FROM FREE FIELDS

TO

AdS

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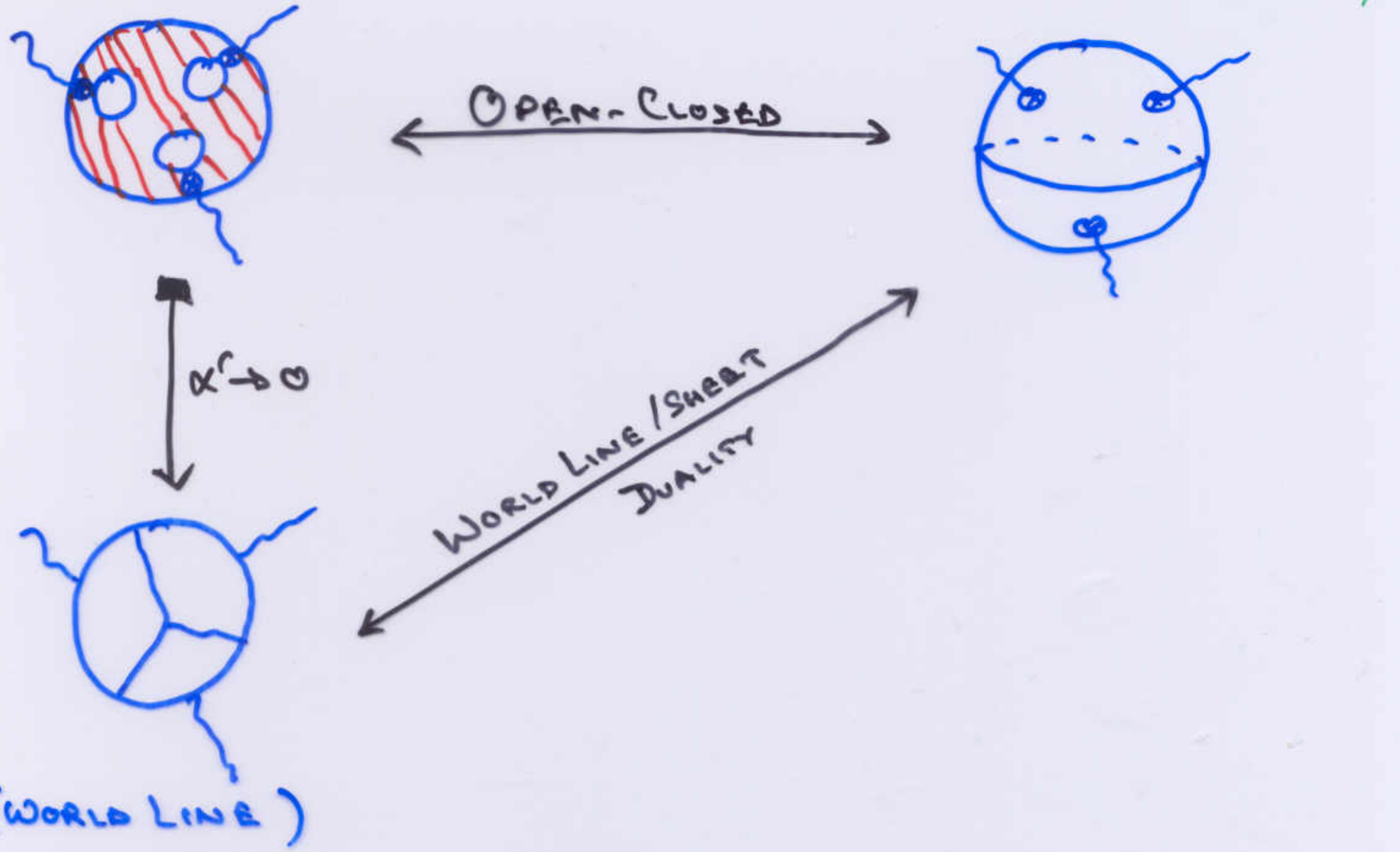
- HOW DO FIELD THEORIES ORGANISE THEMSELVES INTO HIGHER DIMENSION CLOSED STRING THEORIES?
- WHY IS $\lambda = 0$ GAUGE THEORY A STRING THEORY?
- WILL TRY TO FIND A WAY TO EXPRESS FREE FIELD THEORY ANSWERS (CORR. FNS.) AS MANIFESTLY CLOSED STRING (AdS) AMPLITUDES

