Dualities and RG flows in 3d $\mathcal{N}$=1 CS-matter theories

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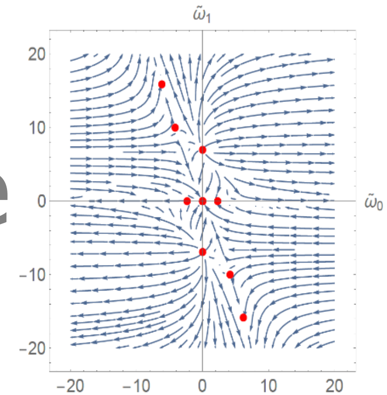


Tenth Crete Regional Meeting on String Theory, Kolymbari, September 19, 2019

Based on OA + Adar Sharon, 1905.07146

Background

- Gauge theory dynamics in 3d is very rich! Many similarities to 4d – confinement, chiral symmetry breaking, conformal fixed points, … Also interesting in its own right: applications to condensed matter, domain walls in 4d theories, etc.
- Special to 3d – can have Chern-Simons term

$$L_{CS} = -\frac{\kappa}{2\pi}\int d^3x\, \varepsilon^{\mu\nu\rho} Tr\left(A_\mu \partial_\nu A_\rho - \frac{2i}{3} A_\mu A_\nu A_\rho\right)$$

in addition to, or instead of, standard gauge coupling. Dimensionless, quantized coupling.

Background

- Classically CS-matter theories conformal for massless matter, and CS coupling doesn't run, so naively get a fixed point for any G, matter content and $\kappa \neq 0$.
- Does YM+CS+matter theory flow to this fixed point ? In many cases, but not always (see Armoni,Dumitrescu,Festuccia,Komargodski). Will not discuss here.
- When scalar fields are present, have 2 options:

1) Allow ϕ^4 couplings and get CS + critical scalar theories;
2) Extra fine-tuning. Then need to worry about classically marginal ϕ^6 (and $\phi^2\psi^2$) couplings.

Background : duality

- Many CS-matter fixed points seem to have dual descriptions. Simplest example :
  $SU(N)_k + \psi \leftrightarrow U(k+\frac{1}{2})_{-N,-N} + \phi, \phi^4$
- Historically these arose from 3 separate directions : (many references)

1. SUSY : 3d $\mathcal{N}$=2 CS-matter theories have Seiberg dualities like in 4d (can flow), e.g.
   $SU(N)_k + Q \leftrightarrow U(k-N+\frac{1}{2})_{-k,-N+1/2} + Q$
   This can be tested by many SUSY tools.
2. Large N : 't Hooft limit of fixed $\lambda = N/k$ has enhanced high-spin symmetry, gravity dual
3. Condensed matter (for specific small N,k)

Background : duality

- The simplest duality

$$SU(N)_k + \psi \leftrightarrow U(k+\frac{1}{2})_{-N,-N} + \phi, \phi^4$$

checked in detail at large N. Seems to hold for all N,k (not necessarily from YMCS) but hard to check. (Mass flows → level-rank duality.)

- The next-simplest case (tricritical)

$$SU(N)_k + \psi, \psi^4 \leftrightarrow U(k+\frac{1}{2})_{-N,-N} + \phi, no\ \phi^4$$

- Related by Legendre transform at large N.
- Add marginal $\frac{\lambda_6}{N^2}\phi^6$. Exactly marginal at large N, duality between family of CFTs.

Background : duality

$$SU(N)_k + \psi, \psi^4 \leftrightarrow U(k+\frac{1}{2})_{-N,-N} + \phi, no\ \phi^4, \frac{\lambda_6}{N^2}\phi^6$$

- At finite N have a beta function for λ_6. Without CS trivial fixed point at λ_6=0, but for non-zero k have non-trivial fixed points for λ_6. At large enough N can argue they exist, and can have a duality. For small N, not clear when have fixed point, and when have stable vacuum.

- Many generalizations to theories with many species of fermions and scalars, different gauge groups, etc.

3d $\mathcal{N}$=1 CS-matter theories

- Today I'll discuss 3d $\mathcal{N}$=1 CS-matter theories. Just 2 supercharges – not enough for any "exact computations" (localization), and no non-renormalization theorems.
- Some motivations :

1. Vacuum stability ensured (for SUSY vacua)
2. Fewer couplings : no analog of ϕ^4 coupling, but have analog of ϕ^6 coupling leading to non-trivial fixed points and dualities (with one fine-tuning).
3. Arise on domain walls of 4d $\mathcal{N}$=1 SUSY theories (e.g. SQCD).

