Infrared Growth in Inflationary Space-Times and the Cosmological Renormalization Group Equation

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Motivations

- Stability of de Sitter/Cosmological constant problem?
- Eternal inflation/measure problem?
- The question of IR safe observables?
- Relation to black holes?
- Systematic understanding

➡ New insights/techniques

Stability of de Sitter

- The two-point function of a massless scalar field in de Sitter is IR divergent

$$\langle \phi^2(x) \rangle = U.V. + \frac{H^2}{4\pi^2} \ln(aH/L)$$

- The IR divergency has prompted many to believe that de Sitter could by itself be unstable
- Relaxation of the cosmological constant...?

[Polyakov, 1982, 2007; Tsamis and Woodard, 1993;...]

Growth of perturbations

- A decay of the cosmological constant is perhaps a too optimistic hope?
- But the growth of perturbations in de Sitter is puzzling!

➔ Is there a physical effect?

Relation to black holes

- Do we have any other indications of something special happening?
- If we drop something into a black hole, its information appears to be lost, but it must be emitted in the Hawking radiation to preserve unitarity
- On the other hand, nothing special happens to the observer falling through the horizon, so if the information carried by him is to be radiated out through the horizon, there is a problem with locality, since it requires spacelike communication of information

➡ Black hole information paradox

Relation to black holes

- It is an indication that a local description must fail on a time scale of order the black hole evaporation time

$$t_{ev} \sim R_{bh} S_{bh} \sim M^3$$

- This is the time scale at which information needs to start to coming out of the black hole

➡ Identifying a source for a perturbative breakdown would indicate no information paradox, only information problem [Giddings, 2007, 2009]

- Similarly one might also expect a breakdown of the perturbative approach in de Sitter on a time scale

$$t_{ds} \sim R_{ds} S_{ds} \sim 1/H^3$$

[Giddings, 2007; Arkani-Hamed et al., 2007]

Eternal Chaotic Inflation

- In chaotic inflation the inflating volume is typically described by a huge total number of e-foldings

➡ Correlation functions in principle plagued by large IR loop contributions

➡ We have to understand how to deal with IR loop contributions in cosmology!

IR safe observables

- We want to define late time cosmological observables independent of fluctuations on scales larger than the observed region
- In particular local late time observables should be independent of the IR cutoff (set by the total inflated volume).

➔ *IR safe observables*

[Giddings & MSS; Byrnes, Gerstenlauer, Hebeck, Nurmi & Tasinito; Urakawa & Tanaka;...]

[In the context of the measure problem, the same issue has been studied by Hartle, Hawking & Hertog; ...]

New insights/techniques

New techniques:

- We developped new semi-classical relations for studying IR loop corrections in de Sitter
- Found an RG equation relating observers with different survey volumes, integrating out IR modes

New insights:

- Better understanding of de Sitter geometry and its perturbative breakdown
- Understanding of how to construct IR safe observables
- A conceptually new way of detecting primordial tensor modes
- A relation between cosmological observables and perturbative breakdown of de Sitter via the RG equation

Outline

1. Semiclassical relations and IR divergences
2. Late-time IR-safe observables and RG equations
3. q-observables and fluctuating geomerty

I. Semiclassical relations and IR divergences

Semiclassical relations

- As an example, consider slow-roll inflation with

$$S = \frac{1}{2}\int \sqrt{-g}\left[\frac{R}{8\pi G} - \partial_\mu\phi\partial^\mu\phi - 2V(\phi)\right]$$

- A perturbative description of the coupled metric and matter perturbations can be derived using the ADM metric

$$ds^2 = -N^2dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

- Where we typically decompose the spatial metric in scalar and tensor perturbations

$$h_{ij} = a^2(t)e^{2\zeta(x,t)}[e^\gamma]_{ij}$$

- One can then define the mode functions, e.g.

$$\gamma_{ij}(x) = \sum_{s=+,\times}\int \frac{d^3k}{(2\pi)^3}\left[b^s_{\mathbf{k}}\epsilon^s_{ij}(\mathbf{k})\gamma_k(t) + b^{s\dagger}_{-\mathbf{k}}\epsilon^{s*}_{ij}(-\mathbf{k})\gamma^*_k(t)\right]e^{i\mathbf{k}\cdot\mathbf{x}} = \int \frac{d^3k}{(2\pi)^3}\gamma_{ij}(k;t,x)$$

Semiclassical relations

- It can be shown that on super horizon scales the occupation number of a given mode is large

$$n_k = \left(\frac{aH}{2k}\right)^2 >> 1 \qquad \text{for} \qquad k << aH$$

➔ the fluctuations becomes classical in accordance with the correspondence principle

- Accordingly the evolution of perturbations on super horizon scales must be entirely classical

Side note

- Classical non-perturbative conservation theorem for cosmological perturbations on large scales [i.e. Langlois and Vernizzi, 2005] are enough!

➡ No need for proving this at quantum level as recently discussed! [I.e. Senatore & Zaldarriaga III, 2012]

Semiclassical relations

- At horizon crossing, the spectrum is determined in terms of the physical momentum, computed by treating the longer-wavelength modes as providing a background metric
- It is convenient to define a *scale-dependent metric fluctuation* at some scale *q*

$$\Gamma_{ij}(q,t,x) = \int_{L^{-1}}^{q} \frac{d^3k}{(2\pi)^3} \left[2\zeta(k;t,x)\delta_{ij} + \gamma_{ij}(k;t,x)\right]$$

- Such that we can define a scale-dependent metric, including longer-wavelength fluctuations at scale *q*

$$h_{ij}(q;x,t) = a^2(t)[\exp\{\Gamma(q,x)\}]_{ij}$$

Semiclassical relations

- In terms of the scale-dependent metric at scale *q*, we can now also define a *scale-dependent physical momentum*

$$\kappa_{q,i}(k,x) = [e^{-\Gamma(q,x)/2}]_{ij} k_j$$

- ➔ IR corrections are incorporated into the spectrum, *P(k),* by writing the tree-level two-point function, $P_0(k)$, instead as a function of κ_k,

$$P(k,x)d^3k = P_0(\kappa_k(k,x))d^3\kappa_k$$

[Giddings & MSS (2010), (2011)]
[See also Hebecker et. al (2010), (2011)]

Example I:
Tensor perturbations in de Sitter

- Consider the two-point function of a free massless test scalar field in de Sitter

$$\langle \sigma_{\mathbf{k}_1} \sigma_{\mathbf{k}_2} \rangle$$

with the spatial metric on the form

$$h_{ij} = a^2 (e^{\gamma})_{ij} \qquad \begin{cases} \zeta = 0 \\ \Gamma_{ij}(q,t,x) = \int_{1/L}^{q} \frac{d^3k}{(2\pi)^3} \gamma_{ij}(k;t,x) \end{cases}$$