STRATEGY:

FOLLOW THROUGH OPEN-CLOSED STRING DUALITY



$$\int [D\mathcal{M}]_{open} \langle V_0(m_1) \dots V_0(m_n) \rangle_{world\ sheet}$$

$$\Big\downarrow \ \alpha' \to 0$$

$$\int [D\mathcal{M}]_{closed} \langle V_0(p_1) \dots V_0(p_n) \rangle$$

$$\int [D\tilde{\mathcal{M}}]_{open} \langle V_0(\tilde{m}_1) \dots V_0(\tilde{m}_n) \rangle_{world\ line} \longleftrightarrow$$

↳ CHANGE OF VARIABLES $\tilde{\mathcal{M}}_{open} \to \tilde{\mathcal{M}}_{closed}$

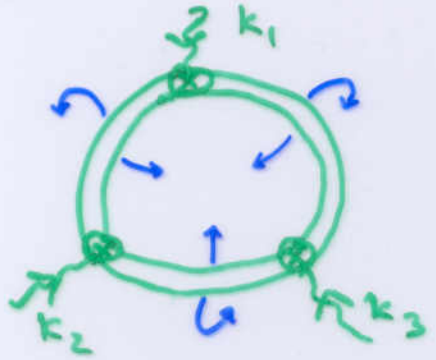
EACH POINT IN OPEN STRING MOD. SP. $\longleftrightarrow$ A CLOSED STRING DIAGRAM

WILL TRY TO CARRY OUT THIS PROGRAM FOR CORRELATORS OF BILINEAR OPERATORS e.g. $\mathrm{Tr}[\Phi\,\partial_{\mu_1}\ldots\partial_{\mu_s}\Phi]$.

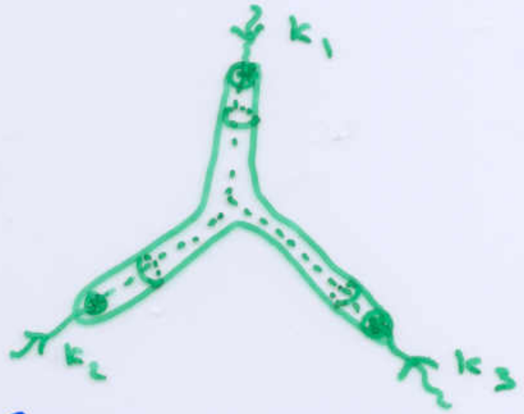
FREE FIELD CORRELATORS OF BILINEARS

- ONLY ANNULUS DIAGRAMS (OR CIRCLE WORLD LINES)

e.g.



(ANNULUS w/ 3 OPEN VERTEX OP.)



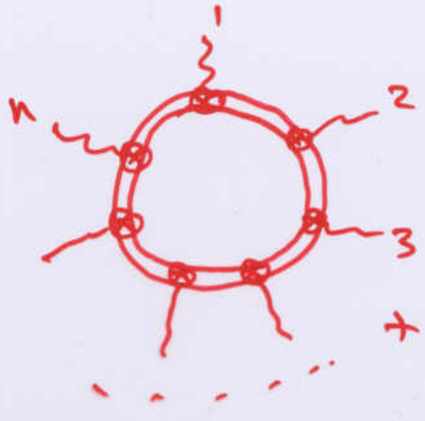
(SPHERE w/ 3 PUNCTURES)

SEE THIS HAPPENING EXPLICITLY FOR 3 PT. FNS. e.g. $\langle \mathrm{Tr}\,\Phi^2(x_1)\,\mathrm{Tr}\,\Phi^2(x_2)\,\mathrm{Tr}\,\Phi^2(x_3)\rangle$

- CLOSED STRING SIDE — DEGENERATE RIEMANN SURFACE $\Rightarrow$ SCHWINGER PARAMETER REPN. OF AdS AMPLITUDES.

