PHASE TRANSITIONS IN DE SITTER

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- Prokopec, arXiv:1110.3187 (2011)
- Janssen, Miao, Prokopec & Woodard, 0904.1151 [gr-qc] JCAP 0905 (2009) 003

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1) DYNAMICS OF SYMMETRY RESTORATION & INFLATIONARY INFRARED

2) MEAN FIELD FOR A REAL SCALAR FIELD

3) MEAN FIELD IN AN O(N) MODEL

4) BEYOND MEAN FIELD AND STOCHASTIC INFLATION

SOME LITERATURE:

Ford & Vilenkin, PRD26 (1982) & PRD33 (1986) 2833

Starobinsky, Yokoyama, PRD50 (1994) 6357 [astro-ph/9407016]

Burgess, Leblond, Holman, Shandera, JCAP 1003 (2010) 033 [arxiv:1005.3551]

Garbrecht & Rigopoulos, [1105.0418 [hep-th]]

Serreau, 1105.4539 [hep-th], PRL 2012

DE SITTER SPACE AND COSMIC INFLATION

- LINE ELEMENT (METRIC TENSOR):

$$ds^2 = -dt^2 + a^2(t)d\vec{x}^2 \quad or \quad g_{\mu\nu} = a^2(\eta)\eta_{\mu\nu}, \quad \eta_{\mu\nu} = diag(-1,\underbrace{1,1,..}_{D-1})$$

- FRIEDMANN (FLRW) EQUATION AND THE SCALE FACTOR

$$H^2 = \frac{\Lambda}{3} \quad \Rightarrow a(t) = a_0 \mathrm{e}^{Ht}, \quad H = \sqrt{\frac{\Lambda}{3}}$$

- More generally, for a power law expansion scale factor reads:

$$\mathrm{a} = \left(\frac{t}{t_0}\right)^{1/\varepsilon} = \left[-(1-\varepsilon)H_0\eta\right]^{-\frac{1}{1-\varepsilon}}, \quad H = H_0 a^{-\varepsilon}, \quad \varepsilon = -\frac{\dot{H}}{H^2} = \mathrm{const.}$$

- inflation: ε=constant<<1 or ε<<1 adiabatic in time (slow roll approx)

HUBBLE EFFECTIVE POTENTIAL

Janssen, Miao, Prokopec & Woodard, 0904.1151 [gr-qc] JCAP (2009)

- **SCALAR : BACKGROUND + FLUCTUATIONS:** $\varphi=\Phi(x)+\phi(x)$
- **EFFECTIVE ACTION:** $\Phi(x) \to \Phi_0, \Gamma_{eff} = V_{eff}[\Phi_0]$
- **IN COSMOLOGY: THE BACKGROUND IS EVOLVING!**

  $\Rightarrow$ **HUBBLE EFFECTIVE POTENTIAL:** $\Phi(x) \to \Phi_0 H(t), \Gamma_{eff} \to V_{hub}[\Phi,H]$

- **Classical equation of motion:**

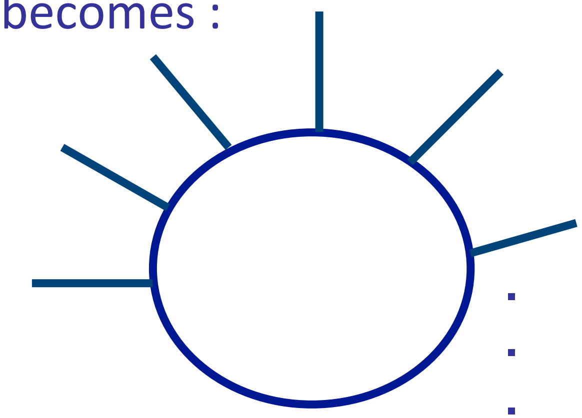
$$\ddot{\Phi} + 3H\dot{\Phi} + m^2\Phi + \xi R\Phi + \frac{\lambda}{6}\Phi^3 = 0$$

- in constant $\varepsilon=-(dH/dt)/H^2$=const. (flat FLRW) spaces this becomes :

$$2\varepsilon^2 - 3\varepsilon + 6\xi(2-\varepsilon) + \frac{\lambda}{6}\Phi_0^2 = 0 \quad (m=0)$$

$$\Rightarrow \Phi_0 = \Phi_0(\varepsilon,\lambda) = \text{const}$$

$\Rightarrow$ **ONE LOOP HUBBLE EFFECTIVE POTENTIAL:** $V_{hub}[\Phi]$



1 LOOP HUBBLE EFFECTIVE POTENTIAL

- **USE IR REGULATED PROPAGATOR (comoving box) FOR CONSTANT SPACES**

Janssen, Miao, Prokopec, Woodard (2008)

$$V'_{\rm hub}(\Phi,H)=\frac{\lambda}{96\pi^2}\left(1-5\varepsilon+3\varepsilon^2\right)H^2\Phi+\left\{\xi+\frac{\lambda(\xi-1/6)}{32\pi^2}\left[\ln\left(\frac{(1-\varepsilon)^2H^2}{\mu^2}\right)+\psi\left(\frac{1}{2}+\nu\right)\right]+\psi\left(N+\frac{5}{2}+\nu\right)\right\}R\Phi$$

