Observables on de Sitter

N. C. Tsamis & R. P. Woodard

Λ-Driven Inflation

hep-ph/9602315, arXiv:1103.5134

- $\mathcal{L} = 1/16\pi G\,(R-2\Lambda)\sqrt{-g}$ for IR gravity
  - $G\Lambda \sim 10^{-10}$ NOT 10^{-122}
  - No scalars, no fine tuning, no special IC's
  - Λ starts inflation & QG back-reaction stops
- Mechanism of back-reaction
  - Inflation rips gravitons from vacuum
  - Their self-gravitation slows expansion
  - Gravity is weak ➔ long phase of inflation

Features of QG Back-Reaction

- It starts at 2 loops
  - Gravitons produced at 1 loop
  - Self-interactions require another loop
- It's slow

  $\varepsilon_L \sim \Lambda^2 \cdot (G\Lambda \ln[a(t)])^{L-1}$ with $a(t) = e^{Ht}$
- It's nearly (negative) vacuum energy
  - $d\varepsilon_L/dt = -3H(\varepsilon_L + p_L)$
  - $\ln[a(t)] = Ht \gg 1 \rightarrow |d\varepsilon_L/dt| \ll H|\varepsilon_L|$
  - Hence $p_L \sim -\varepsilon_L$

Tedious Arguments against QG Back-Reaction

- It's not causal
  - Factors of ln[a(t)] from past light-cone
- $R = 4\Lambda$ from the Einstein equations
  - Gravitational COLLAPSE obeys this too
- IR gravitons can't do anything
  - Small $\neq$ Zero and (big) x (small) $\neq$ small
- Effect must be self-limiting
  - Late effect from early times
- There ought to be a classical picture
  - There is!

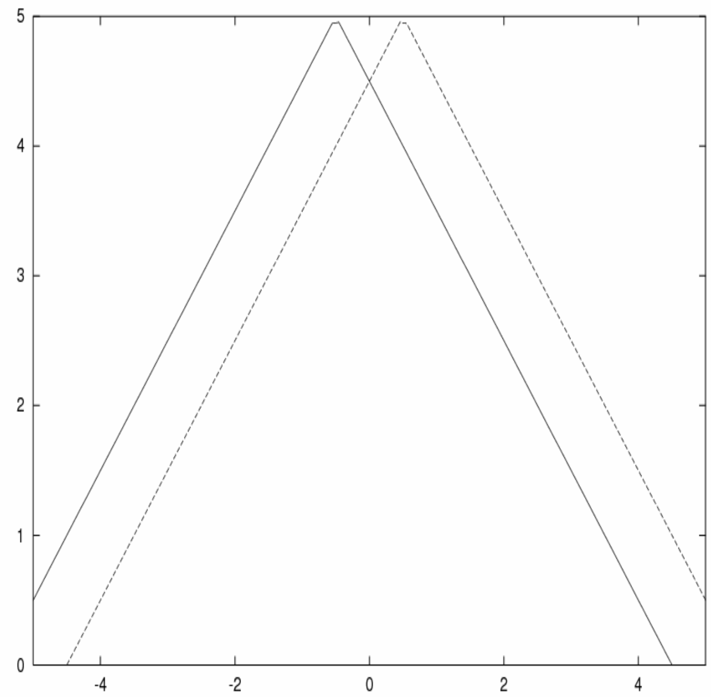
Worries about Fluctuations

No Large Spatial Fluct.

- Nearby points share most past L-Cone
- Especially far past!

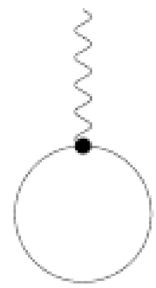
What about $\delta\varepsilon(t,x)$?

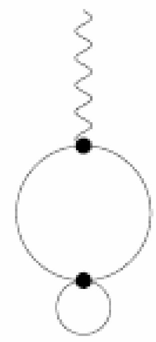
- Past light-cones not quite identical
- Small fluctuations in local H(t,x)



Back-Reaction CERTAIN in φ-Driven Inflation

- Inflation from $V_{eff} \neq V$
  - $\lambda\varphi^4 \rightarrow \lambda\varphi^4 + \lambda^2\varphi^4 \ln(\varphi)$
- Secular at 2 loops
- Can have either sign
  - Bosons: $+\lambda^2\varphi^4 \ln(\varphi)$
  - Fermions: $-\lambda^2\varphi^4 \ln(\varphi)$
- $\pm\lambda^2\varphi^4 f(\lambda\varphi/H)$ in de Sit.
  - Not even local generally
- No gauge issue