WORLD-LINE FORMALISM

$$\frac{1}{N^2}\left\langle \mathrm{Tr}\,\Phi^2(k_1) \ldots\ldots \mathrm{Tr}\,\Phi^2(k_n)\right\rangle_{conn} =$$



+ VARIOUS ORDERINGS

$=$

Moduli of insertion

$$= \int [d\mathcal{M}] \left\langle e^{ik_1\cdot x(\tau_1)} \ldots\ldots e^{ik_n\cdot x(\tau_n)} \right\rangle$$

$$\left[\int [d\mathcal{M}] = \int_0^\infty \frac{d\tau}{\tau} \int_0^\tau \prod_{i=1}^{n} d\tau_i \quad , \quad \langle \ldots \rangle = \int [\mathcal{D}x]\, e^{-\int_0^\tau d\tau' \dot{x}^2} \ldots \right]$$

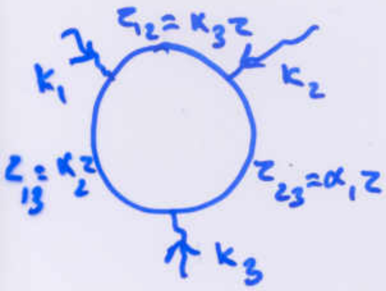
$$= \delta(\Sigma k_i) \int_0^\infty \frac{d\tau}{\tau^{d/2+1}} \int_0^\tau \prod_{i=1}^{n} d\tau_i \, \exp\left[-\frac{1}{2}\sum_{i<j} \vec{k}_i\cdot\vec{k}_j\, G(\tau_i,\tau_j)\right]$$

$$G(\tau_i,\tau_j) = -\frac{\tau_{ij}(\tau-\tau_{ij})}{2\tau} \qquad (\tau_{ij} \equiv |\tau_i - \tau_j|)$$

GREEN'S FN. ON WORLD LINE (CIRCLE)

Note AdS measure $\left(\tau \sim z^2 \Rightarrow \frac{dz}{z^{d+1}}\right)$

3-pt. fn: $\Gamma(\vec{k}_1,\vec{k}_2,\vec{k}_3) =$



$$= \delta(\Sigma k_i)\int_0^\infty \frac{dz}{z^{d/2+1}} z^3 \int_0^1 \prod_{i=1}^{3} d\alpha_i\, \delta\Big(\sum_{i=1}^{3}\alpha_i - 1\Big) \times$$

$$\exp\Big[-\tfrac{1}{2}z\big(k_1^2\alpha_2\alpha_3 + k_2^2\alpha_3\alpha_1 + k_3^2\alpha_2\alpha_1\big)\Big]$$

In Position Space – Recognise This As A Glued Up Version Of The Original Open String Amplitude.

$$\Gamma(\vec{x}_1\vec{x}_2\vec{x}_3) = \int \prod_{i=1}^{3} d^d k_i\, e^{i\vec{k}_i\cdot\vec{x}_i}\, \Gamma(\vec{k}_1\vec{k}_2\vec{k}_3)$$

$$= \int [d\mathcal{M}_3]\int d^d z \prod_{i=1}^{3}\langle \vec{x}_i|\exp[\tfrac{1}{2}z\alpha_j\alpha_k\Box_i]|\vec{z}\rangle$$

$$\int \frac{dz}{z^{d/2+1}} z^3 \int_0^1 \prod_{i=1}^{3} d\alpha_i\, \delta\Big(\sum_{i=1}^{3}\alpha_i - 1\Big)$$

From c.o.m. integral

$$\boxed{\langle\vec{x}|e^{t\Box}|\vec{y}\rangle \equiv \frac{1}{t^{d/2}}e^{-\frac{(\vec{x}-\vec{y})^2}{2t}}}$$

– Free Particle Propagator In d-dim

PICTORIALLY:

CLAIM: RHS is really a bulk-boundary propagator in AdS, in Schwinger (closed string) parametrisation.

TO SEE THIS, LOOK AT THE USUAL AdS BULK-BDY. PROPAGATORS A LITTLE DIFFERENTLY

FOR A SCALAR (w/ $m^2 = -2(d-2) \leftrightarrow \Delta = d-2$)

THE BULK-BDY. PROPAGATOR SATISFIES

$$\left(-z_0^2 \partial_{z_0}^2 + (d-1) z_0 \frac{\partial}{\partial z_0} - z_0^2 \Box - 2(d-2)\right) k = 0.$$

$z_0^2 = t, \quad k = t\tilde{k}$

$$\left[2(d-6)\frac{\partial}{\partial t} - \Box - 4t\frac{\partial^2}{\partial t^2}\right]\tilde{k} = 0$$