- In this case we can write the previous eq. as

$$\left\langle \sigma(\vec{k}) \sigma(\vec{k}') \right\rangle_{\bar{\gamma}} = \left\langle \sigma(e^{-\bar{\gamma}/2} \vec{k}) \sigma(e^{-\bar{\gamma}/2} \vec{k}') \right\rangle_0 = \langle \sigma(\vec{\kappa}) \sigma(\vec{\kappa}') \rangle_0$$

where we defined the local proper momentum

$$\vec{\kappa} = e^{-\bar{\gamma}/2} \vec{k}$$

- One can then Taylor expand the local proper momentum

$$k^2 \to \kappa^2 = k_i (e^{-\gamma})_{ij} k_j = k_i k_j - \gamma_{ij} k_i k_j + \frac{1}{2} \gamma_{il} \gamma_{lj} k_i k_j + \ldots$$

Tensor perturbations in de Sitter

- Then expanding the correlation function in the shifted local proper momentum:

[Giddings & MSS (2010)]
[See also Hebecker et. al (2010), (2011)]

$$\begin{aligned}\langle\sigma_{k_1}\sigma_{k_2}\rangle_\gamma &= \langle\sigma_{k_1}\sigma_{k_2}\rangle_0 \\ &+ \left(-\gamma_{ij}k_ik_j + \frac{1}{2}\gamma_{il}\gamma_{lj}k_ik_j\right)\frac{\partial}{\partial k^2}\langle\sigma_{k_1}\sigma_{k_2}\rangle\bigg|_0 + \frac{1}{2}(\gamma_{ij}k_ik_j)^2\left(\frac{\partial}{\partial k^2}\right)^2\langle\sigma_{k_1}\sigma_{k_2}\rangle\bigg|_0 + \ldots\end{aligned}$$

- Averaging over all soft graviton modes:

$$\begin{aligned}\left\langle\langle\sigma_{k_1}\sigma_{k_2}\rangle_\gamma\right\rangle &= \langle\sigma_{k_1}\sigma_{k_2}\rangle \\ &+ \frac{1}{2}k_ik_j\langle\gamma_{il}\gamma_{lj}\rangle\frac{\partial}{\partial k^2}\langle\sigma_{k_1}\sigma_{k_2}\rangle + \frac{1}{2}k_ik_jk_kk_l\langle\gamma_{ij}\gamma_{kl}\rangle\left(\frac{\partial}{\partial k^2}\right)^2\langle\sigma_{k_1}\sigma_{k_2}\rangle\ .\end{aligned}$$

➡ Equivalent to averaging over a "*large box*"

Tensor perturbations in de Sitter

- In order to evaluate the average over soft gravitons, expand

$$\gamma_{ij}(x) = \sum_{s=+,\times} \int \frac{d^3k}{(2\pi)^3} \left[b^s_{\mathbf{k}} \epsilon^s_{ij}(\mathbf{k}) \gamma_k(t) + b^{s\dagger}_{-\mathbf{k}} \epsilon^{s*}_{ij}(-\mathbf{k}) \gamma^*_k(t) \right] e^{i\mathbf{k}\cdot\mathbf{x}}$$

- The mode function is similar to the one of a free massless scalar in de Sitter

$$\gamma_k(\eta) = \frac{H}{\sqrt{k^3}}(1 + ik\eta)e^{-ik\eta}$$

- and the variance is similarly IR divergent

$$\langle \gamma^2(x) \rangle = \frac{1}{4} \langle \gamma_{ij}(x)\gamma_{ij}(x) \rangle \approx 2 \left(\frac{H}{2\pi} \right)^2 \int_{a_i H}^{a_* H} \frac{dq}{q} = -2 \left(\frac{H}{2\pi} \right)^2 \log(\Lambda_{IR}/a_* H)$$

Tensor perturbations in de Sitter

- Using

$$\frac{k_i k_j}{k^2}\langle\gamma_{il}\gamma_{lj}\rangle = \frac{4}{3}\langle\gamma^2(x)\rangle \qquad \frac{k_i k_j}{k^2}\frac{k_k k_l}{k^2}\langle\gamma_{ij}\gamma_{kl}\rangle = \frac{8}{15}\langle\gamma^2(x)\rangle$$

- The average becomes

$$\left\langle\langle\sigma_{k_1}\sigma_{k_2}\rangle_\gamma\right\rangle = \left\{1+\frac{2}{3}\langle\gamma^2(x)\rangle_*\left[\frac{2}{5}k^4\left(\frac{\partial}{\partial k^2}\right)^2 + k^2\frac{\partial}{\partial k^2}\right]\right\}\langle\sigma_{k_1}\sigma_{k_2}\rangle$$

- Contains terms prop. to variance, which grows large
- But, using

$$\langle\sigma_{k_1}\sigma_{k_2}\rangle \approx (2\pi)^3\delta^3(k_1+k_2)\frac{H^2}{2k^3}$$

[Giddings & MSS (2010)]

➡ The average vanishes in *pure* de Sitter

Tensor perturbations in de Sitter

Corollary:

- No breaking of scale invariance in the correlation function

➡ These one loop IR effects cancels

Examples:

- Consider non-scale invariant two-point function

$$\langle \dot{\sigma}_{k_1} \dot{\sigma}_{k_2} \rangle \approx (2\pi)^3 \delta^3(k_1 + k_2) \frac{H^4 \eta^4}{2} k$$

➡ non-vanishing IR divergent contribution

$$\left\langle \langle \dot{\sigma}_{k_1} \dot{\sigma}_{k_2} \rangle_\gamma \right\rangle = \langle \dot{\sigma}_{k_1} \dot{\sigma}_{k_2} \rangle \left[1 + \frac{4}{15} \langle \gamma^2(x) \rangle_* \right]$$

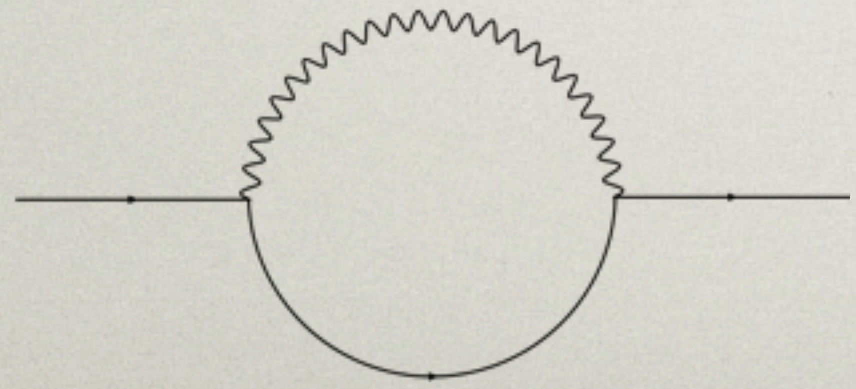
- Similarly one could considered $\langle \gamma_{k_1} \gamma_{k_2} \gamma_{k_3} \rangle$

