D-branes on Calabi Yau Manifolds

Lecture at Kolyampari, Crete
June 27, 2003.
Suresh Govindarajan
IIT Madras

Plan:

1. Motivation and Goals
2. Calabi-Yau Manifolds at large volume.
3. Worldsheet Aspects.
4. D-branes on $\mathbb{C}^3/\mathbb{Z}_3$ and its blow up.
5. D-branes on the quintic — a Landau-Ginzburg description
6. A superpotential computation in the top. B-model
7. Conclusion.

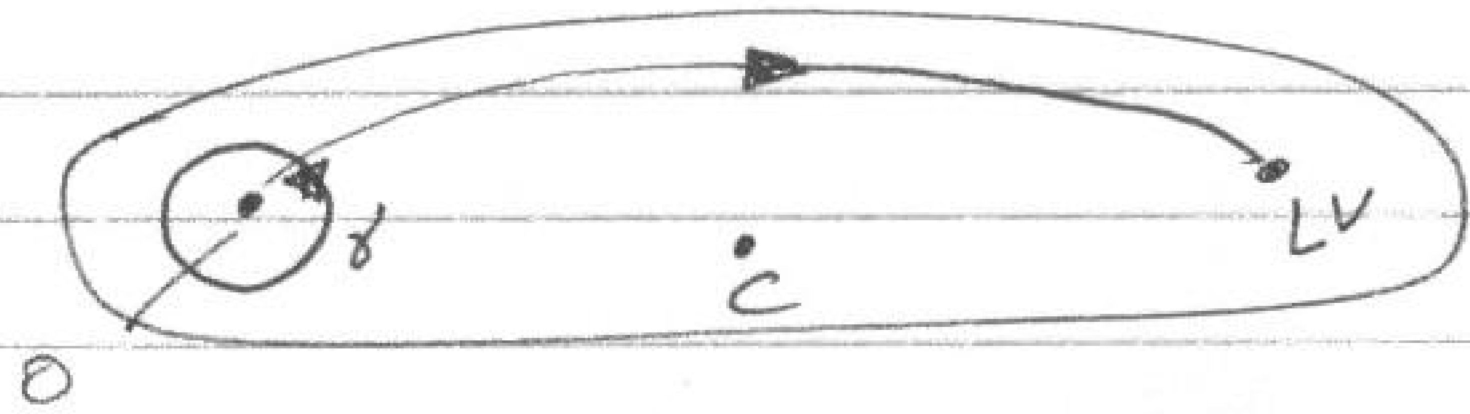
Credits: Douglas, Fiol, Römelsberger, Brunner, Lawrence, Aspinwall, Jayaraman, Tomasiello, Sarkar, Hori, Vafa, Lerche, Kaste, Lazaroiu, Hofman, ...
Diaconescu

These branes must be related to B-branes on $\mathbb{P}^2$. What is their identification?

<u>Subtlety:</u> The periods and hence central charges are multivalued functions of the complexified Kähler modulus.

They undergo <u>monodromy</u> as we move in Kähler moduli space.

<u>e.g.</u>



$\gamma : B_i \longrightarrow B_{i+1}$ $\quad \mathbb{Z}_3$ action.

<u>Keeping in mind that the answer depends on the path chosen,</u> for the path shown above

<u>vector bundle</u>

$B_1 \longrightarrow$ a 4-brane wrapping $\mathbb{P}^2$ $\rightarrow \mathcal{O}_{\mathbb{P}^2}$.

$B_2 \rightsquigarrow$ "4 branes" $\rightarrow$ ~~[illegible]~~ $\Omega_{\mathbb{P}^2} \otimes \mathcal{O}$

$B_3 \rightsquigarrow$ "4 branes" $\rightarrow \mathcal{O}_{\mathbb{P}^2}(-1) = \Omega^2_{\mathbb{P}^2}$

<u>Note:</u> chern classes $\longleftrightarrow$ RR charge

- $\mathcal{O}_{\mathbb{P}^2}$ $\qquad$ 4brane with charge 1

$ch_0 = 1$ $\qquad$ $n_4 = 1,\ n_2 = 0,\ n_0 = 0$

<u>Example 2</u> Quintic = degree 5 hypersurface in $\mathbb{P}^4$.