SOLUTION — VERY CLOSE TO HEAT KERNEL

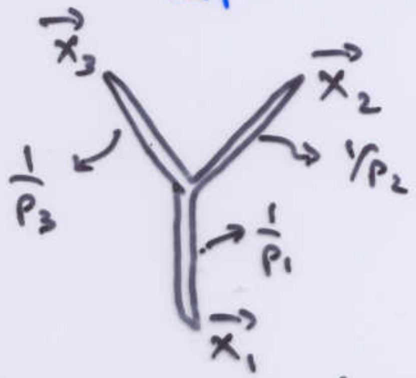
$$k = t\tilde{k} = t\int_0^\infty dp\, p^{\frac{d}{2}-3} e^{-p}\, e^{\frac{t}{4p}\Box}$$

$$k(\vec{x}, \vec{z}, z_0 = t^{1/2}) \equiv \langle \vec{x} | k | \vec{z} \rangle$$

$\therefore$ 3 pt. fn. of Scalars in AdS is

$$\propto \int d^d z \int_0^\infty \frac{dz_0}{z_0^{d+1}} \prod_{i=1}^{3} K(\vec{x}_i, \vec{z}, z_0)$$

$$= \int_0^\infty \frac{dt}{t^{d/2+1}}\, t^3 \int d^d z \int_0^\infty \prod_{i=1}^{3} dp_i\, p_i^{d/2-3}\, e^{-\sum p_i} \prod_{i=1}^{3} \langle \vec{x}_i | e^{\frac{t\Box}{4p_i}} | \vec{z} \rangle$$

$=$ 

(DEGENERATE CLOSED STRING RIEMANN SURFACE)

p_i – SCHWINGER PARAMETERS = "MODULI" OF CLOSED RIEMANN SURFACE

NOTE CLOSE SIMILARITY w/ WORLD LINE ANSWER. JUST NEED TO MAP OPEN STRING MODULI TO CLOSED STRING "MODULI"

CHANGE OF VARIABLES

$$t = \tau \rho \prod_{i=1}^{3} \alpha_i \qquad (p_i = \rho\, \alpha_i)$$

AND THE TWO ARE IDENTICAL.

NOTE CHANGE IS INDEPENDENT OF EXT. MOMENTA, DIMENSION d etc.

3 Pt. Fn. Of More General Bilinears

$$\int [d\mathcal{M}_3] \langle \dot{x}_{\mu_1} \ldots \dot{x}_{\mu_s}(\tau_1) e^{ik_1 \cdot x(\tau_1)} \; \dot{x}_{\nu_1} \ldots \dot{x}_{\nu_r}(\tau_2) e^{ik_2 \cdot x(\tau_2)} \; \dot{x}_{\rho_1} \ldots \dot{x}_{\rho_s}(\tau_3) e^{ik_3 \cdot x(\tau_3)} \rangle$$

$$= \delta(\Sigma k) \int [d\mathcal{M}_3]\, f(\tau_i) \underbrace{k^{i_1}_{\mu_1} \ldots k^{i_L}_{\mu_L}}_{} \exp\left[-\frac{1}{2} \sum_{i=1}^{3} \vec{k}_i \cdot \vec{k}_j \, G(\tau_i, \tau_j)\right]$$

