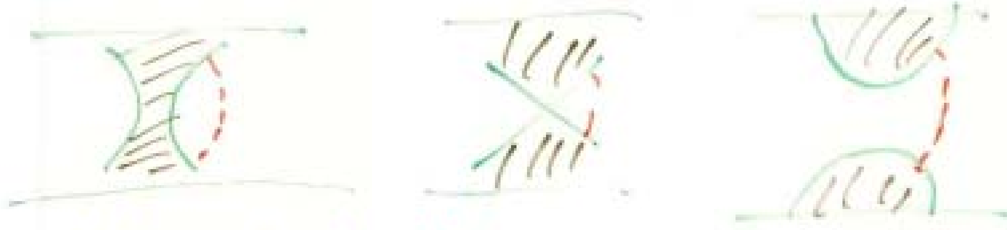
Criticality:

It so happens that pure gravity in $d \le 10$ has a finite Billiard table (equiv. AE_d is hyperbolic), and vice versa for $d \ge 11$.

More explicitly: On KASNER sphere satisfy $1 + p_a - p_b - p_c > 0$ $\forall a, b, c$ all different (see expr. for R_r)

The solution set is non-empty iff $d \ge 11$.

CONES



Euclid. topology of ---- : $S^2 \times S^m$

CONIFOLD-LIKE TRANSITION

The cone soln:

$$ds^2 = d\rho^2 + \frac{n-1}{D-2}\rho^2 d\Omega^2_{S^m} + \frac{n-1}{D-2}\cdot\rho^2\, d\Omega^2_{S^n}$$

Dyn. system:

$$ds^2 = d\rho^2 + e^{2a(\rho)}\, d\Omega^2_{S^m} + e^{2b(\rho)}\, d\Omega^2_{S^n}$$

Linearize
$$\left.\begin{aligned} a &= a_0 + \varepsilon/m \\ b &= b_0 - \varepsilon/n \end{aligned}\right\}$$ the constraint forces a relation

Expand I (action)

$$I \sim \int \rho^{D-1} d\rho \left[\frac{2(D-2)}{\rho^2}\varepsilon^2 - \varepsilon'^2\right] + O(\varepsilon^3)$$

Canonical $\rho \to \hat{\rho} = \hat{\rho}(\rho)$

$$I \sim \int d\hat{\rho}\left[\frac{2}{d-2}\frac{1}{\hat{\rho}^2}\varepsilon^2 - (d\varepsilon/d\hat{\rho})^2\right]$$

$$\Rightarrow V_{eff} = -\frac{2}{d-2}\, 1/\hat{\rho}^2$$

New criterion for BKL type?

- BKL helps analyze cone equations. $d\tau$ $\beta_1, \beta_2 \to \beta_i$ generalization
- BKL eq. have cone soln, but imaginary

$$\beta \sim \log t \sim \log \tau$$

$\tau \sim t^{a}$

· OUTSIDE BKL limit

* different from the $\beta \sim \tau$ free KASNER motion

- CAN ONE GENERALIZE THE CRITERION OF

CONE STABILITY

TO REPLACE THE ANALYSIS OF BILLIARD TABLE?