3d $\mathcal{N}$=1 actions

- Simplest superfield has one real scalar and one real fermion, in superspace $\Phi = \phi + \theta\psi + \theta^2 F$
- Start from $U(N)_\kappa$ theory with a single (complex) superfield in the fundamental representation.
- $S = L_{CS} + \int d^3x\, d^2\theta \left(-\frac{1}{2} D^\alpha \overline{\Phi} D_\alpha \Phi + \frac{\pi\omega}{\kappa}(|\Phi|^2)^2\right)$ where L_{CS} same as before after integrating out gaugino. This includes component interactions:

$$\frac{4\pi^2\omega^2}{\kappa^2}(|\phi|^2)^3 - \frac{2\pi(1+\omega)}{\kappa}|\phi|^2|\psi|^2 - \frac{2\pi\omega}{\kappa}(\bar{\psi}\phi)(\bar{\phi}\psi)$$

$$+ \frac{\pi(1-\omega)}{\kappa}((\bar{\psi}\phi)(\bar{\psi}\phi)\text{+c.c})$$

- For $\omega = 1$ have enhanced $\mathcal{N}$=2 SUSY.

Duality at large N

- $S = L_{CS} + \int d^3x\, d^2\theta \left(-\frac{1}{2} D^\alpha \overline{\Phi} D_\alpha \Phi + \frac{\pi\omega}{\kappa}(|\Phi|^2)^2\right)$
- For large N (fixed $\lambda = N/k$) the beta function of ω vanishes, so have a line of RG fixed points labeled by (λ, ω).
- As without SUSY, many exact computations can be done at leading order in 1/N. The computations suggest a duality: (Jain et al)

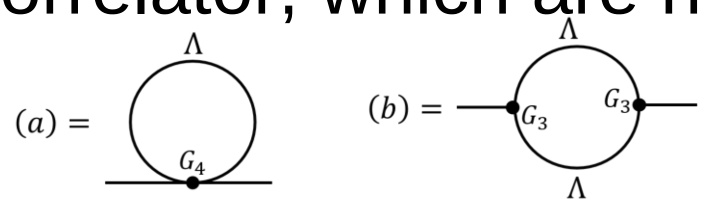
$$\lambda \to \lambda - sign(\lambda), \qquad \omega \to \frac{3-\omega}{1+\omega}, \qquad m \to -\frac{2m}{1+\omega}$$

where the last equation is the transformation of a SUSY mass parameter $\int d^3x\, d^2\theta\, m|\Phi|^2$.

- Are there fixed points, dualities for finite N ?

The beta function

- $S = L_{CS} + \int d^3x\, d^2\theta \left(-\frac{1}{2} D^\alpha \bar{\Phi} D_\alpha \Phi + \frac{\pi\omega}{\kappa} (|\Phi|^2)^2 \right)$
- For finite N cannot compute the beta function exactly. Hope to do it at leading order in 1/N.
- If we change variables to $J = |\Phi|^2$, then to do this we need to know $\langle JJ \rangle, \langle JJJ \rangle$ and $\langle JJJJ \rangle$ at leading order in 1/N, and some correlator at subleading order in 1/N. We computed at leading order $\langle JJ \rangle, \langle JJJ \rangle$, and $\langle JJJJ \rangle$ for collinear momenta. But need to know leading $\langle JJJJ \rangle$ for general momenta (bootstrap ?) and some subleading correlator, which are not yet known.