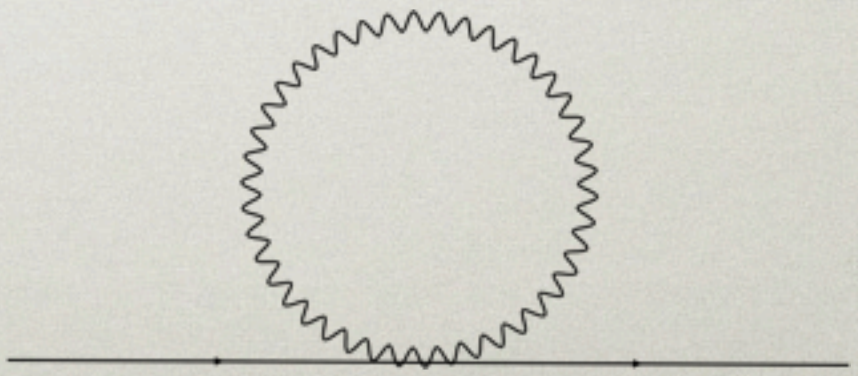
and it would have large IR corrections in pure de Sitter

Consistency Check

Full *in-in* calculation:

- As a check, we can calculate $\langle \sigma_{k_1} \sigma_{k_2} \rangle$ and $\langle \dot{\sigma}_{k_1} \dot{\sigma}_{k_2} \rangle$ using the full *in-in* QFT approach in the "*large box*"
- The contributing diagrams are





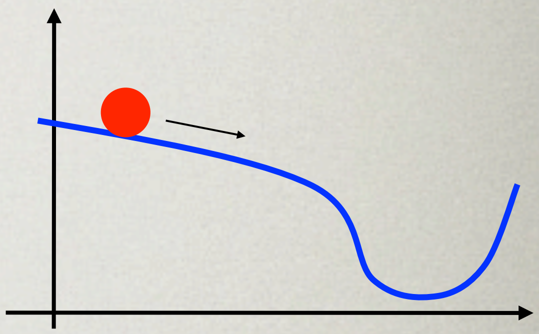
Consistency Check

- It can be evaluated efficiently using the *"Cosmological Diagrammatic rules"*

[Giddings & MSS, arXiv:1005.3287]

➡ The IR limit of the full *in-in* calculation reproduces exactly the results of the semiclassical relations (even for each single diagram)

Example II: Slow-roll



- Quasi de Sitter: time-translation invariance broken by the slow-rolling of a scalar

$$S = \frac{1}{2}\int \sqrt{-g}\,[R - \partial_\mu\phi\partial^\mu\phi - 2V(\phi)]$$

- Symmetry breaking parametrized by slow-roll parameters

$$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2 \quad , \quad \eta = \frac{V''}{V}$$

➡ Spectrum of perturbations not scale invariant

$$\langle \zeta_{k_1}\zeta_{k_2}\rangle = \delta(\mathbf{k}_1+\mathbf{k}_2)\frac{1}{2\epsilon}\frac{H^2}{2k^3}\left(\frac{k}{aH}\right)^{n_s-1} \quad , \qquad n_s - 1 = 2\eta - 6\epsilon$$

➡ Large IR effects...?

Tensor loops

- Expanding again on a background of soft gravitons

$$\langle\zeta_{k_1}\zeta_{k_2}\rangle_\gamma = \langle\zeta_{k_1}\zeta_{k_2}\rangle_0 + \left(-\gamma_{ij}k_ik_j + \frac{1}{2}\gamma_{il}\gamma_{lj}k_ik_j\right)\frac{\partial}{\partial k^2}\langle\zeta_{k_1}\zeta_{k_2}\rangle\Big|_0 + \frac{1}{2}(\gamma_{ij}k_ik_j)^2\left(\frac{\partial}{\partial k^2}\right)^2\langle\zeta_{k_1}\zeta_{k_2}\rangle\Big|_0 + \dots$$

- and averaging over all modes in the "large box"

$$\left\langle\langle\zeta_{k_1}\zeta_{k_2}\rangle_\gamma\right\rangle = \left\{1 + \frac{2}{3}\langle\gamma^2(x)\rangle_*\left[\frac{2}{5}k^4\left(\frac{\partial}{\partial k^2}\right)^2 + k^2\frac{\partial}{\partial k^2}\right]\right\}\langle\zeta_{k_1}\zeta_{k_2}\rangle$$

- we now obtain a IR divergent result

$$\left\langle\langle\zeta_{k_1}\zeta_{k_2}\rangle_\gamma\right\rangle = \left[1 + \frac{n_s-4}{3}\frac{n_s-1}{5}\langle\gamma^2(x)\rangle_*\right]\langle\zeta_{k_1}\zeta_{k_2}\rangle$$

[Giddings & MSS (2010)]

- where again

$$\langle\gamma^2(x)\rangle \approx 2\left(\frac{H}{2\pi}\right)^2\int_{a_iH}^{a_*H}\frac{dq}{q} = -2\left(\frac{H}{2\pi}\right)^2\log(\Lambda_{IR}/a_*H)$$

Tensor loops

- Similarly one can easily compute the IR effect of tensors on tensors

$$\left\langle \langle \gamma_{k_1} \gamma_{k_2} \rangle_\gamma \right\rangle = \left[1 + \frac{n_t - 3}{3} \frac{n_t}{5} \left\langle \gamma^2(x) \right\rangle_* \right] \langle \gamma_{k_1} \gamma_{k_2} \rangle$$

Scalar loops

- Long wavelength background scalar mode, $\bar{\zeta}$, shifts the momentum $k^2 \to k_{\bar{\zeta}}^2 = (e^{-\bar{\zeta}}k)^2$
- Expanding on the shifted momentum yields

$$\begin{aligned}\langle \zeta_{k_1}\zeta_{k_2}\rangle_{\bar{\zeta}} &= \left[1 + \bar{\zeta}\frac{\partial}{\partial\bar{\zeta}} + \frac{1}{2}\bar{\zeta}^2\frac{\partial^2}{\partial\bar{\zeta}^2} + \cdots\right]\left[e^{-6\bar{\zeta}}\langle\zeta(e^{-\bar{\zeta}}k_1)\zeta(e^{-\bar{\zeta}}k_2)\rangle\right] \\ &= \langle \zeta_{k_1}\zeta_{k_2}\rangle_0 - (n_s - 1)\,\bar{\zeta}\,\langle \zeta_{k_1}\zeta_{k_2}\rangle\big|_0 + \left(\frac{1}{2}(n_s-1)^2 + \alpha_s\right)\bar{\zeta}\bar{\zeta}\,\langle \zeta_{k_1}\zeta_{k_2}\rangle\big|_0 + \ldots\end{aligned}$$