- Useful to note that $\mathbb{P}^4 =$ blowup of $\mathbb{C}^5/\mathbb{Z}_5$.
- Natural to first consider Fractional branes on $\mathbb{C}^5/\mathbb{Z}_5$
- Obtain branes on $\mathbb{P}^4$
- and then restrict to the quintic.

<u>Nice result</u>: The results match boundary states in the Gepner model [Recknagel–Schomerus] Douglas, ...

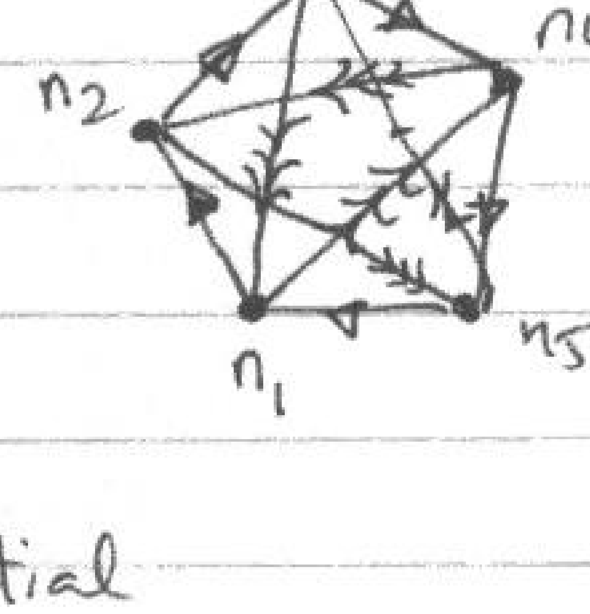
Fractional branes on $\mathbb{C}^5/\mathbb{Z}_5$: (no central charge restriction for this)

- $(B_1 \quad B_2 \quad B_3 \quad B_4 \quad B_5)$
- (at LV) $\boxed{B_i = (-)^{i-1}\, \Omega^{i-1}_{\mathbb{P}^4} \otimes \mathcal{O}_{\mathbb{P}^4}(i-1)[i-1]}$ ← Douglas–Fiol–Römelsberger

  rk $(1 \quad 4 \quad 6 \quad 4 \quad 1)$

- Natural set of branes

  $(n_1 \; n_2 \; n_3 \; n_4 \; n_5)$



<u>links</u>: chiral matter

$X^i_{a,a+1}$

$Y^{[ijk]}_{a,a+3}$ w/ superpotential

$Z_{a,a}$. $\mathcal{W} = \mathrm{tr}(X^i X^j Y^{klm})\epsilon_{ijk\ldots}$

• $(1,1,1,1,1)$ corresponds to D0-brane on $\mathbb{P}^4$.

RESTRICTING TO THE QUINTIC

Quintic is a ~~...~~ 3-dim complex mfd.

$\Rightarrow$ it has $\underline{\underline{4}}$ RR charges. (couple to D0, 2, 4, 6 brane)

But we have ~~(1,1,1,1)~~ 5 charges from $(B_1 \cdots B_5)$

$\Rightarrow$ they <u>cannot</u> be independent after restriction to the quintic.

<u>The Relation:</u> $(1,1,1,1,1)$ is a 0 brane on $\mathbb{P}^4$.

$\Rightarrow$ it has <u>no</u> RR charge on quintic.

"null vector" in the lattice of RR charges given by $n_i\,[B_i]$

$[\;\;]$ ← K-theory classes / RR charges

<u>Implications:</u>

$$[(1,1,1,0,0)] + [(0,0,0,1,1)] = 0$$

expect $[(1,1,1,0,0)] = -[(0,0,0,1,1)]$

on the quintic

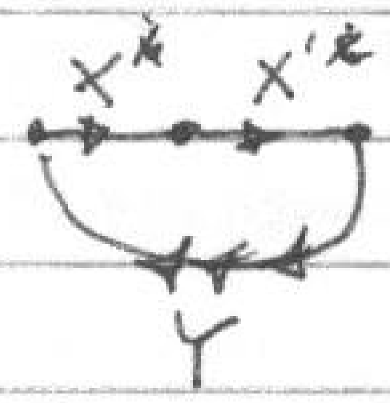
Similarly $[(1,1,1,1,0)] = -[(0,0,0,0,1)]$

Can we prove these equivalences?