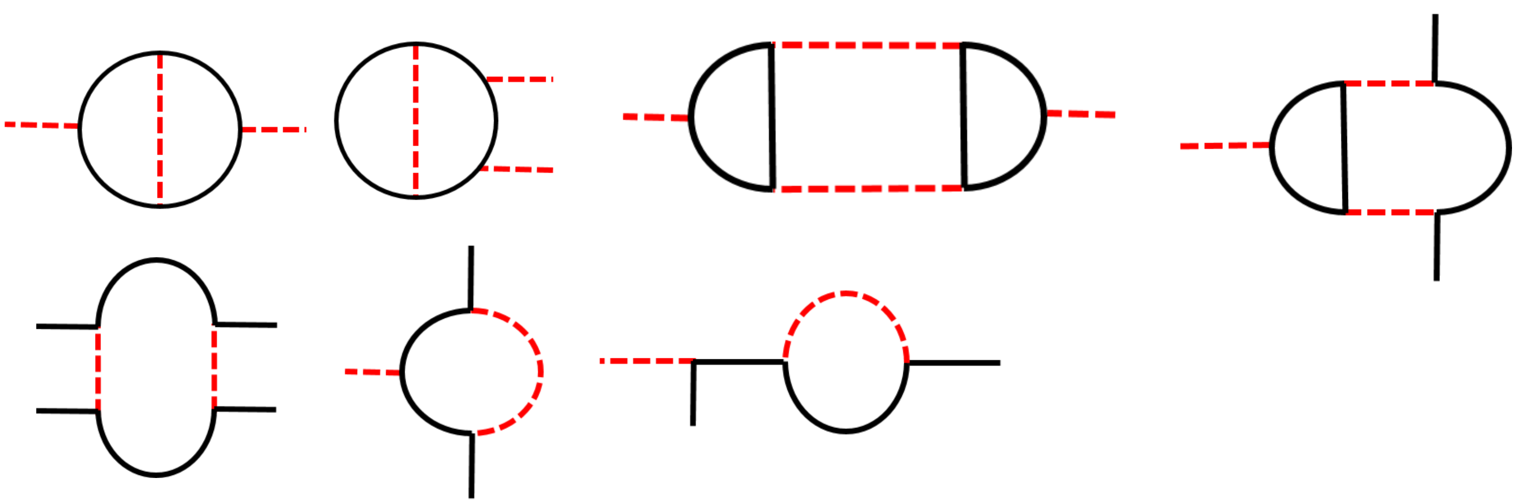


The beta function of Φ^4

- For $\lambda = 0$ we did this – should use $\widetilde{\omega} = \lambda\omega$,

$$S = \int d^3x\, d^2\theta \left(-\frac{1}{2} D^\alpha \overline{\Phi} D_\alpha \Phi + \frac{\widetilde{\omega}}{N} (|\Phi|^2)^2 \right)$$

- The contributing diagrams to $\langle \Phi^4 \rangle$ at order 1/N are (in superspace) :



The beta function of Φ^4

- $S = \int d^3x\, d^2\theta \left(-\frac{1}{2} D^\alpha \overline{\Phi} D_\alpha \Phi + \frac{\widetilde{\omega}}{N} (|\Phi|^2)^2 \right)$
- Summing all these diagrams we find

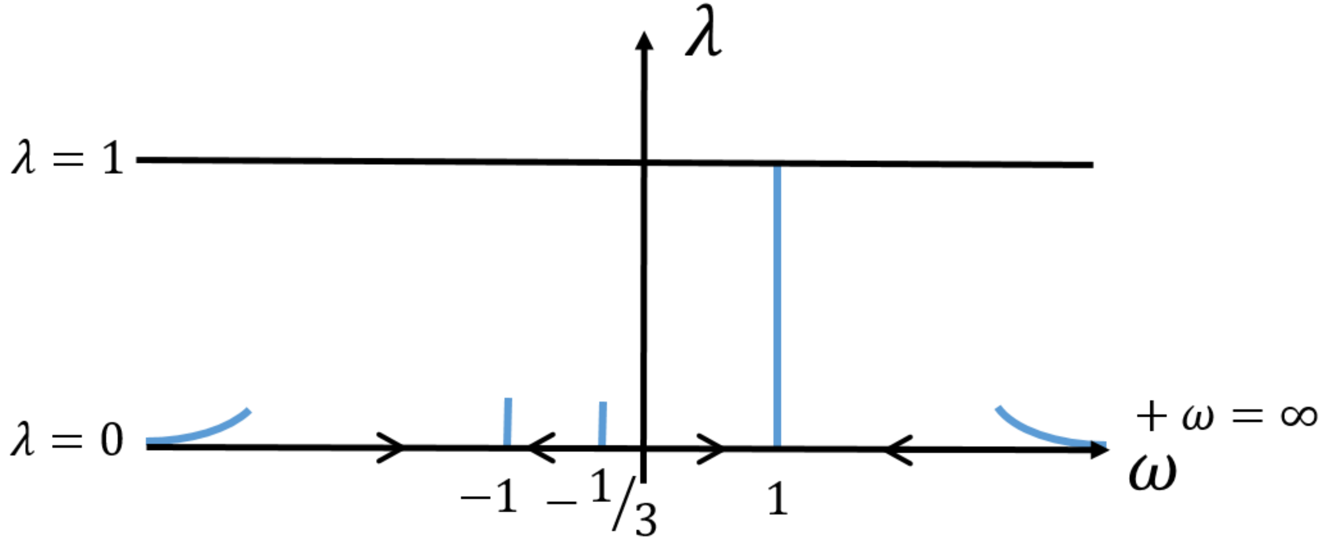
$$\beta(\widetilde{\omega}) = \frac{1}{N} \frac{16\, \widetilde{\omega}^3}{\pi^2} \frac{48 - \widetilde{\omega}^2}{(\widetilde{\omega}^2 + 16)^2} + O\left(\frac{1}{N^2}\right)$$