$\alpha_s = dn_s/d\ln(k)$

- and averaging over soft scalar modes in the "large box" gives

$$\Big\langle \langle \zeta_{k_1}\zeta_{k_2}\rangle_{\bar{\zeta}} \Big\rangle \simeq \langle \zeta_{k_1}\zeta_{k_2}\rangle_0 + \left(\frac{1}{2}(n_s-1)^2 + \alpha_s\right)\langle \zeta_{k_1}\zeta_{k_2}\rangle_0 \left\langle \zeta^2(x)\right\rangle_*$$

- where as usual, the variance diverge in the IR

[Giddings & MSS (2010)]

$$\left\langle \zeta^2(x)\right\rangle_* \approx \frac{1}{2\epsilon}\frac{H^2}{(2\pi)^2}\int_{a_i H}^{a_* H}\frac{dq}{q} = -\frac{1}{2\epsilon}\frac{H^2}{(2\pi)^2}\log(\Lambda_{IR}/a_* H)$$

Scalar loops

- In this way, we can similarly compute the scalar IR loop correction to the tensors

$$\left\langle \langle \gamma_{k_1} \gamma_{k_2} \rangle_{\bar{\zeta}} \right\rangle \simeq \langle \gamma_{k_1} \gamma_{k_2} \rangle_0 + \left(\frac{1}{2} (n_t)^2 + \alpha_t \right) \langle \gamma_{k_1} \gamma_{k_2} \rangle_0 \left\langle \zeta^2(x) \right\rangle_*$$

$$n_t = -2\epsilon \qquad \alpha_t = dn_t / d\ln(k)$$

[Giddings & MSS (2010)]

Issues with the perturbative expansion

- The variance grows like

$$\langle \gamma^2(x) \rangle \sim H^2 \log(a) \sim H^3 t$$

- So it becomes order one on a time scale given by

$$t \sim 1/H^3$$

➡ We enter a non-perturbative regime on a time-scale anticipated from the parallel with the understandings of black hole information problem!

II. Late-time IR-safe observables and RG equations

Local interpretation

- We have seen that IR effects, can be derived semiclasically by defining a scale-dependent metric fluctuation

$$\Gamma_{ij}(q,t,x) = \int_{L^{-1}}^{q} \frac{d^3k}{(2\pi)^3}\left[2\zeta(k;t,x)\delta_{ij} + \gamma_{ij}(k;t,x)\right]$$

- In terms of which, we can define a *scale-dependent physical momentum*

$$\kappa_{q,i}(k,x) = [e^{-\Gamma(q,x)/2}]_{ij}k_j$$

- And scale-dependent metric

$$h_{ij}(q;x,t) = a^2(t)[\exp\{\Gamma(q,x)\}]_{ij}$$

➔ IR corrections are incorporated into the spectrum by writing the tree-level two-point function as a function of κ,

$$P(k,x)d^3k = P_0(\kappa_k(k,x))d^3\kappa_k$$

[Giddings & MSS (2010), (2011)]
[See also Hebecker et. al (2010), (2011)]

Local interpretation

- Thinking of our observable universe as a small box of size *1/q* ^{0}, we can locally absorb constant long wavelength fluctuations in a coordinate transformation

$$\tilde{x}_i^q = [e^{\Gamma(q,x)/2}]_{ij} x_j$$

making the metric locally flat on scales comparable to *1/q* $_0$

- Thus the physical momentum in our small box is

$$p_{0,i} = [\exp\{-\Gamma(q_0, x_0)/2\}]_{ij} k_j / a(t_0)$$

➔ The *scale-dependent physical momentum* in the small box becomes

$$\kappa_k(k,x)^2 = a_0^2 [e^{\Gamma(q_0,t_0,x_0)-\Gamma(a_0 p_0,t_r,x)}]_{ij} p_{0,i} p_{0,j}$$

- And we can compute the spectrum like before

$$P(p_0) d^3 p_0 = P_0 \left[a_0 \left([e^{\Gamma(q_0,t_0,x_0)-\Gamma(a_0 p_0,t_r,x)}]_{ij} p_{0,i} p_{0,j} \right)^{1/2} \right] d^3 \kappa_k$$

IR safety

Working in terms of the observer's local physical momentum p_0 , where wavelength longer than $1/q_0$ are *scaled out*, the observed spectrum depends on

$$\Gamma_{0,ij}(a_0 p_0, t_0, x) = \Gamma_{ij}(a_0 p_0, t_r, x) - \Gamma_{ij}(q_0, t_0, x_0)$$

which is *IR safe* – IR cutoff dependence is eliminated, and the observer's horizon size $1/q_0$ instead functions as an IR cutoff

[Giddings & MSS, 2011]

Observing
the beginning of the end

- This doesn't imply that there are no IR large effects
  - For sufficiently large hierarchy between the scale p_0 of the observed fluctuation, and the horizon scale q_0, there can be large effects
- The size of the spectral distortion can be estimated by Taylor expanding as usual

$$P_\zeta(p_0,x)=\left\{P_{\zeta 0} + \left[-\Gamma_{0,ij}(k,x)+\frac{1}{2}\Gamma_{0,il}(k,x)\Gamma_{0,lj}(k,x)\right]k_ik_j\frac{\partial P_{\zeta 0}}{\partial k^2} + \frac{1}{2}[\Gamma_{0,ij}(k,x)k_ik_j]^2\left(\frac{\partial}{\partial k^2}\right)^2 P_{\zeta 0}+\cdots\right\}\Bigg|_{k=a_0p_0} a_0^3 e^{-Tr\Gamma_0(a_0p_0,x)/2}$$

- The linear term is the largest, and will lead a new type of statistical inhomogeneities/ anisotropies at short scales