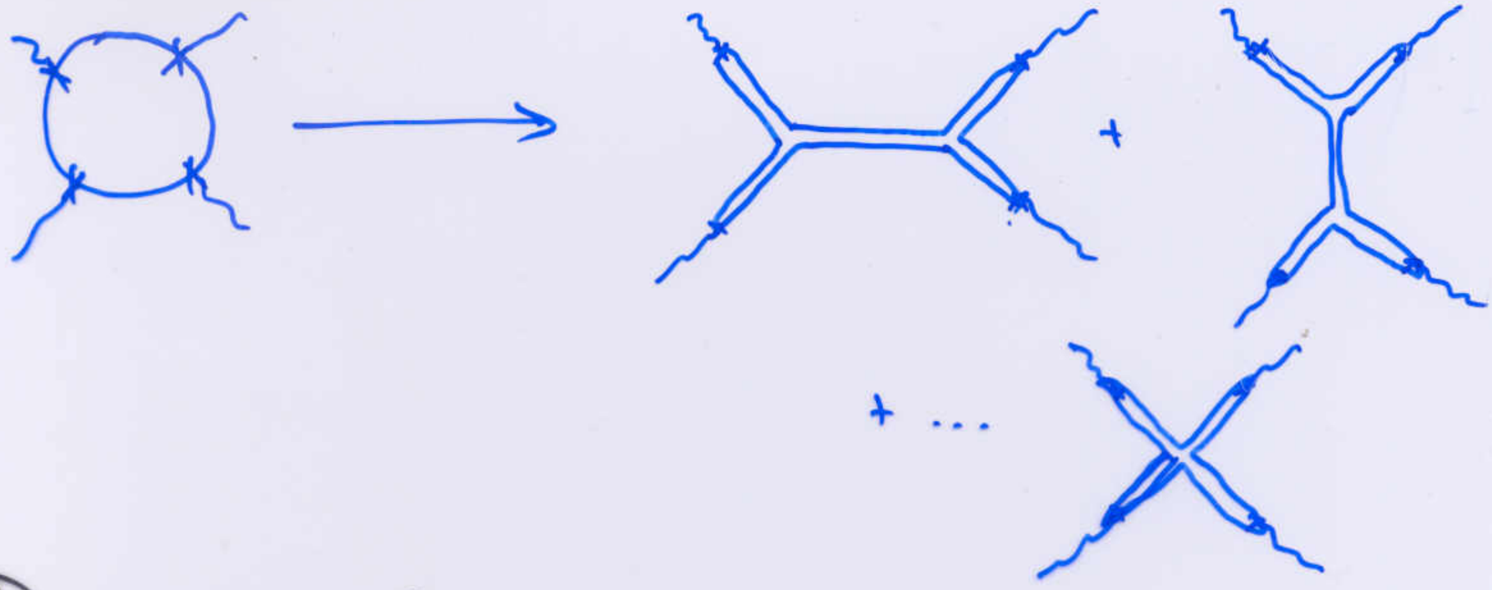
↳ Tensor Structure In Ext. Momenta

Essentially Same Gaussian Integral As Before.

On AdS Side Again Bulk To Bdy. Propagators (for higher spins) Have Very Similar Structure – Scalar Answer With Some Multiplicative Tensor Structure. Seems To Generalise The Agreement Above, When One Makes The <u>Same</u> Change Of Variables On Moduli Space As Above.

Higher Pt. Fns. Of Bilinears

e.g. 4 Point Function



Different Regions Of Open String Mod. Sp. Give Different Degenerations Of Closed String Surface $\longleftrightarrow$ Different Channels In Particle Picture

Reason:

$$\left\langle \prod_{i=1}^{4} e^{ik_i\cdot X_0(\tau_i)} \right\rangle = \left\langle e^{ik_1\cdot X(\tau_1)} e^{ik_2\cdot X(\tau_2)} \prod_{3,4} e^{ik_i\cdot X(\tau_i)} \right\rangle$$

If We Are In The Region Where τ_1, τ_2 Are Adjacent Use Exact Operator Prod.

$$e^{ik_1\cdot X(\tau_1)} e^{ik_2\cdot X(\tau_2)} = e^{i(k_1\cdot X(\tau_1) + k_2\cdot X(\tau_2))} \exp\left[-\tfrac{1}{2} k_1\cdot k_2 \, G(\tau_1,\tau_2)\right]$$

EXPANDING IN TERMS OF SEPARATION $\tau_1 - \tau_2 = \Delta\tau$ GIVES VERTEX OP. OF HIGHER SPIN $\dot{x}_{\mu_1} \ldots \dot{x}_{\mu_s}\left(\frac{\tau_1+\tau_2}{2}\right) e^{i(k_1+k_2)\cdot x\left(\frac{\tau_1+\tau_2}{2}\right)} (\Delta\tau)^s$

THIS INDICATES EXCHANGE OF HIGHER SPIN PARTICLES AND A FACTORISATION OF 4 PT. FN. INTO PROD. OF 3-PT. FNS.

THIS WAS S-channel (k_1+k_2) - exchanged momenta. Other channels from other regions of mod. sp. $(\tau_1, \tau_2 \tau_3, \tau_4)$