KONISHI ANOMALY Approach To Gravitational F-Terms

based on 0304227
with
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Introduction

- Consider $U(N)$ $\mathcal{N}=1$ SYM with a single adjoint chiral Multiplet Φ and an arbitary super potential

$$V(\Phi) = \sum_{k=1}^{n+1} \frac{g_k}{k} \mathrm{Tr}(\Phi^k) \qquad g_2 > 0$$

- $\rightarrow$ F-terms
  - Feynman diagram contribution to F-terms obtained by integrating out the chiral multiplet reduce to planar graphs
  - The diagrams reduce to the planar graphs of an associated matrix model with

$$V(M) = \frac{\hat{N}}{g_m} \sum_k \frac{g_k}{k} \mathrm{Tr}(M^k) = \frac{\hat{N}}{g_m} V(M)$$

  - F-terms are given by

$$\int d^2\theta \left[N_i \frac{\partial F}{\partial S_i} + \frac{1}{2} w^{\alpha}{}_i W_{\alpha j} \frac{\partial^2 F}{\partial S_i \partial S_j} \right]$$

$S_i = w^\alpha w_\alpha$ chiral superfield whose lowest component is the gaugino bilinear in the i^{th} broken phase

$$U(N) \rightarrow \prod_{i=1}^{n} U(N_i)$$

$W_{\alpha i}$ $U(1)$ chiral gauge superfield of $U(N_i)$

- The proof of the above statements
  - . Diagramatic
  - . Anomaly Equations

- F-Terms in gravitational background
  - The Feynman diagrams contributing to gravitational F-Terms reduce to genus 1 diagrams of the corresponding matrix model

F-Term $\int d^2\theta \; G^2 F_1(S)$ S: gaugino bilinear

$$G^2 = G^{\alpha\beta\gamma} G_{\alpha\beta\gamma}$$

$$G_{\alpha\beta\gamma} = \Psi_{\alpha\beta\gamma} + \theta^\delta R_{\alpha\beta\gamma\delta} + \cdots$$

N = 1 Weyl Multiplet

- Diagramatically: shown by Ooguri & Vafa

- We show genus 1 $\left(\frac{1}{N^2}\right)$ corrected resolvant equation for the matrix model agrees with the gravitational corrected anomaly equation in the gauge Theory.

Outline of Talk

- Ingredients in the proof
    - Chiral Ring
    - Generalized Konishi anomaly
- Genus One Corrections
    - Connected Two point functions of gauge Invariant chiral operators
- Comparison with Matrix Model
- Conclusions

Chiral Ring

- Consists of all operators annihilated by $\bar{D}_{\dot\alpha}$

- Relations in chiral ring are defined modulo $\bar{D}_{\dot\alpha}$ exact terms

- Restricting to the chiral ring is an efficient way of focussing on F-terms
  as $\bar{D}_{\dot\alpha}$ exact terms in the ring can be written as

$$\int d^4x\, d^2\theta\, d^2\bar\theta\; S(\theta,\bar\theta)$$

- For $\mathcal{N}=1$ in absence of gravity chiral ring relations are

$$[W_\alpha, \Phi] = 0 \qquad \{W_\alpha, W_\beta\} = 0$$

- $$D^{\dot\alpha}(D_{\alpha\dot\alpha} W_\beta) = [D^{\dot\alpha}, D_{\alpha\dot\alpha}] W_\beta + \cancel{D_{\alpha\dot\alpha} D^{\dot\alpha} W_\beta}$$
$$= \{W_\alpha, W_\beta\} \quad \text{(Bianchi Indentity)}$$

$$D_{\alpha\dot\alpha} = \frac{\partial}{\partial x^{\alpha\dot\alpha}} + [D_{\dot\alpha}(e^{-V} D_\alpha e^{V}), \cdot\,]$$

$$D_{\dot\alpha} e^{-V} D_\alpha e^{V} \sim \sigma^\mu_{\alpha\dot\alpha} A_\mu + \bar\theta_{\dot\alpha}\lambda_\alpha$$

$$[D^{\dot\alpha}, D_{\alpha\dot\alpha}] = Ad(W_\alpha)$$

$$W_\alpha = \lambda_\alpha + \theta^\alpha F_{\alpha\beta} + \cdots$$

- Using the ring relations all gauge Invariant operators can be arranged as

$$\mathrm{Tr}(W^2\varphi^k) \rightarrow R = -\frac{1}{32\pi^2}\mathrm{Tr}\left(\frac{W^2}{z-\varphi}\right)$$

$$\mathrm{Tr}(W_\alpha\, \varphi^k) \rightarrow \omega_\alpha = \frac{1}{4\pi}\mathrm{Tr}\left(\frac{W_\alpha}{z-\varphi}\right)$$

$$\mathrm{Tr}(\varphi^k) \rightarrow T = \mathrm{Tr}\left(\frac{1}{z-\varphi}\right)$$