Consider I $(1,1,0,0,0)$

$$1 \xrightarrow{X^i} 1 \qquad \text{no superpotential.}$$

II: $(0,0,1,1,1)$



$$W = X^i X'^j Y^{klm} \epsilon_{ijklm}.$$

• Counting dim of moduli space.

look for sol^{n}s of $dW = 0$.

• I no ~~moduli~~ superpotential.

X^i are all moduli $i = 1,2,3,4,5$

$5 - 1 = 4$ //

D-term conditions.

• II $dW = 0$

$$X^{[i} X'^{j]} = 0$$

$$X^i Y^{klm} \epsilon_{ijklm} = 0$$

$$X'^i Y^{klm} \epsilon_{ijklm} = 0.$$

a sol^n: $X = X' = x_0$ $Y = 0$

"$\delta^{i,1}$"

Consider inf. def. about sol^n.

$$X^i = X_0^i + x^i$$

$$X'^i = X_0^i + x'^i$$

$$Y^{ijk} = 0 + y^{ijk}.$$

linearized ~~eq~~ EOM.

$$X_0^{[i} x^{j]} - X_0^{[i} x'^{j]} = 0$$

$$X_0^i y^{jkl} \varepsilon_{ijklm} = 0$$

Let $X_0^i = \delta^{i,1}$

sol^n $\quad x = x'$ — 4 moduli as before

$y^{1lm} = 0$ — 6 moduli.

10 moduli

Clearly, the moduli spaces are not the same. Why? No input from the restriction. So the calc. is essentially a statement on $\mathbb{P}^4$ where we know they ~~a~~ are not the same.

Put in the restriction?

How?

Consider the top-B model. Here things are "independent" of Kähler moduli. However, complex str. moduli can appear. [Douglas, SG, Jayaraman & Tomasiello]

Claim: The cubic superpotential W is deformed to a higher order one by complex moduli. We saw that complex moduli appear via.

$$W = C_{ijklm} Z^i Z^j Z^k Z^l Z^m .$$

Try to write a term involving X^i, Y^{ijk}, ϵ_{ijklm}, C_{ijklm} that is a single trace = "closed loops in the quiver".

$$\Delta W \sim \underbrace{\epsilon_{i_1 i_2 i_3 i_4 i_5} C_{j_1 j_2 j_3 j_4 j_5} X^{j_2} X^{j_3} Y^{i_1 i_2 i_3} X^{j_4} X^{j_5} Y^{i_4 i_5 j_1}}_{\text{sextic term}}$$

In fact, this lifts the extra six moduli that came from the y's.

- This is an explicit an non-trivial realisation of a conjecture that lifting of moduli can be described via a superpotential [Douglas].
- Might seem obvious from our studies of D-brane probes but seems to have interesting mathematical consequences as well!
- However, not all moduli are lifted (when necessary) by this potential.

The explicit calculation $\qquad \Phi = \phi + \theta^+\psi_+ + \theta^-\psi_- + \theta^2 F$

Consider ~~the~~ an $(2,2)$ LG model with 5 chiral fields. ϕ^i

$$\mathcal{L} = \int d^4\theta\, K(\Phi,\bar{\Phi}) + \int d^2\theta\, W(\Phi) + c.c.$$

In the top. B model (after twisting).

$$\mathcal{L}^{top}_{bulk} = \int d^2\theta\, W(\Phi) + Q\text{-exact pieces.}$$

Choose $W(\Phi) = \frac{1}{5} C_{i_1 i_2 i_3 i_4 i_5}\, \phi^{i_1}\phi^{i_2}\phi^{i_3}\phi^{i_4}\phi^{i_5}$

Fields	ϕ	$\tau \equiv \psi_+ - \psi_-$	$\xi \equiv \psi_+ + \psi_-$	F
Ghost nos.	$0+\frac{2}{5}$	$-1+\frac{2}{5}$	$-1+\frac{2}{5}$	$-2+\frac{2}{5}$

U(1) is shifted by using quasi hom. of W.

