HOLONOMY AND SYMMETRY IN M-THEORY

CMU hep-th/0305039

DUFF + LIU 0303140

- WHAT ARE SYMMETRIES OF M-THEORY?
- CLASSIFICATION OF SUSY VACUA?

CAN ASK FOR $D=11$ SUGRA OR M-THEORY

HOW MANY SUPERSYMMETRIES CAN A SOLUTION PRESERVE?

SOLUTIONS KNOWN WITH

0, 1, 2, 3, 4, 5, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 32 SUSYS.

OTHERS POSSIBLE?

HOLONOMY OF SUPERCOVARIANT DERIVATIVE PLAYS KEY ROLE.

FORMULATION IN WHICH HOLONOMY GROUP $\rightsquigarrow$ LOCAL "TANGENT SPACE" SYMMETRY?

cf: RIEMANNIAN HOLONOMY $SO(10,1)$

LOCAL TANGENT SPACE SYMMETRY $SO(10,1)$

LOCAL SYMMETRY

DIMENSIONAL REDUCTION OF $D=11$ SUGRA ON $T^7 \longrightarrow D=4$ $N=8$ SUGRA

SYMMETRY $\quad \underset{\text{GLOBAL}}{E_7} \times \underset{\text{LOCAL}}{SU(8) \times SO(3,1)}$

SCALARS IN $E_7 / SU(8)$

HOLONOMY FOR $\mathcal{M}_4 \times \mathcal{M}_7$

RIEMANNIAN HOL $\subset SO(3,1) \times SO(7)$

GENERALISED HOL $(\tilde{D}) \subset SO(3,1) \times SU(8)$

REFORMULATION OF $D=11$ SUGRA FOR 4/7 SPLIT

LOCAL $SO(10,1)$ REPLACED BY LOCAL $SO(3,1) \times SU(8)$ [de WIT, NICOLAI]

- $SU(8)$ A LOCAL SYMMETRY OF M-THEORY?
- DIFFERENT GROUPS FOR $d/11-d$ SPLIT
  BACKGROUND INDEPENDENT PICTURE?
- $SU(8)$ SYMMETRY: CONVENIENT RE-WRITING, OR ESSENTIAL FEATURE?

BACKGROUND INDEPENDENCE OF LOCAL

SYMMETRY GROUP IS INDEPENDENT OF BACKGROUND.

$\mathcal{G}$ CONTAINS $SU(8)$, $SO(16)$, $SO(32)$, $SO(16,16)$ ARISING FOR SPECIAL CHOICES, AND $\mathcal{G} \subseteq GL(32,\mathbb{R})$

$$\rightarrow \quad \mathcal{G} = SL(32,\mathbb{R})$$

THEORY CAN BE WRITTEN WITH LOCAL $\mathcal{G}$ IF NEW DEGREES OF FREEDOM INTRODUCED.

CF GRAVITY $\quad g_{MN} \quad \frac{1}{2}d(d+1)$

VIELBEIN $\quad e_M{}^A \quad d^2 = \frac{1}{2}d(d+1) + \underbrace{\frac{1}{2}d(d-1)}_{\text{PURE GAUGE}}$

LOCAL $SO(d-1,1)$ SYMMETRY

EXTRA $\frac{1}{2}d(d-1)$ d.o.f. CAN BE GAUGED AWAY FIXING LOCAL LORENTZ SYMMETRY.

PURE GRAVITY $\quad$ EXTRA SYMMETRY, d.o.f. REDUNDANT, INESSENTIAL

GRAVITY + FERMIONS $\quad$ VIELBEIN FORM ESSENTIAL FOR COUPLING TO SPINORS

D=11 SUGRA $\quad SL(32,\mathbb{R})$ $\quad$ INESSENTIAL

M-THEORY $\quad SL(32,\mathbb{R})$ $\quad$ ESSENTIAL !

$D = 11$ SUPERGRAVITY

$$e_M{}^A, \quad \Psi_M, \quad A_{MNP}, \quad F_{(4)} = d\,A_{(3)}$$

SUSY: $\delta \Psi_M = \tilde{D}_M \epsilon$

$$\tilde{D}_M \epsilon = D_M \epsilon - \frac{1}{288}\left(\Gamma_M{}^{NPQR} - 8\delta_M^N \Gamma^{PQR}\right) F_{NPQR}\,\epsilon$$

IN BOSONIC $\Psi = 0$ BACKGROUND.

