N=1 Open String Boundary Couplings and D-brane Worldvolume Covariance

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Problem:

Dp-brane worldvolume action:

$$S(A_\mu, \phi^i, G_{MN}, B_{MN}) = \text{DBI} + \text{CS}$$

$\phi^i = 0$: $\sim \int \sqrt{\det(G+B+F)} + \int C \wedge e^{B+F} \quad \cdots$

(covariant, geometric)

$\phi^i \neq 0$, non-Abelian:

(Myers, Garousi, Douglas, Taylor, Raamsdonk
okawa, Ooguri, ...)

A { * similar to a static gauge expression, but:
* Cannot be obtained from a GCT invariant action.

B { * non-geometric structure.

A: Resolved by studying open string boundary couplings.

B: (SFH, Minasian, hep-th/0008149)

Solution (qualitative):

* the correct worldvolume theory is invariant under GCT & some kind of non-Abelian GCT

(NGCT : not known ; Douglas 97, Jan de Boer, Schalm, hepth/0108161 Raamsdonk, hepth/0305145)

* appropriate NGCT gauge ⇒ known non-covariant theory obtained by T-duality (Myers).

* covariance can be restored only by NGCT, not GCT.

* consistency : in abelian theory, NGCT = GCT ⇒ covariance maintained.

* correct NGCT found by studying open string boundary couplings (N=1, non Abelian)

⇒ covariant non-Abelian worldvolume action

what is the full NGCT?
NGCT invariant action? | open

N=1 Open String Boundary Conditions

Action:

$$S = \frac{1}{2}\int_{\Sigma} d^2\sigma \int d^2\theta\, E_{MN}(\mathbb{X})\, D_+\mathbb{X}^M D_-\mathbb{X}^N + \frac{i}{4}\int_{\partial\Sigma} d\tau\, B_{MN}(\psi_+^M\psi_+^N + \psi_-^M\psi_-^N)$$

$$(E_{MN} = G_{MN} + B_{MN})$$

Boundary Vectors: $(\mathcal{N}^{(X)}, \mathcal{N}^{(\psi)}, \mathcal{D}_{(X)}, \mathcal{D}_{(\psi)})$

$$\mathcal{N}_L^{(X)} = E_{LM}\partial_- X^M - E_{LM}^T \partial_+ X^M - i\psi_-^M \partial_M E_{LN}\psi_-^N + i\psi_+^M \partial_M E_{LN}^T \psi_+^N - i\eta\, \psi_+^M \partial_L E_{MN}\psi_-^N \Big|_{\partial\Sigma}$$

$$\mathcal{N}_L^{(\psi)} = E_{LN}\psi_-^N - \eta E_{LN}^T \psi_+^N \Big|_{\partial\Sigma}$$

$$\mathcal{D}_{(X)}^L = \partial_\tau X^L \Big|_{\partial\Sigma} \qquad \left(\mathcal{N}_L^{(X)} \underset{\tau\leftrightarrow\sigma}{\not\longleftrightarrow} P_L\right)$$

$$\mathcal{D}_{(\psi)}^L = \psi_-^L + \eta\,\psi_+^L \Big|_{\partial\Sigma}$$

Supersymmetry:

$$\delta_{susy}\mathcal{N}_M^{(\psi)} = -i\bar\epsilon\,\mathcal{N}_M^{(X)} \qquad \delta_{susy}\mathcal{N}_M^{(X)} = -2\bar\epsilon\,\partial_\tau \mathcal{N}_M^{(\psi)}$$

$$\delta_{susy}\mathcal{D}_{(\psi)}^M = -2i\bar\epsilon\,\mathcal{D}_{(X)}^M \qquad \delta_{susy}\mathcal{D}_{(X)}^M = -\bar\epsilon\,\partial_\tau \mathcal{D}_{(\psi)}^M$$

$$\left\{ \text{Flat space:} \quad \begin{array}{ll} \mathcal{N}^{(X)} \to -2\partial_\sigma X, & \mathcal{N}^{(\psi)} \to \psi_- - \eta\psi_+ \\ \mathcal{D}_{(X)} \to \partial_\tau X, & \mathcal{D}^{(\psi)} \to \psi_- + \eta\psi_+ \end{array} \right\}$$

D-brane Embedding:

$$X^M(\zeta) \quad , \quad \partial_\alpha X^M \quad , \quad a^M_{\hat{a}}$$

$$\left(V_M : \quad V_\alpha = \partial_\alpha X^M V_M \ , \ V_{\hat{a}} = a^M_{\hat{a}} V_M \right)$$

$$\left(a^M_{\hat{a}} G_{MN} \partial_\alpha X^N = 0 \right)$$

Boundary Conditions:

Neumann : $\quad \partial_\alpha X^M \mathcal{N}^{(X)}_M = 0 \ , \ \partial_\alpha X^M \mathcal{N}^{(\psi)}_M = 0$

Dirichlet : $\quad a^{\hat{a}}_M \mathcal{D}^M_{(X)} = 0 \ , \ a^{\hat{a}}_M \mathcal{D}^M_{(\psi)} = 0$

$$\boxed{a^M_{\hat{a}} \mathcal{N}^{(X,\psi)}_M \neq 0 \ , \quad \partial_\alpha X^M G_{MN} \mathcal{D}^N_{(X,\psi)} \neq 0}$$

Worldvolume fields couple to the non-vanishing part of boundary vectors.

Boundary Couplings:

$$\sim \int_{\partial\Sigma} d\tau \left(A_\mu \partial_\tau X^\mu + \phi_i \partial_\sigma X^i \right)$$

$$\left(N: \; \partial_\sigma X^M \Big|_{\partial\Sigma} = 0 , \quad D: \; \partial_\tau X^i \Big|_{\partial\Sigma} = 0 \right)$$

Covariant description:

Intrinsic worldvolume fields: $\mathbb{A}_\alpha$, $\Phi^{\hat{a}}$

Space-time extensions: $A^M = \partial_\alpha X^M \mathbb{A}^\alpha$

$$\phi^M = a^M_{\hat{a}} \, \Phi^{\hat{a}}$$

$$\left(\text{Compare: } V^M = \partial_\alpha X^M \mathbb{V}^\alpha + a^M_{\hat{a}} \mathbb{V}^{\hat{a}} \right)$$

Bosonic Open String Boundary Coupling:

(Also from T-duality)

$$S^{DP}_{\partial\Sigma} = \int d\tau \left(A_M \mathcal{D}^M + \phi^M \mathcal{N}_M \right)$$

$$= \int d\tau \left[\underbrace{\left(A_M + \phi^L B_{LM} \right)} \partial_\tau X^M - \phi_M \partial_\sigma X^M \right]$$

$\boxed{\text{$\phi$-terms are generated by the shift } X^M \longrightarrow X^M + \phi^M \text{ in the bulk action } S_\Sigma}$

N=1 Abelian Boundary Couplings

$$S^{Dp}_{\partial\Sigma} = \int_{\partial\Sigma} d\tau \left\{ A_M \mathcal{D}^M_{(x)} - \frac{i}{4}\mathcal{D}^M_{(\psi)} F_{MN} \mathcal{D}^N_{(\psi)} + \frac{1}{2}\phi^M \mathcal{N}^{(x)}_M - \frac{i}{2}\mathcal{D}^M_{(\psi)} \partial_M \phi^N \mathcal{N}^{(\psi)}_N \boxed{- \frac{i}{4}\mathcal{D}^M_{(\psi)} \mathcal{N}^{(\phi)}_M} \right\}$$

$$\boxed{- \frac{i}{4}\mathcal{D}^M_{(\psi)} \mathcal{N}^{(\phi)}_M} = 0$$

(last term : $F \to F + B$)

ϕ-terms follow from the supersymmetric shift of the bulk action :

$$X^M \longrightarrow X^M + \phi^M(\xi)$$

$$\psi^M_\pm \longrightarrow \psi^M_\pm + \psi^\alpha_\pm \partial_\alpha \phi^M(\xi)$$

Covariant form

$$S^{Dp}_{\partial\Sigma} = \int d\tau \Big\{ A^{(\phi)}_M \partial_\tau X^M - \frac{i}{4}\mathcal{D}^M_{(\psi)} F^{(\phi)}_{MN} \mathcal{D}^N_{(\psi)} - \phi_N \partial_\sigma X^N$$

$$+ i\psi^\alpha_+ \hat{\nabla}^+_\alpha \Phi^{\hat{a}} \psi_{+\hat{a}} - i\psi^\alpha_- \hat{\nabla}^-_\alpha \Phi^{\hat{a}} \psi_{-\hat{a}}$$

$$- \frac{i}{2}\eta \phi^M \left(\psi^\alpha_+ \psi^\beta_- H_{M\alpha\beta} + \psi^{\hat{a}}_+ \psi^{\hat{b}}_- H_{M\hat{a}\hat{b}} \right)$$

$$\begin{cases} A^{(\phi)}_M = A_M + \phi^L B_{LM} \\ \hat{\nabla}^\pm_\alpha \Phi^{\hat{a}} : \text{torsionful normal bundle cov. der.} \end{cases}$$