- In presence of gravity

$$[W_\alpha, \Phi] = 0 \; ; \; [W_\alpha, W_\beta] = 2G_{\alpha\beta\gamma}W^\gamma$$

again: $D^{\dot\alpha}(D_{\alpha\dot\alpha}W_\beta) = [D^{\dot\alpha}, D_{\alpha\dot\alpha}]W_\beta$

but $D_{\alpha\dot\alpha}W_\beta = \left(\frac{\partial}{\partial x^{\alpha\dot\alpha}} + \sigma^\mu_{\alpha\dot\alpha}\,\omega_{\mu ab}\,\sigma^{ab} + \sigma^\mu_{\alpha\dot\alpha}\,\mathrm{Ad}\,A_\mu\right)W_\beta$

(lowest components)

$$\sigma^\mu_{\alpha\dot\alpha}\,\Omega_{\mu ab}(\sigma^{ab})_{\beta\gamma} \sim \sigma^\mu_{\alpha\dot\alpha}\,\omega_{\mu ab}\,\sigma^{ab} + \bar\Theta_{\dot\alpha}\,\Psi_{\alpha\beta\gamma}$$

gravitino field strength

- Again can organize fields into $R, \omega_\alpha T$

- Other relations in the Ring

$$G_{\alpha\beta\gamma}\,\omega^\gamma = 0$$

$$G^2 w_\alpha = 0$$

$$G^4 = 0$$

- Gravitational corrections truncate at order G^2

Generalized Konishi Anomaly

- To obtain equations which completely determine the operators R, T ω_α we need Ward Identities. Provided by the Konishi Anomaly
- Consider the variation

$$\delta\Phi = \varepsilon\Phi \qquad \delta\bar{\Phi} = -\varepsilon\delta\Phi$$

conserved current

$$0 = \langle \underbrace{\bar{D}^2(\bar{\Phi}e^V\Phi)}_{\text{Holomorphic contribution}} \rangle = \underbrace{\frac{1}{32\pi^2}\mathrm{Tr}_{Ad}\, W^\alpha W_\alpha}_{\text{Anomaly}} + \underbrace{\mathrm{Tr}\,\Phi\,\frac{\partial V}{\partial\Phi}}_{\text{classical variation}}$$

Consider Θ^2 with $\bar{\Theta}^2$ component

$$\partial_\mu \mathrm{Tr}(\bar{\psi}\bar{\sigma}^\mu\psi - \psi\sigma^\mu\bar{\psi}) = \frac{1}{32\pi^2}\mathrm{Tr}_{Ad}(F\wedge F) + \frac{N^2}{32\pi^2}\cdot\frac{1}{24}R\wedge R$$

In presence of gravity

superfield

$$+\frac{1}{32\pi^2}\mathrm{Tr}_{Ad}\, W^\alpha W_\alpha + \frac{N^2}{32\pi^2}\cdot\frac{1}{3}\left(G^{\alpha\beta\gamma}G_{\alpha\beta\gamma}\right)$$

For a generalized variation $\delta\Phi_{ij} = \varepsilon f_{ij}(\Phi, W)$

one gets

$$0 = A_{ij\,;kl}\frac{\delta f_{ji}}{\delta\Phi_{kl}} + \mathrm{Tr}(fV')$$

$$A_{ij;kl} = \frac{1}{32\pi^2}\Big\{(W^2)_{kj}\delta_{il} + (W^2)_{il}\delta_{kj} - 2(W^\alpha)_{kj}(W_\alpha)_{il} + \frac{1}{3}\delta_{kj}\delta_{il}G^2\Big\}$$

Using The generalized Anomaly & the Equations in the Chiral Ring We get

$$\langle R^2(z)\rangle - \langle \mathrm{Tr}(V'(\Phi) R)\rangle = 0$$

$$2\langle R(z) W_\alpha(z)\rangle - \langle \mathrm{Tr}(V'(\Phi) W_\alpha(z)\rangle = 0$$

$$2\langle R(z) T(z)\rangle - \langle \mathrm{Tr}\, V'(\Phi)\, T(z)\rangle + \langle W^\alpha(z) W_\alpha(z)\rangle - \frac{G^2}{3}\langle T(z)\, T(z)\rangle = 0$$

Note: In flat space correlation fns of Chiral gauge Invariant operators factorize.