IN $\Psi = 0$ BACKGROUND NO. OF SUSYS PRESERVED IS NUMBER OF KILLING SPINORS: ν

$$\tilde{D}_M \epsilon = 0$$

MAJORANA SPINORS: 32 REAL COMPONENTS

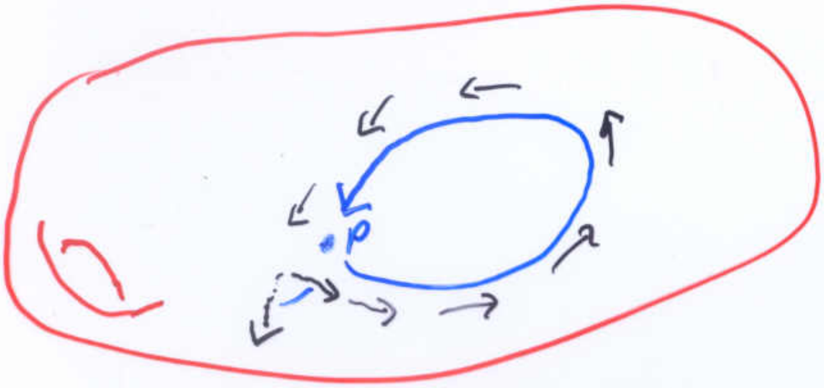
$$0 \leq \nu \leq 32$$

$(-++\ldots+)$, Γ_M REAL

$$\Gamma_{MN\ldots P} = \Gamma_{[M}\Gamma_N \ldots \Gamma_{P]}$$

$\tilde{D}_M$: CONNECTION ON SPIN-BUNDLE

HOLONOMY



PARALLEL TRANSPORT
ROUND CLOSED
CURVE C $x(t)$
$\dot{x}^M \tilde{D}_M \epsilon = 0$

GIVES LIN TRANS $\epsilon^A \to L^A{}_B \epsilon^B$

$\{ L^A{}_B(P, C) \}$ FOR ALL CURVES $\to$
HOLONOMY GROUP $\mathcal{H}(\tilde{D})$

RIEMANNIAN HOLONOMY
$$\mathcal{H}(D) \subseteq Spin(10,1)$$

GENERALISED HOLONOMY
$$\mathcal{H}(\tilde{D}) \subseteq GL(32, \mathbb{R})$$

KILLING SPINORS REQUIRE SPECIAL HOLONOMY

e.g. IN CASE $F = 0$, $\mathcal{H}(\tilde{D}) = \mathcal{H}(D)$
$\mathcal{H}(D) \subseteq Spin(7), G_2, SU(2), SU(3), SU(4), SU(5), \ldots$

STRUCTURE GROUP $\mathcal{G}$ FOR A CLASS OF CONFIGURATIONS: MAXIMAL HOLONOMY

DUFF & LIU: CONFIGS W. $d/11-d$ SPLIT

$$\mathcal{G}(D) \subset \begin{cases} SO(d-1,1) \times SO(11-d) \\ ISO(d-1) \times ISO(10-d) \end{cases} \quad \text{OR}$$

$$\rightarrow \mathcal{G}(\tilde{D}) \subset \begin{cases} SO(d-1,1) \times G_s(11-d) & \text{SPACELIKE} \\ SO(d) \times G_t(11-d) & \text{TIMELIKE} \\ ISO(d-1) \times G_n(11-d) & \text{NULL} \end{cases}$$

n	$G_s(n)$	$G_t(n)$	$G_n(n)$
1	1	1	1
2	$SO(2)$	$SO(1,1)$	R
3	$SO(3)\times SO(2)$	$SO(2,1)\times SO(1,1)$	$ISO(2)\times R$
4	$SO(5)$	$SO(3,2)$	$(SO(3)\times SO(2)) \ltimes R^6$
5	$SO(5)\times SO(5)$	$SO(5,\mathbb{C})$	$SO(5) \ltimes R^{10}$
6	$USp(8)$	$USp(4,4)$	$SO(5)\times SO(5) \ltimes R^6$
7	$SU(8)$	$SU^*(8)$	$USp(8) \ltimes R^{27}$
8	$SO(16)$	$SO^*(16)$	$(SU(8)\times U(1)) \ltimes R^{56}$
9	$SO(16)\times SO(16)$	$SO(16,\mathbb{C})$	$SO(16) \ltimes R^{120}$
10	$SO(32)$	$SO(16,16)$	$[SO(16)\times SO(16)] \ltimes R^{256}$

SAME AS LOCAL SYMM GROUPS FOR TOROIDAL RED^N:

SPACELIKE RED^N ON T^n : $E_n / G_s(n)$

TIMELIKE ON $T^{n-1,1}$: $E_n / G_t(n)$ $\quad (n \leq 8)$

NULL : $T^{n-1,0}$ $\quad E_n / G_n(n)$

DUFF-LIU ANSATZ

$d/11-d$

$x^\mu \quad y^i$

$$g^{(11)}_{MN} = \begin{pmatrix} \Delta\, g_{\mu\nu} & 0 \\ 0 & g_{ij} \end{pmatrix} \qquad A^{11}_{ijk} = \phi_{ijk}$$

$$\psi_\mu \propto \left(\Psi^{(11)}_\mu - \frac{1}{d-2}\gamma_\mu \Gamma^i \Psi^{(11)}_i\right), \quad \lambda_i \propto \Psi^{(11)}_i$$

WARPED PRODUCT, FIELDS INDEP OF y^i

$$\delta\psi_\mu = \hat{D}_\mu \epsilon, \qquad \delta\lambda_i = X_i\,\epsilon$$

KILLING SPINORS:

$\hat{D}\epsilon = 0 \iff \mathcal{H}(\hat{D})$ "SPECIAL"

AND $\quad X_i\,\epsilon = 0$

$\mathcal{H}(\hat{D})$ MUCH EASIER TO ANALYSE

$$\mathcal{H}(\hat{D}) \subseteq Spin(d-1,1) \times G_s(11-d)$$

$\rightarrow$ DUFF-LIU STRUCTURE GROUPS

$$\mathcal{H}(\hat{D}) \subseteq \mathcal{H}(\tilde{D})$$

MANY EXAMPLES WITH $\mathcal{H}(\tilde{D}) \neq \mathcal{H}(\hat{D})$

STRUCTURE GROUP FOR GENERAL BACKGROUNDS

$\mathcal{G} \subseteq GL(32,\mathbb{R})$ AND $\mathcal{G}$ CONTAINS $SO(32)$, $SO(16,16)$

$$\tilde{D} = \partial + \omega.\Gamma_2 + F(\Gamma_3 + \Gamma_5)$$

$\mathcal{G}$ GENERATED BY $\Gamma_2, \Gamma_3, \Gamma_5$

$\Rightarrow \mathcal{G} = SL(32,\mathbb{R})$ GENS $\{\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5\}$

NOT $GL(32)$ AS $1 = \Gamma_{11}$ DOESN'T OCCUR ON RHS OF COMMUTATORS

$$\Rightarrow \mathcal{H}(\tilde{D}) \subseteq SL(32,\mathbb{R})$$

ϵ : $\underset{\sim}{32}$ OF $SL(32,\mathbb{R})$

ν KILLING SPINORS : ν SINGLETS

$$\Rightarrow \mathcal{H}(\hat{D}) \subseteq SL(32-\nu,\mathbb{R}) \ltimes \mathbb{R}^{\nu(32-\nu)}$$

$$\begin{array}{c} \updownarrow \nu \\ \updownarrow 32-\nu \end{array} \left(\begin{array}{c|c} \mathbb{1}_\nu & B \\ \hline 0 & A \end{array}\right) \begin{pmatrix} \epsilon_1 \\ \vdots \\ \epsilon_\nu \\ \hline 0 \\ \vdots \\ 0 \end{pmatrix} = \begin{pmatrix} \epsilon \\ \hline 0 \end{pmatrix} \qquad \begin{array}{l} A \in SL(32-\nu) \\ B \in \mathbb{R}^{\nu(32-\nu)} \end{array}$$

31 KILLING SPINORS : $\mathcal{H} \subseteq \mathbb{R}^{31}$

WHICH SUBGROUPS $\mathcal{H} \subseteq SL(32,\mathbb{R})$

ACTUALLY ARISE AS HOLONOMIES ?

TOPOLOGICAL OBSTRUCTIONS TO EXISTENCE OF KILLING SPINORS

PAPADOPOULOS + TSIMPIS

- NO OBSTRUCTION FOR $\nu < 22$ KILLING SPINORS
- THERE IS AN OBSTRUCTION TO THE EXISTENCE OF $\nu = 22$ $(= 32 - 11 + 1)$ KILLING SPINORS

FOR $\nu \geq 22$, EULER CLASS OF A CERTAIN BUNDLE HAS TO VANISH

LOCAL SYMMETRY.