- A stable triple fixed point at $\widetilde{\omega} = 0$, two unstable ones at $\widetilde{\omega} = \pm\sqrt{48}$, and a stable one at $\widetilde{\omega} = \infty$.
- Naively have separate fixed points (related by parity) at $\widetilde{\omega} = \pm\infty$, but we claim they should be identified since at large $\widetilde{\omega}$ can rewrite interactions using an extra singlet superfield H as

$$S = \int d^3x\, d^2\theta \left(H|\Phi|^2 - \frac{N}{4\widetilde{\omega}} H^2 \right)$$

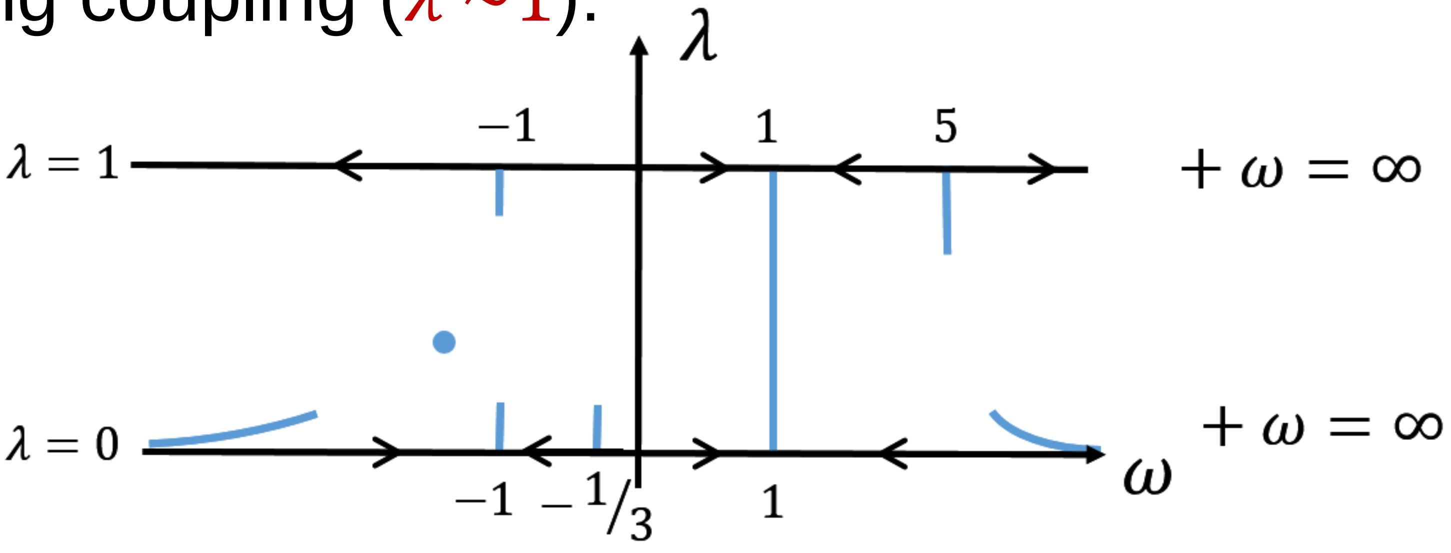
The fixed points

- $S = L_{CS} + \int d^3x \, d^2\theta \left(-\frac{1}{2} D^\alpha \bar{\Phi} D_\alpha \Phi + \frac{\pi\omega}{\kappa} (|\Phi|^2)^2 \right)$
- At weak coupling triple fixed point at $\widetilde{\omega} = 0$ splits into three fixed points with $\widetilde{\omega} \propto \lambda$, or finite ω :