Here apriori eg. $\langle R^2\rangle \neq \langle R\rangle\langle R\rangle$

<u>strategy</u>: To solve above equations

→ write $\langle R^2\rangle = \langle R\rangle^2 + \langle RR\rangle_c$

$$\langle RR\rangle_c \propto G^2$$

→ Expand $\langle R\rangle = \langle R_0\rangle_0 + G^2\langle R_1\rangle$

→ Substitute In above Eqns & match order by order

eg $\langle R_0\rangle^2 - V'(z)\langle R(z)\rangle - \frac{f(z)}{4}$
$+ G^2\left[(2R_0 - V'(z))\langle R_1\rangle\right] + \langle RR\rangle_c = 0$

where $\langle \mathrm{Tr}\, V'(\Phi - V'(z)\, R\rangle = -\frac{1}{4} f_{n-1}(z)$.

$f(z)$: polynomial of degree $n-1$

→ $f(z)$ is fixed by demanding $\frac{1}{2\pi i}\int_{C_i} R(z)dz = S_i$

$i = 1 \cdots n$

- Conditions For Other Operators

$$\frac{1}{2\pi i}\int_{C_i} w_\alpha(z)dz = w_{\alpha i} \quad ; \frac{1}{2\pi i}\int_{C_i} T(z)dz = N_i$$

Genus One Corrections

- How to obtain equations for the connected correlators:

KONISHI ANOMALY

Consider for e.g. $\langle R(z)\, T(\omega)\rangle_c$

Variation $f_{ij} = R_{ij}(z)\, T(\omega)$

Jacobian

$$\frac{\delta f_{ji}}{\delta \Phi_{kl}} = \frac{\delta R_{ji}(z) T(\omega)}{\delta \Phi_{kl}} + R_{ji}(z)\, T_{mk}(\omega)\, T_{lm}(\omega)$$

1st Term

with classical piece $\langle [R^2 - \mathrm{Tr}(V'(\Phi) R(z))] \cdot T(\omega)\rangle$

$$= \langle R^2 - \mathrm{Tr}\, \cancel{V'(\Phi) R(z)}\rangle \langle T(\omega)\rangle$$

$$+ 2\langle R(z)\rangle \langle R(z)\, T(\omega)\rangle_c + \langle RRT\rangle_c$$

$$- \langle \mathrm{Tr}\, V'(\Phi)\, R(z) T(\omega)\rangle_c .$$

2nd Term $\quad -\frac{1}{3} G^2 \langle \mathrm{Tr}(R(z)\, T(\omega)\, T(\omega)\rangle \quad = -\frac{G^2}{3}\partial_\omega\left(\frac{R(z)-R(\omega)}{z-\omega}\right)$

Net Eq 9

$$O(G^4)$$

$$\left[2R(z) - I(z)\right] \langle R(z) T(\omega)\rangle_c + \cancel{\langle R(z) T(z) T(\omega)\rangle_c}$$

$$- \frac{1}{3} G^2 \partial_\omega \left(\frac{R(z) - R(\omega)}{z-\omega} \right) = 0$$

where : $I(z) A(z) = \frac{1}{2\pi i} \oint_{C_2} dy \frac{V'(y) A(y)}{y-z}$

NOTE: • $\langle R(z) T(\omega)\rangle_c \neq 0$
$\propto G^2$

→ • The Equations for the various Connected 2pt fns are consistent

e.g $M(z) \langle R(z) T(\omega)\rangle_c = \frac{1}{3} G^2 \partial_\omega \left(\frac{R(z) - R(\omega)}{z-\omega} \right)$

$M(z) = 2R(z) - I(z).$

also get $M(\omega) \langle R(z) T(\omega)\rangle_c = \frac{1}{3} G^2 \partial_z \left(\frac{R(z) - R(\omega)}{z-\omega} \right)$

Nontrivial Condition

$$\frac{G^2}{3} M(\omega) \partial_\omega \left(\frac{R(z) - R(\omega)}{z-\omega} \right) = \frac{G^2}{3} M(z) \partial_z \left(\frac{R(z) - R(\omega)}{z-\omega} \right)$$

is satisfied.