SUPERGRAVITY: SCALARS IN G/H.

e.g. IIB $SL(2,R)/U(1)$

dim(G/H) PHYSICAL SCALARS

2 REAL: $\phi = \phi_1 + i\phi_2$

G ACTS NON-LINEARLY
RIGID SYMMETRY

$$\phi \to \frac{a\phi + b}{c\phi + d}$$

CAN INTRODUCE EXTRA SCALARS
$\chi \in H$, SO COMPLETE SET

1 EXTRA REAL
χ,
3 SCALARS

TAKES VALUES IN G.
REPRESENT BY $\mathcal{V} \in G$

e.g. $\mathcal{V} = \exp\begin{pmatrix} \chi & \phi \\ \bar{\phi} & -\chi \end{pmatrix}$

WITH LOCAL H SYMMETRY

$$\mathcal{V}(x) \longrightarrow h(x)\, \mathcal{V}(x)\, g^{-1}$$

G NOW ACTS LINEARLY ON $\mathcal{V}$.

CAN FIX H GAUGE BY SETTING $\chi = 0$.

LINEAR G TRANSFORMATION ACCOMPANIED BY COMPENSATING H TRANS TO MAINTAIN GAUGE.

FOR $G/H = E_n / G_s(n)$, CAN USE

EITHER PHYSICAL GAUGE $\chi = 0$, OR
INTRODUCE χ AND LOCAL H SYMMETRY.

THUS LOCAL H "INESSENTIAL" IN SUGRA

REFORMULATIONS OF $D=11$ SUGRA WITH LOCAL $G(n)$

USE AN $n/11-n$ SPLIT

PARTIALLY FIX LOCAL LORENTZ

$$SO(10,1) \longrightarrow SO(10-n,1) \times SO(n)$$

INTRODUCE EXTRA D.O.F. AND ENLARGE LOCAL SYMMETRY $SO(n) \longrightarrow G(n)$

FOR CLASSICAL SUGRA, AN INESSENTIAL BUT SUGGESTIVE RE-WRITING.

4/7 de WIT & NICOLAI

3/8 NICOLAI

5/6, 6/5 DRABANT, TOX, NICOLAI

WEST REFORMULATIONS

D=11 SUGRA IN SUPERSPACE:

SUPERDIFFEO $(11,32)$

CONTAINS $GL(11/32)$, $OSp(1/64)$

REFORMULATE AS NON-LINEAR REALIZATION (FOLLOWING OGIEVETSKII)

GLOBAL $GL(11/32)$, $OSp(1/64)$

LOCAL $SO(10,1)$

GLOBAL SYMMETRIES "MANIFEST"

$GL(32,\mathbb{R}) \supset SL(32,R)$

$Sp(64,R) \supset GL(32,R) \supset SL(32,R)$

AGAIN, "INESSENTIAL", BUT SUGGESTIVE REWRITING

$GL(32,\mathbb{R})$: AUTOMORPHISM OF SUSY ALGEBRA

Q_α : $\underset{\sim}{32}$

M-THEORY vs SUGRA

M-THEORY HAS SUSY VACUA CORRESPONDING TO SUGRA SOLUTIONS WITH NO KILLING SPINORS!

"SUSY WITHOUT SUSY" [DUFF, LU, POPE]

e.g. $AdS_5 \times S^5$ IN IIB, 32 SUSY'S

S^5: HOPF FIBRATION, S^1 BUNDLE OVER $\mathbb{C}P^2$

T-DUALISE:

$$S^5 \longrightarrow CP^2 \times S^1 , \quad B_2 \neq 0$$

IIA SUGRA SOLUTION

$$\mathcal{M} = AdS_5 \times CP^2 \times S^1$$

BUT CP^2 HAS NO SPIN STRUCTURE

CAN'T DEFINE SPINORS ON CP^2 OR $\mathcal{M}$

NO KILLING SPINORS, NO SPINORS AT ALL!

T-DUALITY SHD GIVE EQUIVALENT DESCRIPTIONS OF SAME CONFIGURATION

WHERE HAVE IIB FERMIONS GONE?

EXPECT SAME SPECTRUM IN AdS_5 FROM IIA,B

IIB: FERMIONS ALL HAVE NON-TRIVIAL DEPENDENCE ON S^1, ALL CARRY MOMENTUM ON S^1

$\Rightarrow$ FERMIONS IN IIA ALL HAVE WINDING NUMBER, SO ARE WINDING MODES

HOPF REDUCTION ON S^1

$D=9$ SUGRA $\quad AdS_5 \times \mathbb{C}P^2$

$F = dA \propto$ KAHLER FORM $\quad A = $ KK VECTOR

REDUCE IIB FERMIONS:—

$D=9$ FERMIONS ALL CHARGED W.R.T. A

NOT SPINORS - i.e. NOT SECTIONS OF $Spin(8,1)$ BUNDLE, AS THERE ARE NONE

SECTIONS OF $Spin(8,1) \times_{\mathbb{Z}_2} U(1)$ BUNDLE

$Spin^c$ STRUCTURE ON $\mathbb{C}P^2$ $(\times AdS_5)$

NO SPINORS COUPLING TO ω, BUT "CHARGED SPINORS" COUPLING TO $\omega + A$

$U(1)$ CHARGE HALF-INTEGRAL

32 "CHARGED KILLING SPINORS"