N=1 Non Abelian Boundary Couplings

Boundary action $\longrightarrow$ "Wilson line" $= \mathrm{tr}\, \mathcal{P}\, e^{i S^{Dp}_{\partial\Sigma}}$

$$S^{Dp}_{\partial\Sigma} = \int_{\partial\Sigma} d\tau \Big\{ A_M \mathcal{D}^M_{(X)} - \frac{i}{4} \mathcal{D}^M_{(\psi)} F_{MN} \mathcal{D}^N_{(\psi)} \boxed{\underbrace{- \frac{i}{4} \mathcal{D}^M_{(\psi)} \mathcal{N}^{(\psi)}_M}_{0 \,=}}$$

$$+ \frac{1}{2} \Phi^M \mathcal{N}^{(X)}_M - \frac{i}{2} \mathcal{D}^M_{(\psi)} \left(\partial_M \Phi^L + i [A_M, \Phi^L] \right) \mathcal{N}^{(\psi)}_L$$

$$+ \frac{1}{4} \mathcal{N}^{(\psi)}_M [\Phi^M, \Phi^N] \mathcal{N}^{(\psi)}_N \Big\}$$

Φ-terms still follow from a non-Abelian supersymmmetric generalization of $\delta X^M = X^M + \Phi^M$:

$$\delta \left(\mathcal{D}^M_{(\psi)} \mathcal{N}^{(\psi)}_M \right) \neq 0 \; + \; \delta_{susy} \lambda^a = - i \epsilon\, \mathcal{D}^L_{(\psi)} [A_L, \lambda^a]$$

$$\Rightarrow \boxed{X^M \rightarrow X^M + \Phi^M_a \lambda^a \, , \; \mathcal{D}^M_{(\psi)} \rightarrow \mathcal{D}^M_{(\psi)} + \mathcal{D}^L_{(\psi)} \left(\partial_L \Phi^M + i [A_L, \Phi^M] \right)}$$

$\Rightarrow$ linear Φ-terms

Covariant expression:

contains $\quad \hat{D}^{\pm}_\alpha \Phi^{\hat{a}} = \hat{\nabla}^{\pm}_\alpha \Phi^{\hat{a}} + i \partial_\alpha X^M [A^{(\Phi)}_M, \Phi^{\hat{a}}]$

$$A^{(\Phi)}_M = A_M + \Phi^L B_{LM}$$

$$\delta B_{MN} = \partial_M \Lambda_N - \partial_N \Lambda_M \;\Rightarrow\; \delta A_M = -\Lambda_M - \Phi^L (\partial_L \Lambda_M - \partial_M \Lambda_L)$$

T-duality & Restoration of Covariance

We have found the covariant worldsheet boundary action for Dp-branes.

A $\left\{ \begin{array}{l} \text{Write this in static gauge:} \\ X^\mu = \xi^\mu, \quad X^i = 0 \Rightarrow A_M \partial_\alpha X^M \to A_\mu \\ \qquad\qquad\qquad\qquad \phi_M \to \phi_i \end{array} \right| \begin{array}{l} \phi^i \neq 0 \\ \phi^\mu \neq 0 \end{array} \Bigg\}$

Obtain static gauge Dp-branes by T-duality:

$$\underset{\text{D9}}{\begin{pmatrix} \tilde{G}, \tilde{B}, \tilde{A}_M \\ \tilde{X}^\mu, \tilde{X}^i \end{pmatrix}} \xrightarrow{\text{T-duality}} \underset{\text{Dp}}{\begin{pmatrix} G, B, A_\mu = \tilde{A}_\mu \\ \phi^i = \delta^{ij} \tilde{A}_j \\ \boxed{\text{No } \phi^\mu!} \end{pmatrix}}$$

$\Rightarrow$

B $\left\{ \int d\tau \cdots \left(\frac{1}{2} \phi^i \mathcal{N}_i^{(X)} - \frac{i}{2} \mathcal{D}^\mu_{(\psi)} \left(\partial_\mu \phi^i + i [A_\mu, \phi^i] \right) \mathcal{N}_i^{(\psi)} + \frac{1}{4} \mathcal{N}_i^{(\psi)} [\phi^i, \phi^j] \mathcal{N}_j^{(\psi)} \right) \right.$

$$\text{B} \xrightarrow{\boxed{\phi^i \longrightarrow \phi^M}} \text{A}$$

The missing ϕ^M's can be insertеd using the Neumann b.c.