The fixed points

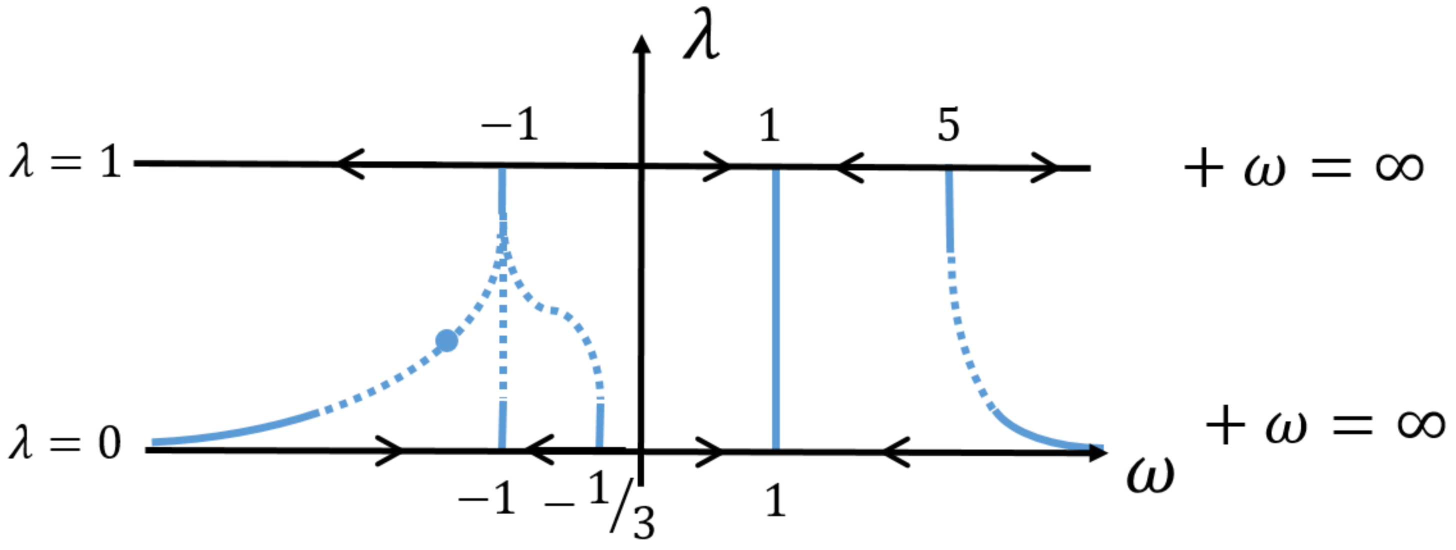
- $S = L_{CS} + \int d^3x\, d^2\theta \left(-\frac{1}{2} D^\alpha \bar{\Phi} D_\alpha \Phi + \frac{\pi\omega}{\kappa} (|\Phi|^2)^2\right)$
- Using the large N duality we can obtain from this the behavior of the fixed points also at strong coupling ($\lambda \sim 1$):



(Have self-dual fixed point with exact moduli space at $\lambda = \frac{1}{2}, \omega = -3$.)