$$\text{charge}[W(\Phi)] = \frac{2}{5}\times 5 = 2$$

LG model with boundary (B-type b.c.'s)

only possible b.c.'s $\boxed{\begin{aligned}\phi^i &= 0\\ \tau^i &= 0\end{aligned}}$ $\qquad$ Warner / SG, Jayaraman, Sarkar

Boundary observables:

$$\Xi^i = \xi^i - 2\bar{\theta}_i\partial_y\phi^i - \theta^2\partial_x\xi^i$$

One can check

$$\{Q, \Xi^i\} = 0$$

fermionic

$$\mathcal{L}^{top}_{bdry} = \int d^4\theta \; X^i \Xi^i + Y^{ijk} \Xi^i \Xi^j \Xi^k + Z \Xi^1 \Xi^2 \Xi^3 \Xi^4 \Xi^5$$

odd terms are projected out by GSO.

- $$\mathcal{L}^{top} = \mathcal{L}^{top}_{bulk} + \mathcal{L}^{top}_{boundary}$$

- $X^i, Y^{ijk} \ldots$ are 5×5 matrices reflecting CP Factors for the 5 fractional branes.

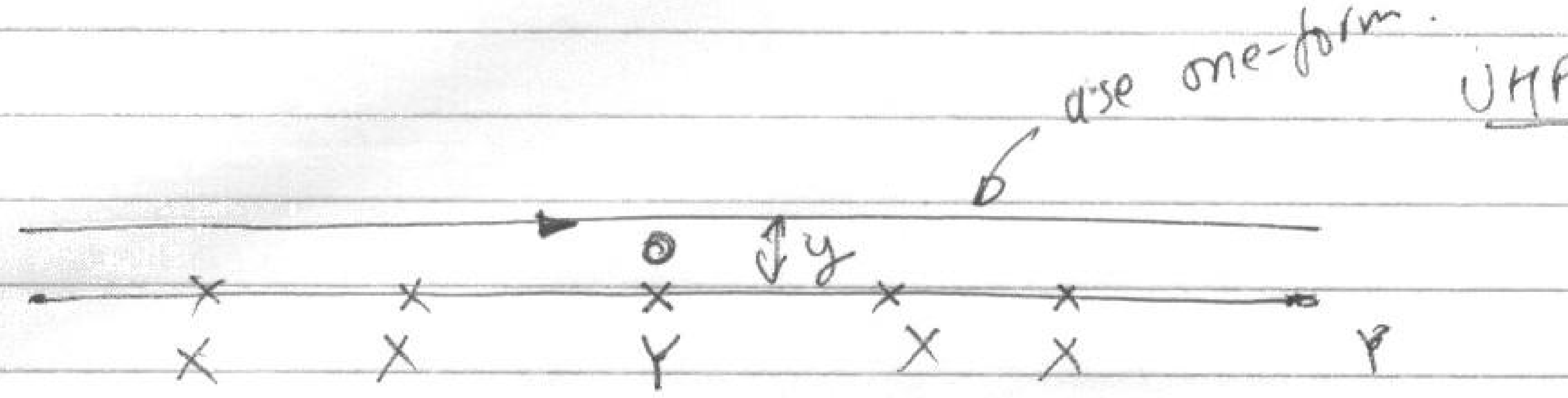
$$W_0 = \mathrm{tr}\left(X^i X^j Y^{klm}\right) \varepsilon_{ijklm}. \qquad \boxed{\text{cubic}}$$

$$\underbrace{\Xi^i \ldots \Xi^n}_{\sim \, \varepsilon^{ijklm}.}$$ soaks ghost number + zero modes

$$W_1 \;\; X \; X \; Y \; X \; X \; Y^{ijk} \; W$$

$$\downarrow \; \downarrow \; \downarrow \; \downarrow \; \downarrow \; \downarrow \; \downarrow$$

1 2 3 4 5 6 7.



$SL(2,R)$ ~~partially~~ fixed by setting one $Y \to \infty$
$Y \to 0$.

- scaling remains
- remaining integrals to be carried out

4 X^s.
1 bulk
5 moduli

- scaling ⇒ answer must be independent of 'y'.
- Integrations can be explicitly done. and answer is non-zero. and agrees with our expectation.
- This is the first in an infinite series whose structure is clear. However, integrals are not obviously calculable.