→ • Solutions to the 2-pt fns. fixed by Physical Requirement

$$\frac{1}{2\pi i} \oint_{C_1} dz\, R(z) = S_i \; ; \; \frac{1}{2\pi i} \oint_{C_1} d\omega\, T(\omega) = N_i \; , \; \frac{1}{2\pi i} \oint_{C_1} dz\, \omega_\alpha(z) = w_{\alpha_1}$$

⇒ Contour Integrals of connected 2pt fns around z & ω branch cuts <u>vanish</u>

Soln is <u>unique</u>

Let $\langle R(z) T(\omega)\rangle_c = G^2 H(z,\omega)$ $H(z,\omega)$ is symmetric

- Solutions for connected 2pt fns

$$\langle R(z)\, R(w)\rangle_c = \langle \omega_\alpha(z)\, T(w)\rangle_c = \langle \omega_\alpha(z)\, R(w)\rangle_c$$

$$= \langle w_{\alpha\beta}(z)\, w_{\beta\gamma}(w)\rangle_c = 0$$

$$\langle R(z) T(w)\rangle_c = G^2\, H(z,w)$$

$$\langle w^\alpha(z)\, w_\alpha(w)\rangle_c = 10 G^2 H(z,w)$$

$$\langle T(z)\, T(w)\rangle_c = G^2 N_i \frac{\partial}{\partial s_i} H(z,w)$$

- Solutions to 1-pt fn at $O(G^2)$

$\langle R\rangle$; $\langle w_\alpha\rangle$ recieve No corrections

Corrections to $T(z)$ consists of 2 Terms

$$T^{(1)}_{(0)}(z) = -\frac{1}{6} N_i \frac{\partial}{\partial s_i} T^{(0)}(z). \qquad (\,O(N^2)\text{ piece}\,)$$

$$T^{(1)}_{1}(z) = \frac{-12}{(2R^0 - V')}\left[H(z,z) + C(z) \right]$$

$C(z)$: polynomial of degree $n-1$ uniquely determined by requirement $\int_{C_i} T_1^{(1)}(z)\, dz = 0$

- $O(N^2)$ genus (0) term can be absorbed in $T^{(0)}$ by field redefinition $s_i \to s_i + \frac{G^2}{6} N_i$

Comparison with the Matrix Model

- Consider a hermetian matrix model

$$S = \frac{\hat{N}}{g_m} \sum_k \frac{g_k}{k} \mathrm{Tr}(M^k)$$

The resolvant $\Omega = \frac{g_m}{\hat{N}} \mathrm{Tr}\left(\frac{1}{z-M}\right)$ satisfies

$$\langle \Omega^2 \rangle \quad - I(z) \langle \Omega(z) \rangle = 0$$

$$\rightarrow \langle \Omega \rangle^2 + \underbrace{\langle \Omega(z)\Omega(z) \rangle_c}_{\sim 1/\hat{N}^2} - I(z) \langle \Omega(z) \rangle = 0$$

. Identical to gauge theory Resolvant $R(z)$

- To compute $1/\hat{N}^2$ corrections we need $\langle \Omega(z)\Omega(z) \rangle_c$

$\rightarrow$ similar procedure

Consider the variation $\delta M_{ij} = \Omega_{ij}(z)\, \Omega(w)$

obtain

$$\left(2\Omega(z) - I(z)\right) \langle \Omega(z)\Omega(w) \rangle_c + \langle \Omega(z)\Omega(z)\Omega(w) \rangle_c^{\;\sim 1/\hat{N}^4}$$

$$- \left(\frac{g_m}{\hat{N}}\right)^2 \partial_w \left(\frac{\Omega(z) - \Omega(w)}{z - w}\right) = 0$$

Identical to eq for $3 \langle R(z)\, T(w) \rangle$

Demanding $\int_{c_i} \langle \Omega(z)\Omega(w) \rangle dz = 0 = \int_{c_i} \langle \Omega(z)\Omega(w) \rangle dw$

Soln is unique .8

$$\langle \Omega(z)\Omega(w) \rangle_c = \left(\frac{g_m}{\hat{N}}\right)^2 3\, H(z, w)$$

- Similarly can show that

$$\boxed{\Omega^{(1)}(z) = \frac{1}{4} T^{(1)}(z)}$$

Conclusions :

- We have used
  - Konishi Anomaly
  - Modified Chiral Ring to show gravitational

  F-terms are captured by genus one diagram of the matrix model.

  crucial Ingredient : lack of factorization

- Including the graviphoton field strength $F_{\alpha\beta}$ In the Ring.

$$\{W_\alpha, W_\beta\} = F_{\alpha\beta} + G_{\alpha\beta\gamma} W^\gamma$$

One can show that the gauge theory resolvant R captures the all genus expansion of the matrix model

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