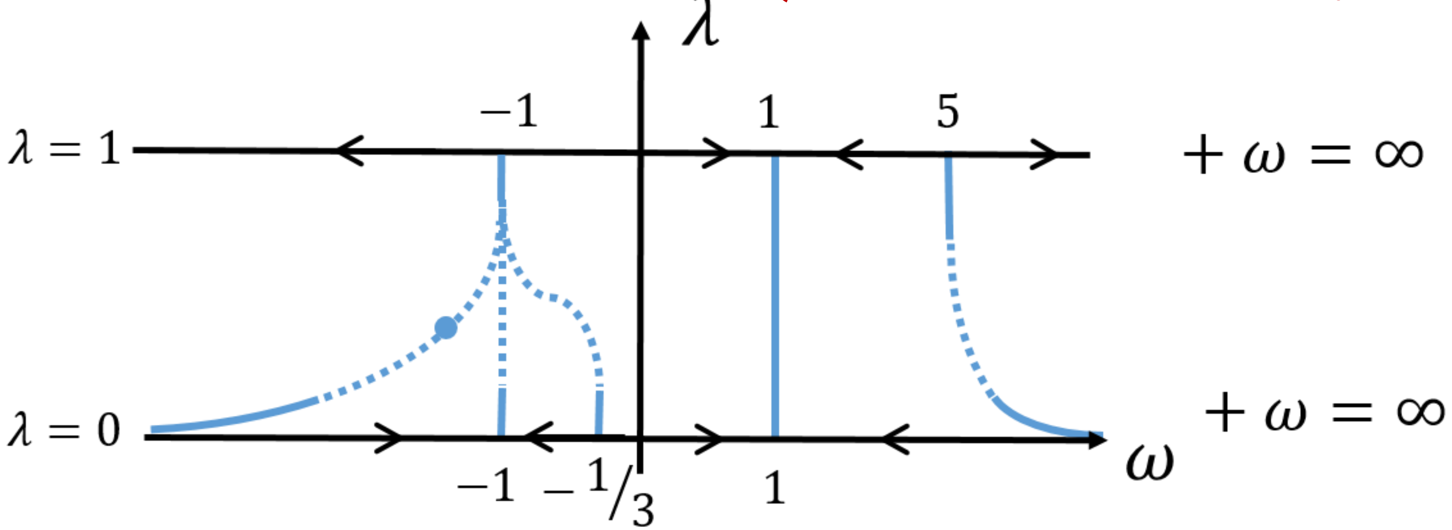
The fixed points

- $S = L_{CS} + \int d^3x\, d^2\theta \left(-\frac{1}{2} D^\alpha \Phi D_\alpha \Phi + \frac{\pi\omega}{\kappa}(|\Phi|^2)^2\right)$
- Using also the fact that number of zeros of $\beta(\omega)$ at order 1/N is bounded by six, obtain following conjecture for RG flows at large finite N :



The fixed points

- $S = L_{CS} + \int d^3x\, d^2\theta \left(-\frac{1}{2} D^\alpha \overline{\Phi} D_\alpha \Phi + \frac{\pi\omega}{\kappa} (|\Phi|^2)^2 \right)$



So at large finite N have 3 stable fixed points, the $\mathcal{N}$=2 point, one near $\omega = -1$ and one near $\omega = \infty$, and expect a duality exchanging the latter two

$$SU(N)_{k+\frac{N-1}{2}} + \Phi \leftrightarrow U(k)_{-N-\frac{k-1}{2},-N+1/2} + \Phi$$

Theories with more flavors

- When we have more flavors Φ_i ($i = 1, \cdots, N_f$) have 2 classically marginal couplings :

$$\int d^3x\, d^2\theta \left(\frac{\pi\omega_0}{\kappa} (\bar{\Phi}_i \Phi_i)^2 + \frac{\pi\omega_1}{\kappa} (\bar{\Phi}_i \Phi_j)(\bar{\Phi}_j \Phi_i) \right)$$

(3 ϕ^6 couplings without SUSY).