$$\begin{tikzcd}
& \text{M} \quad AdS_5\times CP^2\times T^2 \\
\text{IIB} \quad AdS_5\times S^5 \arrow[r, leftrightarrow, "T"] \arrow[dr, "\text{HOPF REDUCTION ON } S^1 \text{ FIBRE}"'] & \text{IIA} \quad AdS_5\times \mathbb{C}P^2\times S^1 \arrow[u, "\text{LIFT}"'] \\
& D=9,\ AdS_5\times \mathbb{C}P^2 \arrow[u, "\text{LIFT}"']
\end{tikzcd}$$

LIFT D=9 TO IIA.

$A \rightarrow B_2$ OF IIA

FERMIONS COUPLING TO A $\longrightarrow$ FERMION WINDING MODES COUPLING TO B_2

HALF-INTEGRAL CHARGES $\longrightarrow$ ½ INTEGRAL "WINDING"

FRACTIONAL STRINGS

32 KILLING "WINDING SPINORS"

NO ORDINARY KILLING SPINORS

LIFT TO D=11

FERMIONIC "MEMBRANES" WRAPPING T^2, COUPLING TO A_3, FRACTIONAL MEMBRANE CHARGE

32 "KILLING WRAPPING SPINORS"

DUALITIES: MEMBRANES $\longrightarrow$ OTHER BRANES

$D=9$, $AdS_5 \times \mathbb{C}P^2$

FERMIONS SECTIONS OF $Spin(8,1) \times U(1)$ BUNDLE.

BUT THIS IS STRUCTURE GROUP $\mathcal{G} = Spin(8,1) \times G_S(2)$ FOR 9/2 SPLIT. THEORY SHOULD BE FORMULATED WITH LOCAL $\mathcal{G}$ SYMMETRY FOR THIS BACKGROUND.

DIMENSIONAL REDUCTION TO d IN GENERAL

$\rightarrow$ FERMIONS ARE CHARGED SPINORS TRANSFORMING UNDER $\mathcal{G} = Spin(d-1,1) \times G_S(11-d)$, SECTIONS OF $\mathcal{G}$ - BUNDLE, BUT

...SPINORS

...OCAL $\mathcal{G}$ SYMMETRY FOR ... $\Rightarrow$ NEED ...

...TION. $Spin^G$ STRUCTURE $Spin(n,1) \times_{\mathbb{Z}_2} G$ ... FORMULA...

INDEPENDENCE REQUIRES LOCAL ... BACKGROUND

CONTAINING ALL $\mathcal{G}$ $\Rightarrow$ ... SYMMETRY

...$L(32,\mathbb{R})$ SYMMETRY ... LOCAL ...

CONCLUSIONS

- CAN CLASSIFY SUSY BACKGROUNDS ACCORDING TO $\mathcal{H}(\tilde{D}) \subseteq SL(32,\mathbb{R})$
- NEED "BERGER": WHICH SUBGROUPS OF $SL(32,\mathbb{R})$ CAN BE HOLONOMIES?
- $\tilde{D} = \hat{D} + Y$, $\mathcal{H}(\hat{D}) \subseteq \mathcal{H}(\tilde{D})$ CAN BE USEFUL AS IN DUFF-LIU
- FOR A $d/11-d$ SPLIT, $\mathcal{H} \subseteq \mathcal{G}$ AND $d=11$ SUGRA CAN BE REWRITTEN WITH LOCAL $\mathcal{G}$ (AT LEAST IN SOME CASES) PERHAPS WITH LOCAL $SL(32,\mathbb{R})$
- M-THEORY: FERMIONS NOT SPINORS BUT CHARGED SPINORS IN GENERAL SECTIONS OF $Spin^{G}(n,1) = Spin(n,1) \times_{\mathbb{Z}_2} G = \mathcal{G}$ BUNDLE

  $\Rightarrow$ NEED FORMULATION WITH LOCAL $\mathcal{G}$, AND FOR BACKGROUND INDEPENDENCE, WITH LOCAL $SL(32,\mathbb{R})$