These are related to the ~~deformation~~ fact that closed strings (moduli) appear as deformations of the A_∞-algebra of open-string operators

Conclusion / Issues ..

1. Ignored issues of stability : topological vs physical branes.

   • Π-stability ; flow of gradings ...
   • spectrum of physical branes.

     [Douglas: spec. of top. branes ~ derived category of ...]

   • have some ~~....~~ understanding of gross features of decay.

2. Changes in choices of branes: related to gen. Seiberg dualities.

3. Detailed structures of D-branes

4. Orientifolds w/ branes
   fluxes (?)

— α —

Motivation + Goals

- What is the spectrum of (BPS) D-branes?
- How does it depend on various moduli?

  e.g. what happens when the CY3 has a size of string scale? [α'-corrections are important].

- Given a D-brane, what is its (super)potential?
- Extended Mirror Symmetry [Kontsevich; Vafa]

  type IIB on CY3 + D-branes $\xleftrightarrow{\text{mirror}}$ type IIA on $\widetilde{CY3}$ + D-branes.

- Lessons for $N=1$ gauge theories and more generally, $N=1$ supersymmetric compactifications.

Calabi-Yau Factology (CY3)

- Ricci-Flat (Kähler) manifolds with SU(3) hol.
  [~ Ω = holomorphic 3 form]
- type II compactifications lead to $\mathcal{N}=2$ supergravity in $d=4$.
- Spectrum: CY3 w/ Hodge nos. $h_{1,1}$ + $h_{2,1}$
  Common: Supergravity Multiplet; Universal Hypermult.

complex str

type IIA	type II B.
$h_{1,2}$ hypermultiplets.	$h_{1,1}$ hypermultiplets
$h_{1,1}$ vector multiplets	$h_{1,2}$ vector multiplets

Kähler

- So two kinds of scalars appear — those from hypermultiplets + those from vector multiplets. They do <u>NOT</u> mix.

  Moduli space = Kähler moduli space × Complex str. moduli space

- <u>Special Geometry</u> (vector multiplets in IIA, say)

  Kähler metric is given by $F(X^A)$ $A=0,\dots,h_{1,2}$

  $$K=-\ln i\left(X^A\bar{F}_A-\bar{X}^A F_A\right)$$

  $$X^A=\int_{\gamma_A}\Omega\quad;\quad F_A=\int_{\gamma_B}\Omega.$$

  "periods"

  $$\Pi^i=\begin{pmatrix}X^A\\F_A\end{pmatrix}$$

D-branes on large Calabi-Yau manifolds

- Useful to think of them as BPS objects carrying ~~[illegible]~~ RR charge.

~~BPS + special geometry ⇒~~

- To be concrete, consider SPACE FILLING branes.

In IIA : has D6-branes in flat space and one can (hopefully) wrap 3-cycles of the CY3 preserving $\frac{1}{2}$ supersymmetry.

In IIB :
- D9 branes wrapping full CY3
- D7 branes wrapping 4 cycle
- D5 branes wrapping 2 cycles.
- D3 branes localised at a point

D(3+2p) branes wrap 2p cycles.

- Spacetime susy preservation ⇒

3 cycles → SPECIAL LAGRANGIAN.

$$\mathrm{Im}(e^{i\alpha}\Omega)|_{\Sigma} = 0$$
$$\omega|_{\Sigma} = 0$$

for even cycles. → cycles must be holomorphic.

- Central charge. [Ferrara et al.] (type IIA)

$$Z(e_A, m^B) \propto (e_A X^A + m^B F_B)$$

3. WORLDSHEET ASPECTS

However, the spacetime description is ~~not~~ limited to cases when the CY3 is large. Useful to have an intrinsic worldsheet description.

Polchinski: D-branes are "conformally invariant boundary conditions." i.e., they are objects on which an open-string can end.