But we still have to prove it for Quantum Gravity

- Back-reaction requires ≥ 2 loops
  - Even 1 loop is tough in de Sitter QG!
- Real interest after perturbative regime
  - $\varepsilon_L \sim \Lambda^2[G\Lambda \ln(a)]^{L-1} \sim \Lambda^2$ for $\ln(a) \sim 1/G\Lambda$
  - Still « $\Lambda/8\pi G \sim \Lambda^2/G\Lambda$
- Thin edge: prove it perturbatively
- What constitutes a proof? (~90% political)
  - Critics tried $\langle g_{\mu\nu}(t,x)\rangle = \#\bar{g}_{\mu\nu}$ from dS invariance
  - But won't accept computing $\langle g_{\mu\nu}(t,x)\rangle$

Case of φ-Driven Inflation
(Geshnizjani & RHB: gr-qc/0204074)

- $\varphi(t,x) = \varphi_0(t) + \delta\varphi(t,x)$, $|\varphi_0| << |\delta\varphi|$
  - ➔ Gradient of φ is timelike
- $u_\mu[\varphi,g](t,x) = -\partial_\mu\varphi(t,x)/[-g^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi]^{1/2}$
- Fix surfaces of simultaneity with $T[\varphi](t,x)$
  - ➔ $\varphi(T(t,x),x) = \varphi_0(t)$
- $\mathcal{H}[\varphi,g](t,x) = 1/3\ D^\mu u_\mu(T,x) = H(t) + \ldots$
  - ➔ No secular back-reaction at one loop
- Different result for $\varphi \rightarrow \Phi$ (spectator scalar)
  - hep-th/0310265

What about Quantum Gravity?

- Same given scalar "clock" $\Phi[g](t,x)$
  - $u_\mu[g](t,x) = -\partial_\mu \Phi(t,x)/[-g^{\alpha\beta}\partial_\alpha \Phi \partial_\beta \Phi]^{1/2}$
  - $T[g](t,x)$ such that $\Phi[g](T,x) = \Phi[\bar{g}](t,x)$
  - $\mathcal{H}[g](t,x) = 1/(D-1)\, D^\mu u_\mu(T,x)$
- But what to use for $\Phi[g](t,x)$?
  - Must have timelike gradient
  - And $\mathcal{H}[g](t,x)$ must be UV & IR finite

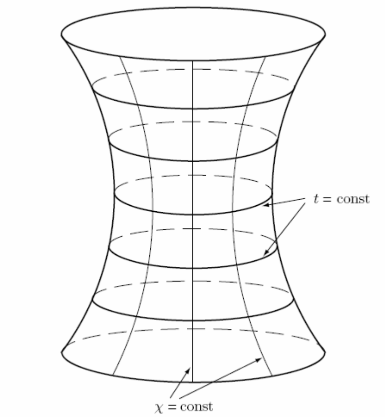
"Expansion" is ambiguous even for classical de Sitter!

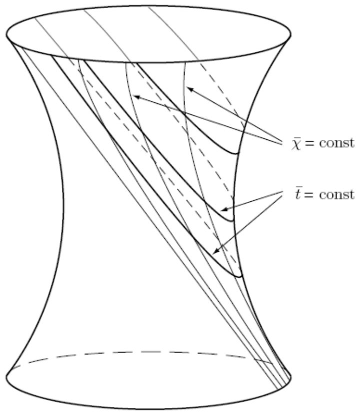
$ds^2 = -dt^2 + \cosh(Ht) d\chi^2$

- $\Phi(t,\chi) = t$
- $u_\mu = -\delta^0{}_\mu$
- $\mathcal{H} = H \tanh(Ht)$

$ds^2 = -dt^2 + e^{2Ht} d\chi^2$

- $\Phi(t,\chi) = t$
- $u_\mu = -\delta^0{}_\mu$
- $\mathcal{H} = H$





Our Solution: Exploit Causality & the Initial Value Surface

- $\mathcal{V}[g](x) = \int d^Dx' \sqrt{-g(x')}\, \Theta[-\ell^2[g](x;x')]$
  - New UV ∞'s from geodesic parameter ints
  - & $\langle h_{\mu\nu}(t',x)\, h_{\rho\sigma}(t'',x)\rangle \rightarrow \infty$ on light-cone
- $1/D_4$ for $D_4 = \Box^2 + 2D_\mu[R^{\mu\nu} - g^{\mu\nu}R/3]D_\nu$
  - Gives $\mathcal{V}[\bar{g}]/8\pi$ for ANY FRW
  - Ok, but complicated
- $-1/\Box$ for $\Box = 1/\sqrt{-g}\, \partial_\mu[\sqrt{-g}\, g^{\mu\nu}\partial_\nu]$
  - $\int_0^t dt'/a'^{D-1} \int_0^{t'} dt''\, a''^{D-1}$ monotonically increasing
  - Simple