Observing the beginning of the end

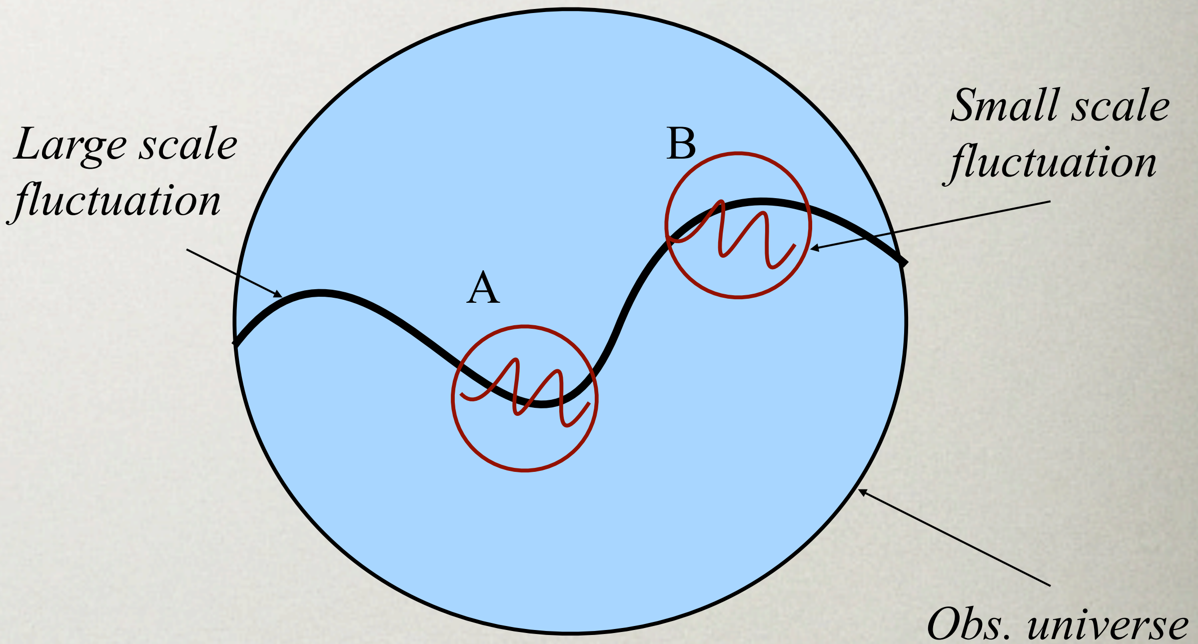
- Equivalently, in case of tensor corrections to the scaler spectrum, the previous general expression reduces to the expansion

$$\langle \zeta_{k_1} \zeta_{k_2} \rangle_\gamma = \langle \zeta_{k_1} \zeta_{k_2} \rangle_0 + \left(-\gamma_{ij} k_i k_j + \frac{1}{2} \gamma_{il} \gamma_{lj} k_i k_j \right) \frac{\partial}{\partial k^2} \langle \zeta_{k_1} \zeta_{k_2} \rangle \Big|_0 + \frac{1}{2} \left(\gamma_{ij} k_i k_j \right)^2 \left(\frac{\partial}{\partial k^2} \right)^2 \langle \zeta_{k_1} \zeta_{k_2} \rangle \Big|_0 + \dots$$

where the long wavelength tensor modes now only get contributions from modes smaller than the observable box, and we still didn't average over the observable box

Observing the beginning of the end

- The statistical inhomogeneities/anisotropies arises from mode-mode coupling at second order in perturbation theory
- If we measure the power of short wavelength modes, it will vary from place to place in the observable universe, due to longer wavelength modes



- The linear effect vanishes on average, but it's size can be estimated by computing the variance, which for tensor modes grows as

$$\langle \gamma^2 \rangle \approx 2 \left(\frac{H}{2\pi} \right)^2 \log[H(t_0)/p_0]$$

Observing the beginning of the end

- The effect is order 10^{-5} and might be observable in measurement 21cm emissions, which are predicted to probe *homogenous* anisotropy down to 10^{-7} level

[Pullen & Kamionkowski (2007)]

[See also Masui and Pen (2010)]

- This effect of tensor modes is imprinted on the scalar spectrum on all scales, while the effect of primordial tensors on *B*-modes relies on relatively large scales

➔ Could be a new way to probe primordial tensor modes

Side Note

- Compare with [Senatore &Zaldarriaga II, 2012]. In Abstract they claim:

> post-inflationary universe and will also change the time of horizon crossing of that mode. We argue that there are no infrared projection effects in physical questions, that there are no effects from modes of longer wavelength than the one of interest. These potential effects cancel when computing

- However later they explain:

> We emphasize again that this does not mean that this longer modes would not affect the measurements of a late time observer. Provided the modes are shorter than the horizon at the time of the observation they will contribute to the late time projection effects (*eg.* gravitational lensing)

➡It's a bit confusing! But the last point is exactly our point.

Cosmological RG equation

- Thinking of *q* as playing the role like a renormalization scale one can differentiate the equation

$$\kappa_{q,i}(k,x) = [e^{-\Gamma(q,x)/2}]_{ij} k_j$$

➔ to find

$$q\frac{\partial}{\partial q}\kappa_{q,i}(x) = -\frac{1}{2}\left(\frac{q}{2\pi}\right)^3 \int d^2\Omega_q \left[2\zeta(\mathbf{q};t,x)\delta_{ij} + \gamma_{ij}(\mathbf{q};t,x)\right]\kappa_{q,j}(x)$$

with the right hand side giving an analogue of the beta functions

- Similarly, differentiating the spectrum

$$P(p_0)d^3p_0 = P_0\left[a_0\left([e^{\Gamma(q_0,t_0,x_0)-\Gamma(a_0p_0,t_r,x)}]_{ij}p_{0,i}p_{0,j}\right)^{1/2}\right]d^3\kappa_k$$

gives an RG equation connecting *large box* and *small box* observables

$$q\frac{\partial}{\partial q}P = \left[q\partial_q\Gamma_{ij}(q,x)p_{0,i}p_{0,j}\frac{\partial}{\partial p_0^2} + \frac{1}{2}q\partial_q Tr\Gamma(q,x)\right]P$$