- Recall, nonlinear sigmal models for CY3 compactifications have (2,2) supersymmetry on the worldsheet. $\boxed{\hat{c}=3}$

~~others~~

$$Q_L,\ \bar{Q}_L,\ j_L = i\sqrt{\hat{c}}\,\partial\phi_L$$

$$Q_R,\ \bar{Q}_R,\ j_R = i\sqrt{\hat{c}}\,\bar{\partial}\phi_R.$$

$$\underbrace{\Omega_L}_{\text{hol 3-form.}} \sim e^{\frac{i}{2}\sqrt{\hat{c}}\,\phi_L} \quad ; \Omega_R \sim e^{\frac{i}{2}\sqrt{\hat{c}}\,\phi_R}.$$

- Presence of a boundary breaks translation inv.

$\Rightarrow$ some supersymmetry must necessarily be broken as well.

TWO INEQUIVALENT WAYS (Ooguri, Oz, Yin ; Warner)

$\boxed{\frac{\sqrt{\hat{c}}}{2}\phi = \varphi}$ Dirichlet b.c

$\Omega_L = e^{i\varphi\pi}\,\Omega_R$ $\Leftarrow$ $j_L = -j_R$ A-type (compatible with A-twist)

or $(\partial+\bar{\partial})\phi = \partial+\bar{\partial}(\phi_L+\phi_R) = 0$ "Dirichlet"

$j_L = +j_R$ B-type

$(\partial-\bar{\partial})\phi = 0$ "Neumann"

- More detailed analysis leads to the following conditions.

  1. The submanifold Σ on which a string can end must be <u>minimal</u>

$$\beta_{\varphi}(\hat{n}^i) = \mathrm{tr}(K^i_{ab}) = 0 \qquad \text{Leigh}$$

one-loop beta f^n
for (normal) scalars.

  2. A-type supersymmetry $\Rightarrow$ Σ = Lagrangian

     B-type supersymmetry $\Rightarrow$ Σ = holomorphic submanifold.

1 + 2 $\Rightarrow$ sL for IIA hol. submfd for IIB.	same as spacetime analysis!

- Worldvolume spectrum. (has $\mathcal{N}=1$ supersymmetry).

  scalars $\in N\Sigma$
  (chiral mult.)

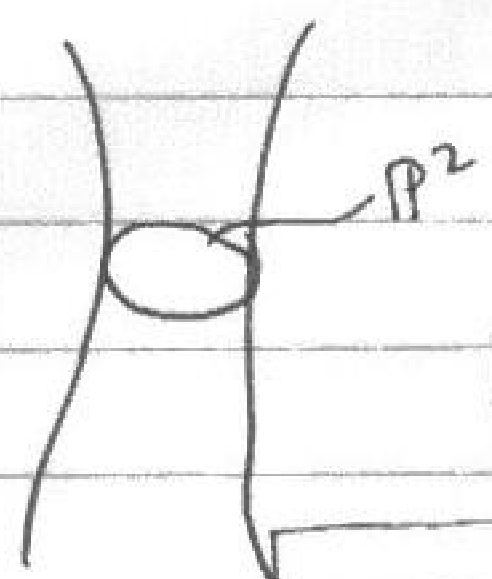
A-type	B-type
gauge fields are <u>flat</u>.	$F^{(2,0)} = F^{(0,2)} = 0$ consequence of B-type susy.
	$F^{(1,1)} \wedge \omega^{n-1} = c\,\omega^n$. $\boxed{\text{DUY}}$ eqns.

$\longleftrightarrow$ Mirror Symmetry. | Gaugefields are hol. connection on v.b^s

In order to be concrete and specific, we will now specialise to the case of B-branes in two examples.

(i) The blowup of $\mathbb{C}^3/\mathbb{Z}_3$. [non-compact CY3]

= total space of the line bundle $\mathcal{O}(-3)$ on $\mathbb{P}^2$.



$$(z_1, z_2, z_3) \sim (\omega z_1, \omega z_2, \omega z_3)$$

$$\boxed{\mathbb{P}^2 : \quad (z_1, z_2, z_3) \sim \lambda (z_1, z_2, z_3)}$$

~~$n_1+n_2+n_3 = 0 \mod 3$~~.
$\omega^3 = 1$.