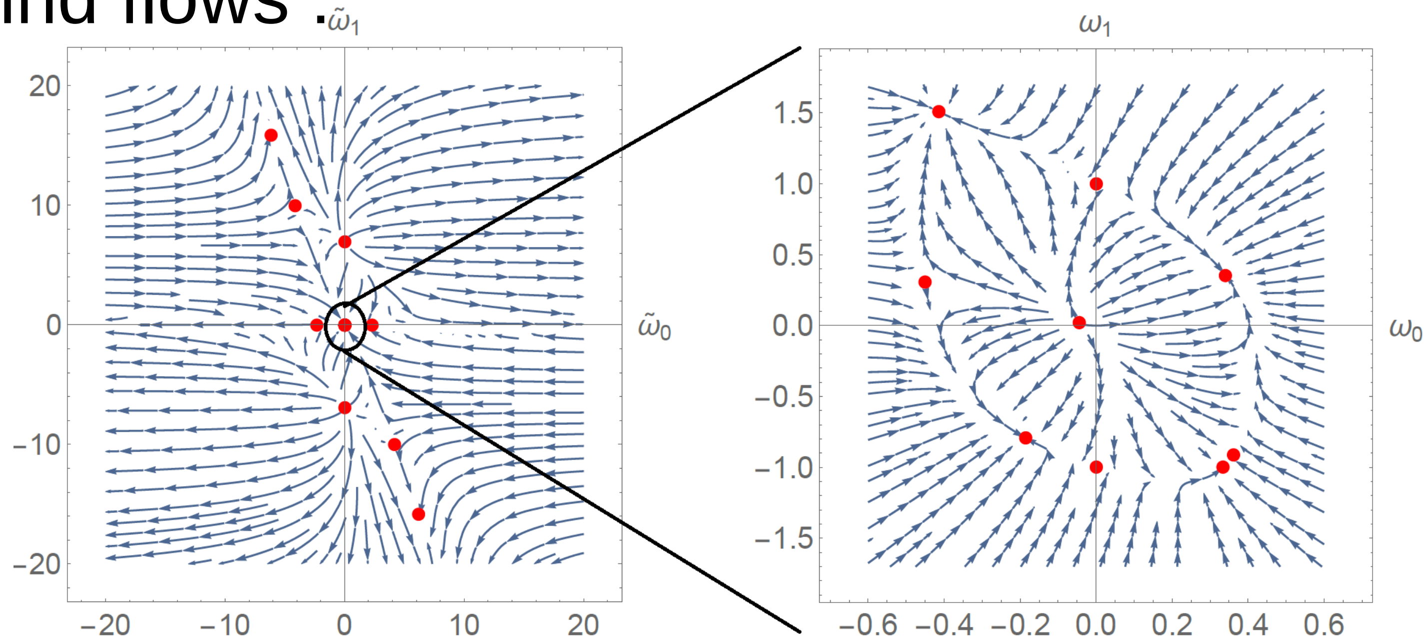
- For infinite N both couplings exactly marginal, can again compute correlators exactly, results suggest a duality:

$$\lambda \to \lambda - sign(\lambda), \omega_1 \to \frac{3-\omega_1}{1+\omega_1}, \omega_2 \to \frac{3-\omega_2}{1+\omega_2}$$

where $\omega_2 = N_f\, \omega_0 + \omega_1$.

Theories with more flavors

- Are there fixed points, dualities for finite N ?
- Can again compute just at weak CS coupling, there find flows :



- Have some IR-stable fixed points, conjectured to be dual. $\mathcal{N}$=2 point at $(\omega_1, \omega_0) = (1,0)$ unstable.

Theories with more flavors

- So conjecture various dualities of the form :

$$SU(N)_{k+\frac{N-N_f}{2}}+N_f\,\Phi \leftrightarrow U(k)_{-N-\frac{k-N_f}{2},-N+N_f/2}+N_f\,\Phi$$