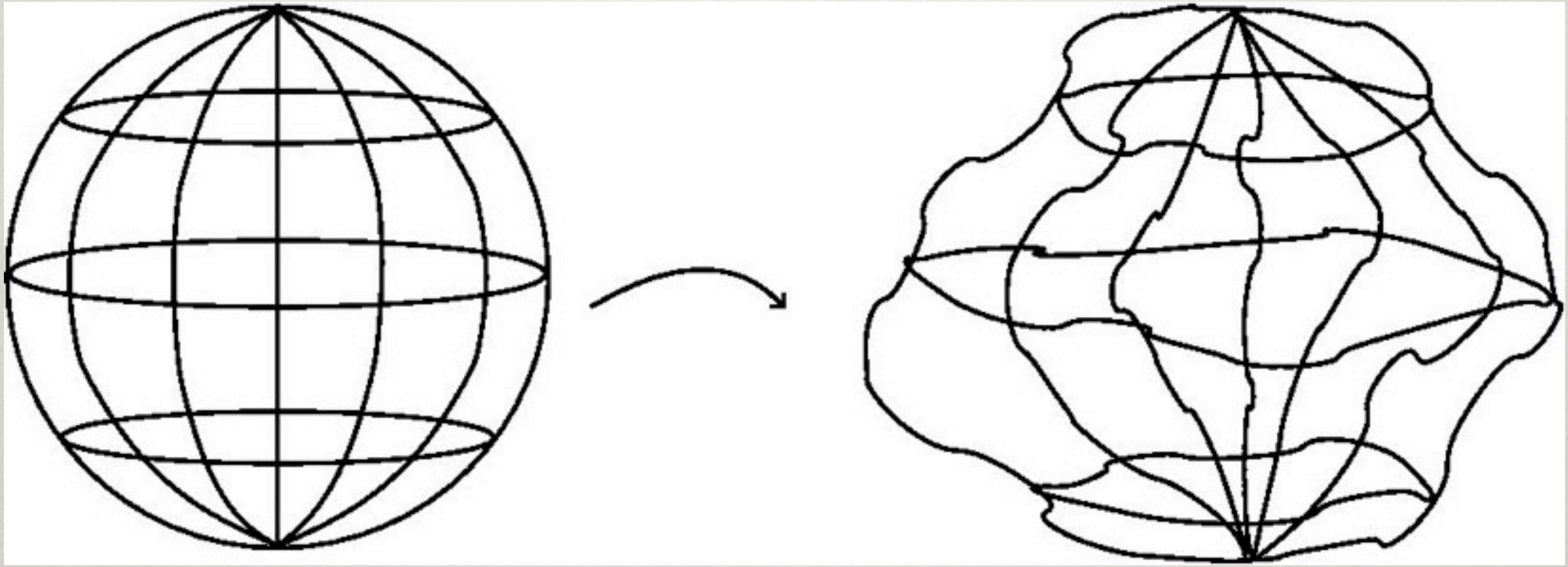
[Giddings & MSS, 2011]

Apparent Moral:

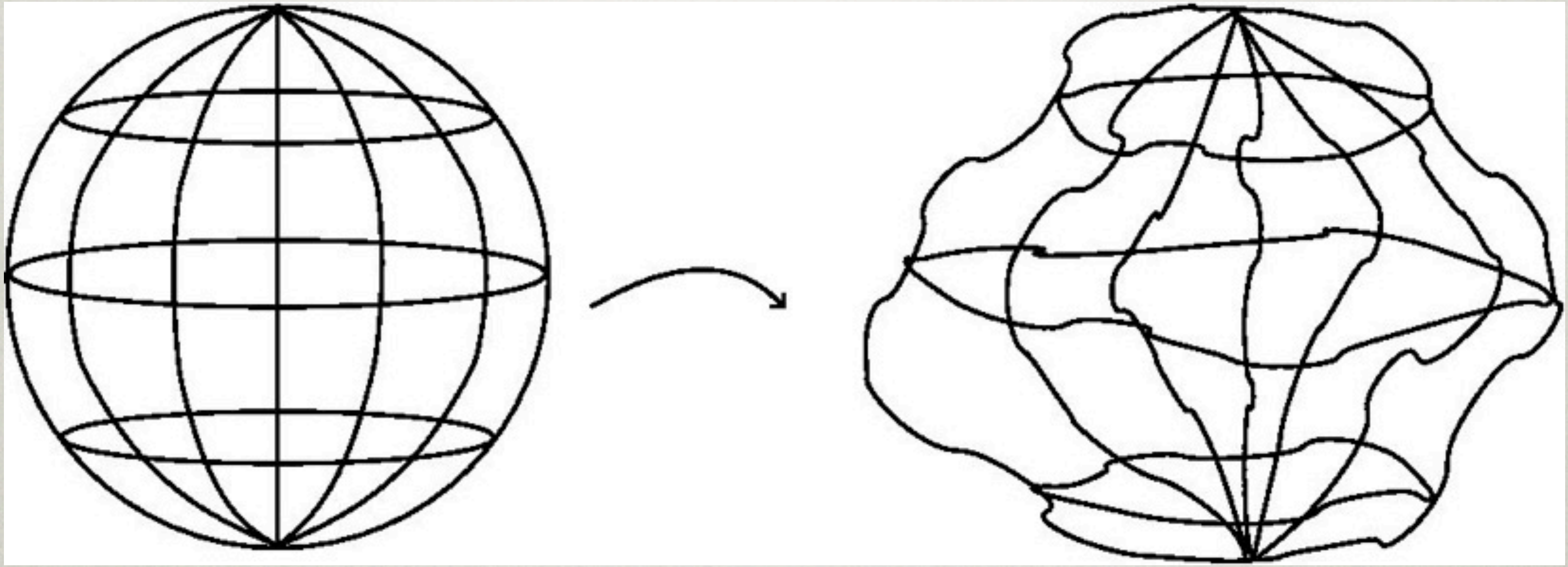
- For local observers: the large effects may be absorbed in the renomalization of locally defined physical momentum/coordinates
- but globally, effects can apparently not be eliminated

III. q-observables and fluctuating geometry

The picture so far...



The picture so far...



Is there a more direct way to verify that this is what happens to the geometry???

Q-observables via geodesic lengths

- It's hard to find gauge invariant local descriptions of the geometry
- We have seen some evidence already that there are physics in the long wavelength fluctuations, and they lead to some lumpiness of the geometry on large scales
- It's a little surprising when usually thinking that inflation redshift away all structure and makes space time smoother – but the no hair theorem of de Sitter is a statement about local physics!

➔ Are the some good "global" observables, which we can use to more directly describe the geometry?

Q-observables via geodesic lengths

- One might consider the geodesic distance between two freely falling particles (or "satellites")
  - ➔ Try to characterize fluctuations in the geometry in terms of the fluctuations in the distance between the satellites
  - ➔ More generally one can try to write correlators in terms of the geodesic distance

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- ➔ More generally one can try to write correlators in terms of the geodesic distance
- However, we know there are no null or time-like geodesics connecting points at large distances
- But even worse, there also no space-like geodesic connecting such points

Q-observables via geodesic lengths

- Instead, one might look for geodesics that lies in spatial slices of a particular time slicing
- This requires a clock
- As a clock one can use the local expansion in the volume element or the inflaton field in the case of inflation or other additional structure...