(ii) The Quintic

$$\mathbb{P}^4 : (z_1, z_2, z_3, z_4, z_5) \sim \lambda (z_1, \cdots, z_5).$$

The Quintic is given by the hypersurface given by an arbitrary degree five polynomial in $\{z_i\}$.

$$W(z_i) = c^{i_1 i_2 i_3 i_4 i_5} z_{i_1} \cdots z_{i_5} = 0$$

$$\frac{5 \times 6 \times 7 \times 8 \times 9}{5 \times 4 \times 3 \times 2 \times 1} = 14 \times 9 = 126 \text{ parameters.}$$

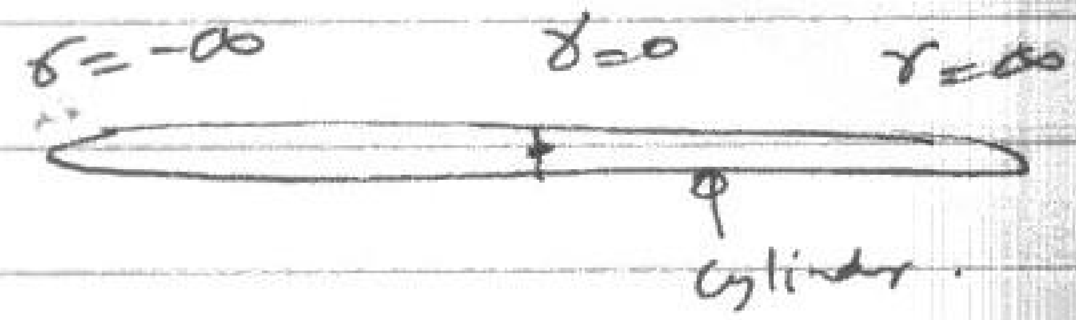
less 25 $GL(5)$ redefs.

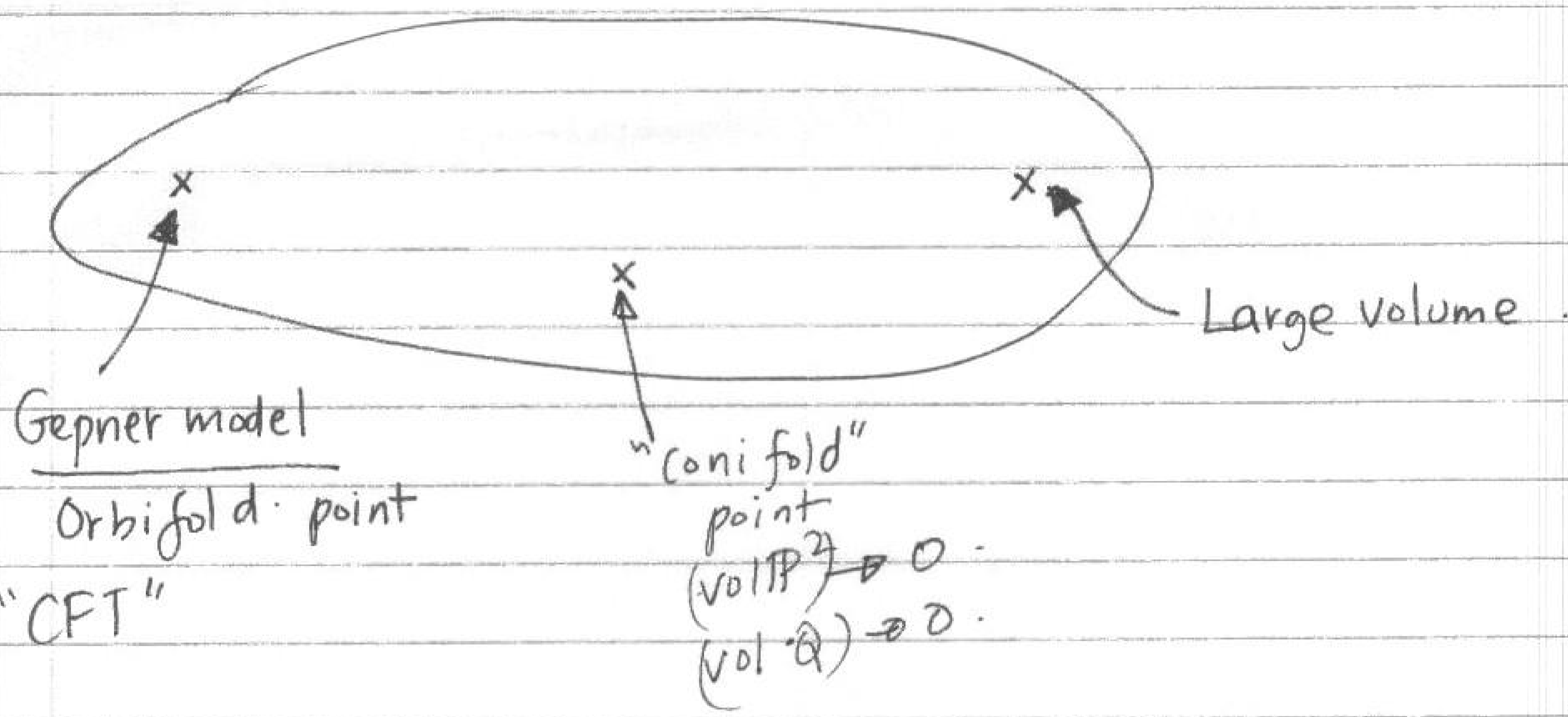
$\underline{101}$ parameters [complex moduli].