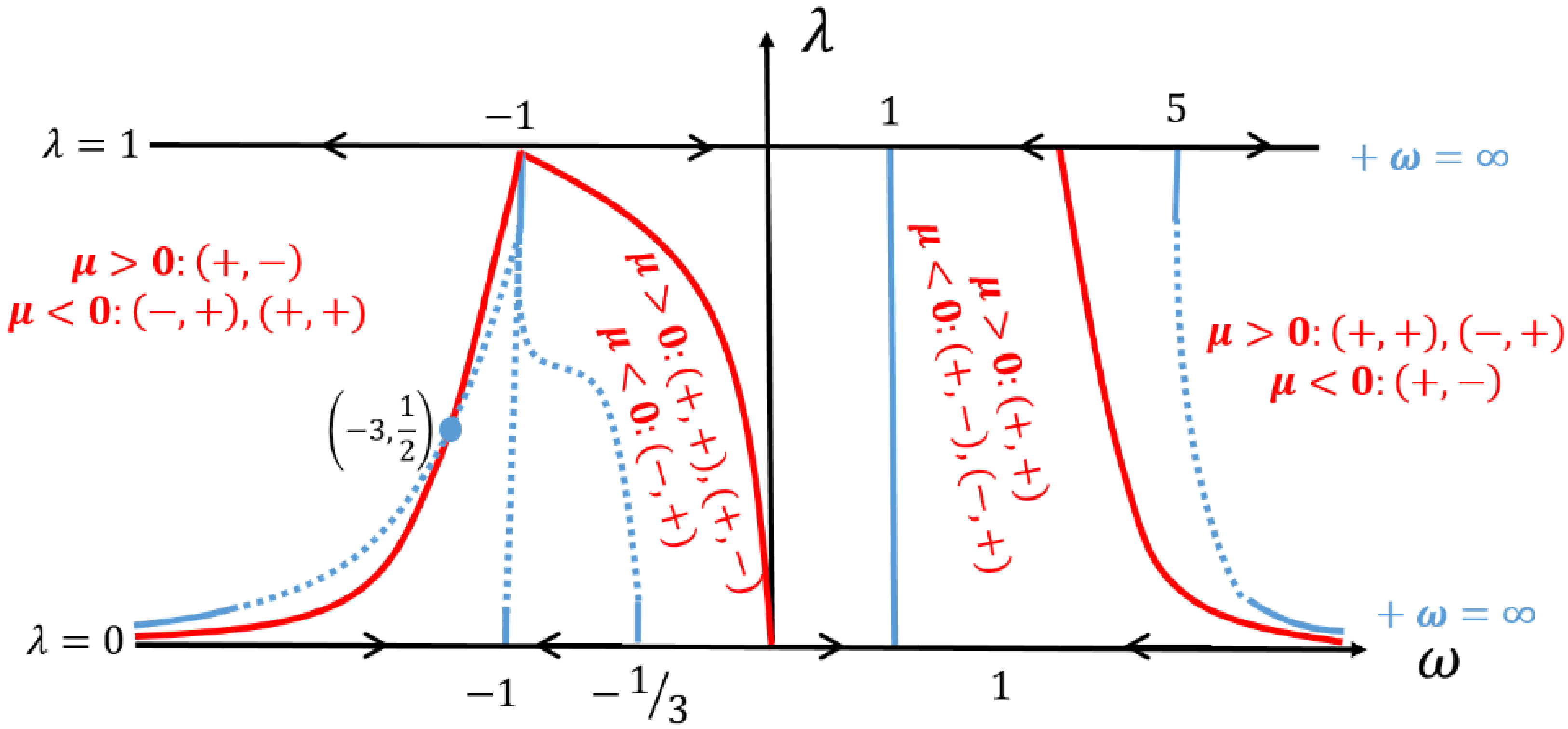
  between various values of (ω_1, ω_0).
- When these values are large, again more natural to describe by adding singlets, in singlet or adjoint of $SU(N_f)$ flavor group :

$$\int d^3x\, d^2\theta\, \left(H\left(\bar{\Phi}_i\Phi_j\right)+x_2H^2+H_{ij}\left(\bar{\Phi}_i\Phi_j\right)+x_1H_{ij}^2\right)$$

  with small (x_1, x_2).
- Interpolation to strong coupling complicated.
- $\lambda=\frac{1}{2}, (\omega_1, \omega_2)=(-3,-3)$ f.p. exact moduli space

Summary

- Analyzed possible fixed points of 3d $\mathcal{N}$=1 $SU(N)_k$ and $SU(N)_k$ with N_f flavors at large N. For $N_f = 1$ have 3 stable fixed points including $\mathcal{N}$=2 point. For $N_f > 1$ have more, $\mathcal{N}$=2 point unstable.
- Conjectured dualities between these fixed points, more precise version of duality conjectures previously made by Jain et al, Bashmakov et al, Benini et al, …
- At self-dual points – fixed points w/exact m.s.
- Also analyzed phase structure near fixed points – different for the 3 $N_f = 1$ fixed points.
- Want to know more about 1/N, finite N ! How ?
- Holography (one-loop in Vasiliev high-spin gravity)?


λ
λ = 1
−1
1
5
+ ω = ∞
μ > 0: (+, −)
μ < 0: (−, +), (+, +)
(−3, 1/2)
μ > 0: (+, +), (+, −)
μ < 0: (−, +)
μ > 0: (+, +)
μ < 0: (+, −), (−, +)
μ > 0: (+, +), (−, +)
μ < 0: (+, −)
+ ω = ∞
λ = 0
ω
−1
−1/3
1

Happy 80^{th} birthday Farhad !