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Spatially flat gauge

$$\phi \equiv \phi(t) + \delta\phi(\mathbf{x}, t) \ , \qquad h_{ij} = a^2(t)(e^{\gamma})_{ij}$$

Comoving gauge

$$\phi \equiv \phi(t) \ , \qquad h_{ij} = a^2(t)e^{2\zeta}(e^{\gamma})_{ij}$$

Fluctuating line integrals

- The spatial distance between two points separated only in the x-coordinate is

$$\mathcal{S}(t) = \int_{X_1(t)}^{X_2(t)} \sqrt{h_{ij} dX^i dX^j} \ ,$$

with the zeroth order path

$$X_0^\mu = (0, x, 0, 0) \ .$$

- In the perturbed metric X_0^μ is no longer a geodesic, and the perturbed distance at second order is

$$\begin{aligned} \mathcal{S}[X_0 + \delta X, h_0 + \delta h] &= \mathcal{S}[X_0, h_0] + \frac{\delta \mathcal{S}}{\delta h}\delta h + \frac{\delta \mathcal{S}}{\delta X}\delta X \\ &+ \frac{1}{2}\frac{\delta^2 \mathcal{S}}{\delta h^2}\delta h^2 + \frac{1}{2}\frac{\delta^2 \mathcal{S}}{\delta X^2}\delta X^2 + \frac{\delta^2 \mathcal{S}}{\delta X \delta h}\delta X \delta h + \ldots \end{aligned}$$

- The condition that the perturbed path is a geodesic, gives the geodesic eq. for the perturbed path

$$0 = \frac{\delta \mathcal{S}}{\delta X}\bigg|_{\substack{X_0+\delta X, \\ h_0+\delta h}} = \frac{\delta^2 \mathcal{S}}{\delta X^2}\delta X^2 + \frac{\delta^2 \mathcal{S}}{\delta X \delta h}\delta X \delta h$$

Fluctuating line integrals

- The geodesic equation becomes

$$\partial_x^2 \delta X^\alpha = -\Gamma^\alpha_{xx} = a^{-2}(t) \left(-\partial_x \delta h_{x\alpha} + \frac{1}{2} \partial_\alpha \delta h_{xx} \right) , \qquad \alpha = y, z ,$$

- This can be solved subjected to right boundary conditions given by points fixed in the geometry (*e.g.* the position of two freely falling satellites)

Example:
Line integral of the torus

- In order to avoid any ambiguity in anchoring the end points of the geodesics in the geometry, we can consider
  lengths of the cycles of the torus

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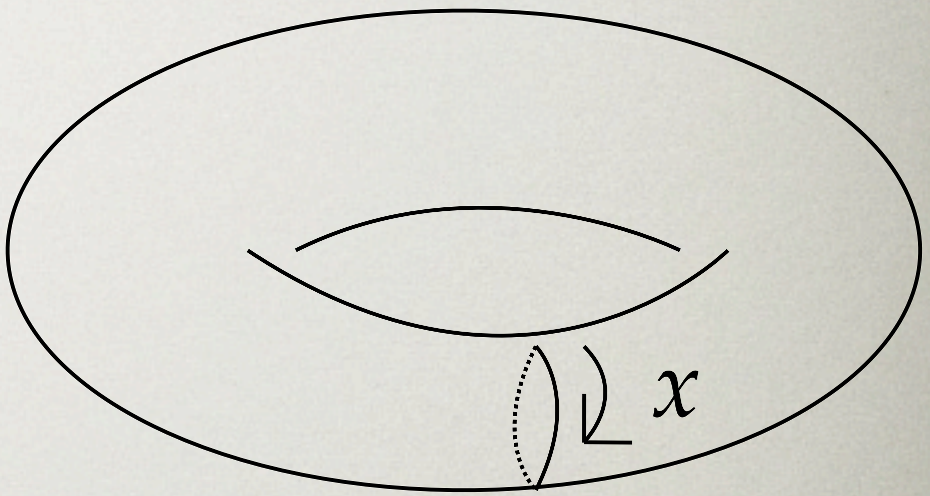
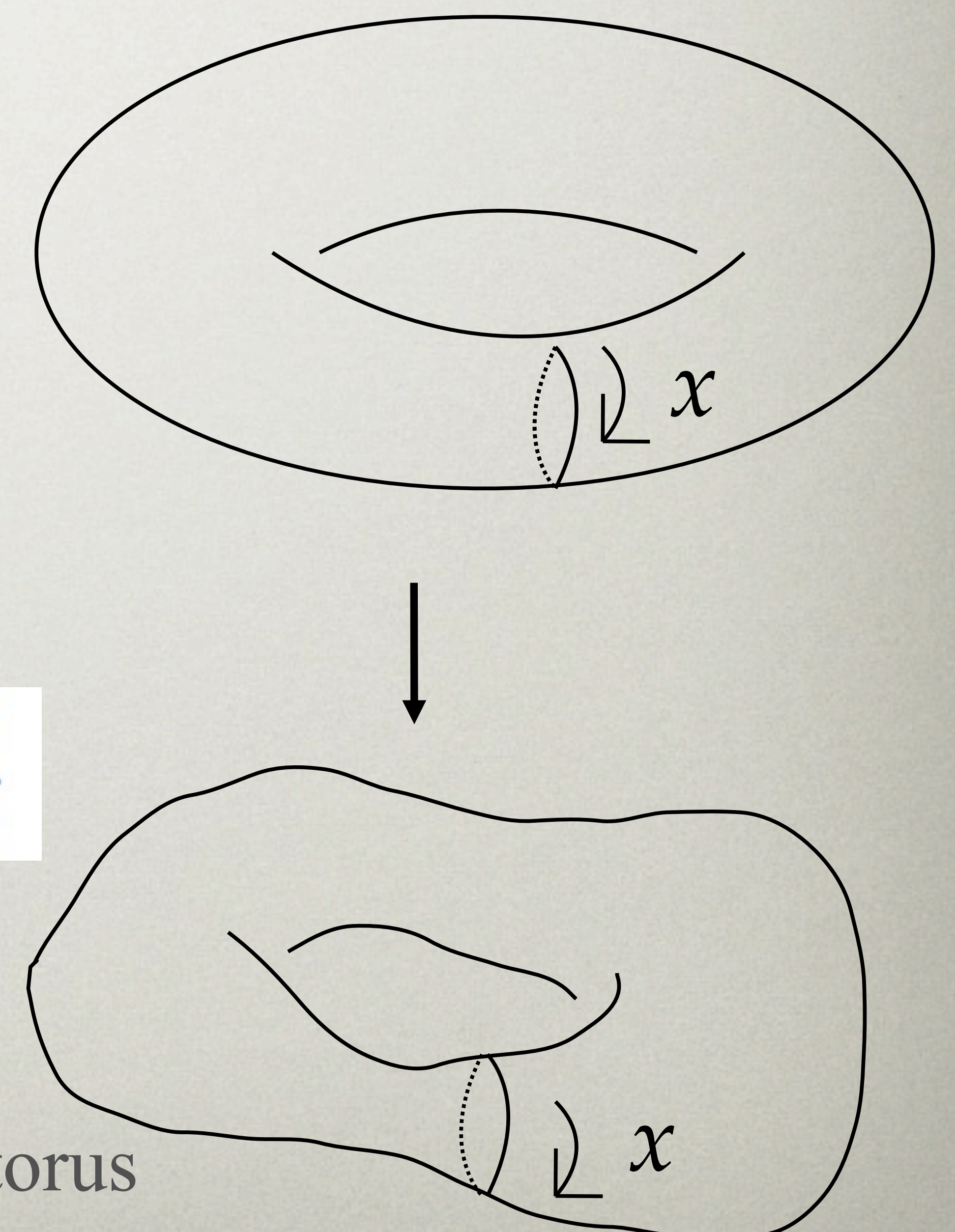
- In order to avoid any ambiguity in anchoring the end poins of the geodesics in the geometry, we can consider lengths of the cycles of the torus
- Naively one might expect the inflated torus to be very smooth