$$\mathcal{N}_\mu^{(X)} = 0 \quad , \quad \mathcal{N}_\mu^{(\psi)} = 0$$

$$\Rightarrow \boxed{\phi^\mu \mathcal{N}_\mu^{(X)} = 0, \qquad \partial\phi^\mu \mathcal{N}_\mu^{(\psi)} = 0, \text{ etc.}}$$

1) the completions

$$\phi^i \longrightarrow \phi^M$$

$$\partial_\alpha \phi^i + i[A_\alpha, \phi^i] \to \partial_\alpha \phi^M + i[A_\alpha, \phi^M]$$

$$\partial_\alpha \phi^i + \omega_{\alpha j}^{i} \phi^j \longrightarrow \hat{\nabla}_\alpha^\pm \phi^{\hat{a}}$$

$$[\phi^i, \phi^j] \longrightarrow [\phi^M, \phi^N]$$

involve no change in physics: symmetry

2) the symmetry is

$$\boxed{X^\mu \longrightarrow X^\mu + \phi^{\mu, a} \lambda^a}$$

$$\psi_\pm^\mu \longrightarrow (\; - \; - \; - \; - \;)$$

3) Open string boundary couplings lead to D-brane worldvolume theory:

Worldvolume theory should exhibit this symmetry

Covariant non-Abelian Worldvolume Action

$$\boxed{\text{DBI} + \text{CS}}$$

Abelian:

$(G+B)_{MN}\,\partial_\alpha X^M \partial_\beta X^N$, $C_{M_1 \ldots M_n}\,\partial_{\alpha_1} X^{M_1} \cdots \partial_{\alpha_n} X^{M_n}$: $\boxed{V_M\,\partial_\alpha X^M}$

$$\Downarrow$$

T-duality $\Rightarrow$ scalars :
can be understood as semi-static
gauge : $X^\mu = \xi^\mu$, $X^i = \phi^i$ $\Rightarrow$ $\boxed{V_\mu + V_i\,\partial_\mu \phi^i}$

Non-Abelian Scalar Couplings (T-duality + ...)

i) $V_M\,\partial_\alpha X^M \longrightarrow V_\mu + V_i\left(\partial_\mu \phi^i + i[A_\mu, \phi^i]\right)$

ii) $C_{\ldots\, ij}\,[\phi^i, \phi^j]$

iii) $V(x^\mu, x^i) \longrightarrow V(x^\mu, x^i + \phi^i) = e^{\phi^i \partial_i}\, V(x^\mu, x^i)$

No ϕ^μ, $\nexists$ global transverse directions x^i;

Not obtainable from a GCT covariant

expression by a choice of gauge!

Resolution:

Open string considrations $\Rightarrow$ a symmetry that renders all these covariant:

i) $V_M \partial_\alpha X^M + V_M\left(\partial_\alpha \Phi^M + i[A_\alpha, \Phi^M]\right)$

ii) $C_{\dots MN}[\Phi^M, \Phi^N]$ $\qquad (\Phi^M = \Phi^{\hat{a}} a^M_{\hat{a}})$

iii) $V(X^M + \Phi^M) = e^{\Phi^M \partial_M} V(X^M)$

(normal bundle connection?)

Systematics:

Assume that X^M and Φ^M always appear in the combination

$$X^M + \Phi^{M,a} \lambda^a$$

(X^M: not GCT vectors, Φ^M: GCT vectors)

$$\left[\begin{array}{l} D_\alpha(X^M + \Phi^M) = \partial_\alpha X^M + D_\alpha \Phi^M \\ [X^M + \Phi^M, X^N + \Phi^N] = [\Phi^M, \Phi^N] \end{array}\right.$$

$$\Rightarrow V_M \partial_\alpha X^M \longrightarrow \boxed{V_M(X+\Phi)\, D_\alpha(X^M + \Phi^M)}$$

* The known non-covariant form of the action arises in the static gauge:

$$\boxed{X^\mu = \xi^\mu \,, \quad X^i = 0}$$

and after the shift:

$$\boxed{X^\mu \longrightarrow X^\mu - \phi^\mu}$$

* Restoration of geometric description:

$$\begin{bmatrix}\text{exterior}\\ \text{multiplication}\end{bmatrix} \Rightarrow \begin{bmatrix}\text{clifford}\\ \text{multiplication}\end{bmatrix}$$

(SFH, Minasian, hepth/0008149)

* complete NGCT invariant action ?