Both cases have <u>one</u> Kähler modulus:

the size of the $\mathbb{P}^2$ / size of the quintic.

The Kähler moduli space is similar in both cases.





~~For the quintic: the Ricci flat metric is not~~ known.

- The quintic as a CFT i.e., Gepner model is given by 5 copies of $k=3$ minimal models $(N=2)$.
- Another related construction is as an LG orbifold.
  - 5 ~~$N=2$~~ (2,2) chiral (anti-chiral) superfields. ϕ ($\bar{\phi}$).
  - superpotential of degree ϕ: $W(\phi) = C^{i_1 \cdots i_5} \phi_{i_1} \phi_{i_2} \cdots \phi_{i_5}$
  - ~~[illegible]~~
  - The CFT is obtained as an IR fixed point
  - Prop. of the chiral operators can be obtained in the top. B-model.

Example 1

D-branes on $\mathbb{C}^3/\mathbb{Z}_3$ (Douglas-Moore)

- Basic branes : "Fractional Branes"

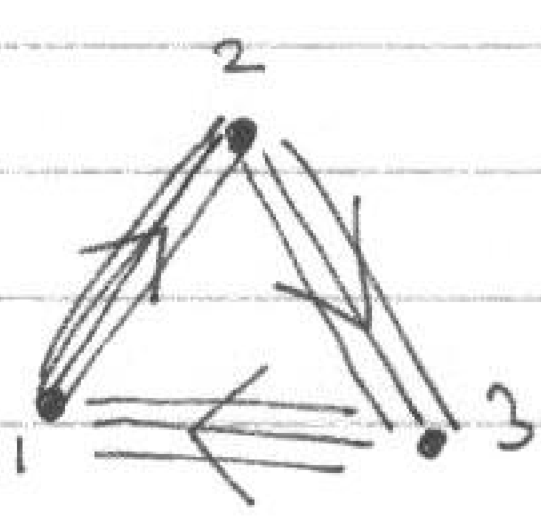
$$B_1,\ B_2,\ B_3.$$

- ~~Each preserves diff. supersymm. in spacetime.~~

- Massless. Spectrum of open strings connecting them.

i —— j (i) for $i=j$ one abelian vector mult.

(ii) chirals in $(i, i+1)$



(near the orbifold limit)

Implies : • The set of branes, are given by

$$(n_1, n_2, n_3). \qquad n_i > 0$$

$\mathbb{Z}_3$-symm

$$\bullet\ W = \mathrm{tr}\left(X^i_{1,2}\ X^j_{2,3}\ X^k_{3,1} \right)$$

W: superpotential; $X^i_{1,2}$: $n_1 \times n_2$ matrix; $X^j_{2,3}$: $n_2 \times n_3$ matrix; $X^k_{3,1}$: $n_3 \times n_1$ matrix.

- $(1,1,1)$ corresponds to a zero brane (pure D3 brane in spacetime)

- The set of physical branes $=$ θ-stable objects.