Schwinger-Keldysh Realization
(For the experts)

- $\Phi[g] = -[(\Box^{-1})_{++} - (\Box^{-1})_{+-}]\ 1$
- $\Box = \Box_0 + \Delta\Box$

  $\Phi = \Phi_0 - [(\Box_0^{-1})_{++}(\Delta\Box)_+\Phi_0 - (\Box_0^{-1})_{+-}(\Delta\Box)_-\Phi_0] + \ldots$
- $g_{\mu\nu} = a^2[\eta_{\mu\nu} + h_{\mu\nu}]$
  - $\Box_0 = a^{-D}\partial_\mu[a^{D-2}\partial^\mu]$
  - $\Delta\Box = \frac{1}{2}\, a^{-2}h_{,\mu}\partial^\mu - a^{-D}\partial_\mu[a^{D-2}h^{\mu\nu}\partial_\nu] + O(h^2)$
- $\Phi_0(\eta) = H^{-2}/(D-1)\ [\ln(a) - (1-a^{-(D-1)})/(D-1)]$
  - $\Phi_0'(\eta) = aH^{-1}/(D-1)\ [1 - a^{-(D-1)}]$

Implementation at 1 Loop

$g_{\mu\nu} = a^2 [\eta_{\mu\nu} + h_{\mu\nu}]$

- $\Phi[g](\eta,x) = \Phi_0(\eta) + \Phi_1(\eta,x) + \Phi_2(\eta,x) + \ldots$
  - $\Phi_1(x) = i\int d^Dx'\, i\Delta(x;x') \{-\frac{1}{2} a^{D-2} h' \Phi_0' + \partial_\mu[h^\mu{}_0\, a^{D-2}\, \Phi_0']\}$
  - Φ_2 not needed
- $T[g](\eta,x) = \eta - \Phi_1(\eta,x)/\Phi_0'(\eta) + \ldots$
  - T_2 not needed because $\mathcal{H}_0 = H$
- $u_\mu[g] = -a([1-\frac{1}{2}h_{00}+\Phi_0'/\Phi_1'+\ldots]\delta_\mu{}^0 + \partial_\mu\Phi_1/\Phi_0' +\ldots)$
- $\mathcal{H}[g] = H + \mathcal{H}_1 + \mathcal{H}_2 + \ldots$
  - $\mathcal{H}_1 = \frac{1}{2}Hh_{00} + [\frac{1}{2}h_{ii}' - h_{0i,i} - \nabla^2\Phi_1/\Phi_0']/(D-1)a$
  - $\mathcal{H}_2 = 3/2\, H(h_{00})^2 + \ldots + h_{00,i}\, \partial_i\Phi_1/\Phi_0' + \ldots - \nabla^2\Phi_2/\Phi_0'$
  - $<\mathcal{H}_1>$ ➔ 1 h + 1 vertex (gr-qc/0506056)
  - $<\mathcal{H}_2>$ ➔ just propagators

Infrared & Ultraviolet

- Infrared
  - $R \times R^{D-1} \rightarrow R \times T^{D-1}$ & release at t=0
  - IR finite, but possibly secular
  - Must perturbatively correct initial state
- Ultraviolet
  - $\mathcal{H}[g](t,x)$ is local for $T[g](t,x) = t$ gauge
  - Likely (composite operator) renormalizable

Conclusions

- Λ-Driven Inflation would be great
  - Solves (old) Λ Problem
  - Provides unique model of inflation
- But must prove QG stops inflation!
  - First step: show it slows perturbatively
  - Need invariant measure of "expansion"
- Prediction: if we're right
  - Everyone will use nonivariant measures
  - And VERY dirty approximations

Our Proposal → Same form as φ-Driven Inflation

- $\Phi[g](t,x) \rightarrow T[g](t,x)$, $u_\mu[g](t,x)$ & $\mathcal{H}[g](t,x)$
  - $\Phi[g] = [(\square^{-1})_{++} - (\square^{-1})_{+-}]\ 1$
- Implementation at 1 loop
  - $\mathcal{H} = H + \mathcal{H}_1 + \mathcal{H}_2 + \ldots$
  - $\mathcal{H}_1 \rightarrow$ 1 vertex, $\mathcal{H}_2 \rightarrow$ just propagators
- Implementation at 2 loops
  - $\mathcal{H} = H + \mathcal{H}_1 + \mathcal{H}_2 + \mathcal{H}_3 + \mathcal{H}_4 + \ldots$
  - $\mathcal{H}_k \rightarrow (4 - k)$ vertices
  - Need state corrections at order h^3 & order h^4
- IR finite but perhaps secular at 2 loops
- UV renormalizable because gauge-local