Example: Line integral of the torus

- In order to avoid any ambiguity in anchoring the end poins of the geodesics in the geometry, we can consider lengths of the cycles of the torus
- Naively one might expect the inflated torus to be very smooth
- But, consider the proper length around the torus along *x*-direction



$$\langle \mathcal{S} \rangle \approx a(t)L \left\{ 1 - \left[\frac{2}{15} + \frac{1}{2} a(t) L H \right] \langle \gamma^2(x) \rangle + \mathcal{O}\left(\langle \gamma^2(x) \rangle^2 \right) \right\}$$

- On average it gets a large contribution from the growing variance whose effect is to shorten the geodesic around the torus

[Giddings & MSS, 2011]

Comoving satellites

- It has been argued that if all correlation functions are written in terms of the local geodesic distance on the reheating surface, then there is no IR problem!

[Hebecker et. al (2010), (2011), Tanaka & Urakawa (2010), (2011)]

- This is not what an actual late time observer measures, but it might still define an interesting IR finite q-observable.
- However, the geodesic distance on the reheating surface between two comoving satellites at large separations also gets large corrections from the perturbations!

Comoving satellites

- To illustrate, lets compute the geodesic distance between two comoving satellites in the perturbed geometry
- Just like in the case of the torus, we need to perturbe the distance to second order – focussing for simplicity on scalar pert.

$$\langle \mathcal{S} \rangle = \langle \mathcal{S}_0 \rangle + \frac{1}{2} a(t) \int_0^L dx \, \langle \zeta^2 \rangle - \frac{a(t)}{2} \int_0^L dx \, \langle \partial_x \delta X_\alpha \partial_x \delta X_\alpha \rangle \ .$$

- Neglecting the perturbations of the path gives

$$\langle \mathcal{S} \rangle = a(t) L \left(1 + \frac{1}{2} \langle \zeta^2 \rangle + \ldots \right) \approx a L e^{\frac{1}{2} \langle \zeta^2 \rangle}$$

Comoving satellites

- Considering the usual two point function

$$\langle\zeta(0)\zeta(\mathbf{x})\rangle = \int \frac{d^3k}{(2\pi)^3}\frac{H^2}{4\epsilon k^3}\left(\frac{k}{aH}\right)^{n_s-1} e^{i\mathbf{k}\cdot\mathbf{x}} .$$

- One can then write instead in terms of the geodesic distance

$$\langle\zeta(0)\zeta(x(\langle\mathcal{S}\rangle))\rangle = \int \frac{d^3\tilde{k}}{(2\pi)^3}\frac{H^2}{4\epsilon\tilde{k}^3}\left(\frac{\tilde{k}}{aH}\right)^{n_s-1} e^{\frac{1}{2}(n_s-1)\langle\zeta^2\rangle} e^{i\tilde{k}_x\langle\mathcal{S}\rangle/a}$$

- Introducing a new explicit time dependence in the usual power spectrum after horizon crossing

$$P_\zeta(\tilde{k}) = P_\zeta^{(0)}(\tilde{k})\left[1+\frac{1}{2}(n_s-1)\left\langle\zeta^2\right\rangle+\ldots\right] ,$$

Side Note

- Compare with [Senatore &Zaldarriaga II, 2012]
- They define the physical volume of freely falling observers, and write spectrum in terms of the physical coordinates tracing out the volume

In fact it is equal to the amplitude of fluctuations that modes of a shorter scale would have in the absence of the effect we are considering. As a result the amplitude of fluctuations due to our effect is directly given by the tilt:

$$\delta \ln(\Delta^2) = \frac{3}{2}(n_s - 1)\Delta^2 N_e \ . \qquad (18)$$

- Factor 3 extra compared with our result is apparently from erroneously using

$$\langle a \rangle = a e^{\delta N} \qquad \Rightarrow \qquad \langle a^3 \rangle = a^3 e^{3\delta N}$$

- However $\langle X \rangle^3 \neq \langle X^3 \rangle$

➡ There is apparently no new effect!

(It's just a different way of writing the conserved spectrum as a sum of two non-conserved quantities)

Comoving satellites

- But remember this was neglecting the fluctuations of the geodesic path
  - Including the fluctuations of the path the line integral becomes

$$\langle S \rangle = a(t)L\left(1 - \frac{A}{36\pi}a(t)HL + \dots\right)$$

  - The corrections to the path length due to fluctuations rapidly becomes comparable to the length of the unperturbed path
  - This happens a few e-folds after the satellite separation has become larger than the horizon

$$a(t)L \sim \epsilon H^{-3}$$

Summary

- We found simple semiclassical relations for deriving the IR loop effects during inflation
- We checked the semiclassical relations with exact *in-in* calculations, which matches the results diagram for diagram
- We developed new *"Cosmological Diagrammatic rules"* that makes the *in-in* calculation more efficient.
- The IR effects can become large in the total inflated volume ("large vol.") in realistic models of inflation
- The time scale for corrections to become large in the "large vol." is $t \sim RS$, which coincides with the time scale on which one expects a breakdown of perturbative physics in the black hole context.

Summary

- We have showed how to define *IR safe* observables for late time *small box* observers like ourselves
- We have showed how it leads to a cosmological RG equation connecting different box sizes (e.g."large box" and "small box" observers)
- We have demonstrated how an observer today might be able to observe the beginning of the end of perturbative de Sitter imprinted in small statistical inhomogeneities/anisotropies at short scales
- Finally, we have found good indications that the geometry of the reheating surface grows very inhomogenous at late times, even within an initially smooth inflating pocket.