$$+\left\{\frac{\lambda}{6}+\frac{\lambda^2}{64\pi^2}\left[\ln\left(\frac{(1-\varepsilon)^2H^2}{\mu^2}\right)+\psi\left(\frac{1}{2}+\nu\right)\right]+\psi\left(N+\frac{5}{2}+\nu\right)\right\}\Phi^3$$

$$+\frac{\left(1-\varepsilon^2\right)\lambda}{32\pi^2}\sum_{n=-1}^{N}\frac{1}{\nu-n-3/2}\left\{\frac{\Gamma(2\nu-n)\Gamma(2\nu-2n)}{2\Gamma(n+1)\Gamma^2(\nu-n+1/2)}\left(4k_0^2\eta^2\right)^{n-\nu+3/2}-\left(\nu^2-\frac{1}{4}\right)\right\}H^2\Phi+O(\hbar^2)$$

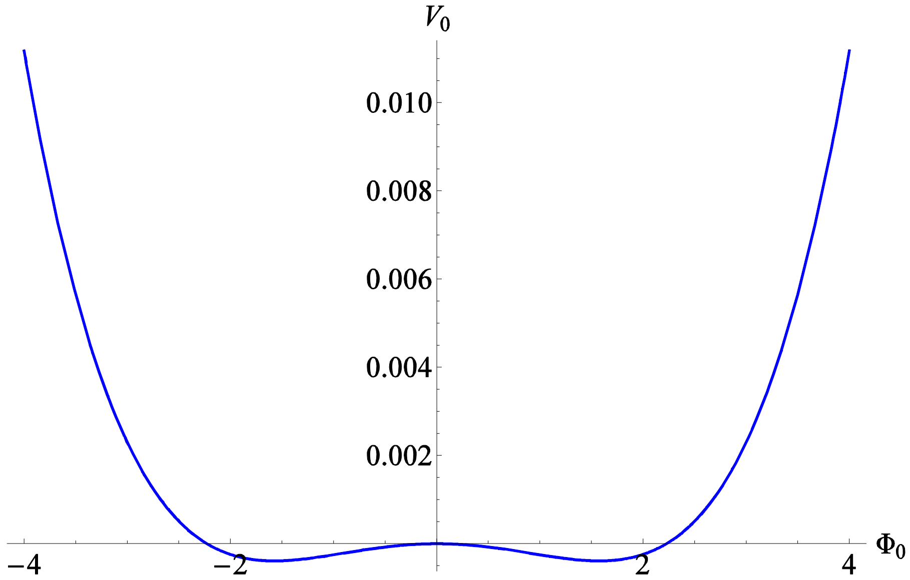
- **where:**

$$\nu_{D=4}^2=\frac{1}{4}-\frac{\left(\xi-1/6\right)R+\lambda\Phi^2/2}{\left(1-\varepsilon\right)^2H^2},\quad k_0^2\eta^2=\frac{k_0^2}{\left(1-\varepsilon\right)^2a^2H^2}$$

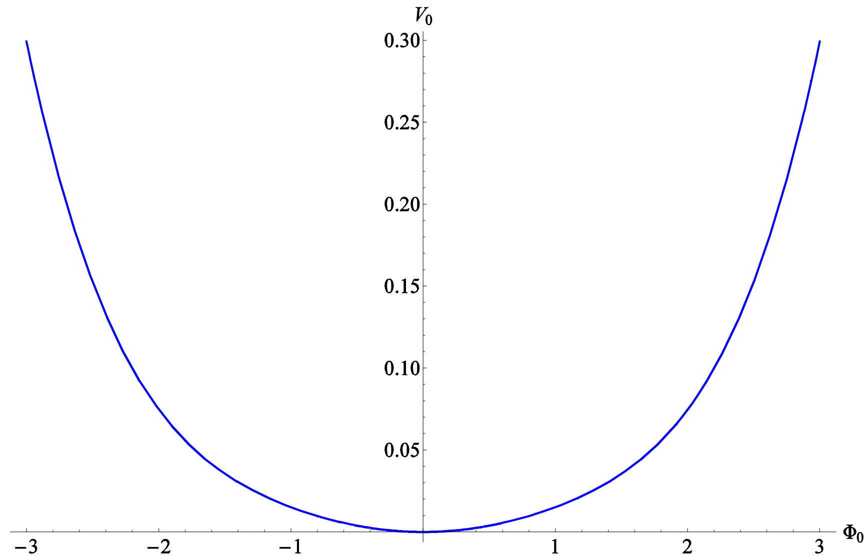
Q: What can we learn from this complicated potential?

- **In the limit when $\lambda\Phi^2 >> H^2 \to 0$, it reproduces the Coleman-Weinberg eff. potential**
- **In the de Sitter limit when $\varepsilon\to 0$, it reproduces the result from Bilandzic & TP (2006)**
- **Quantum effects break the classical scaling symmetry both by μ and by k_0**

EVOLUTION OF V_{eff} WITH TIME: SYMMETRY RESTORATION



⇒

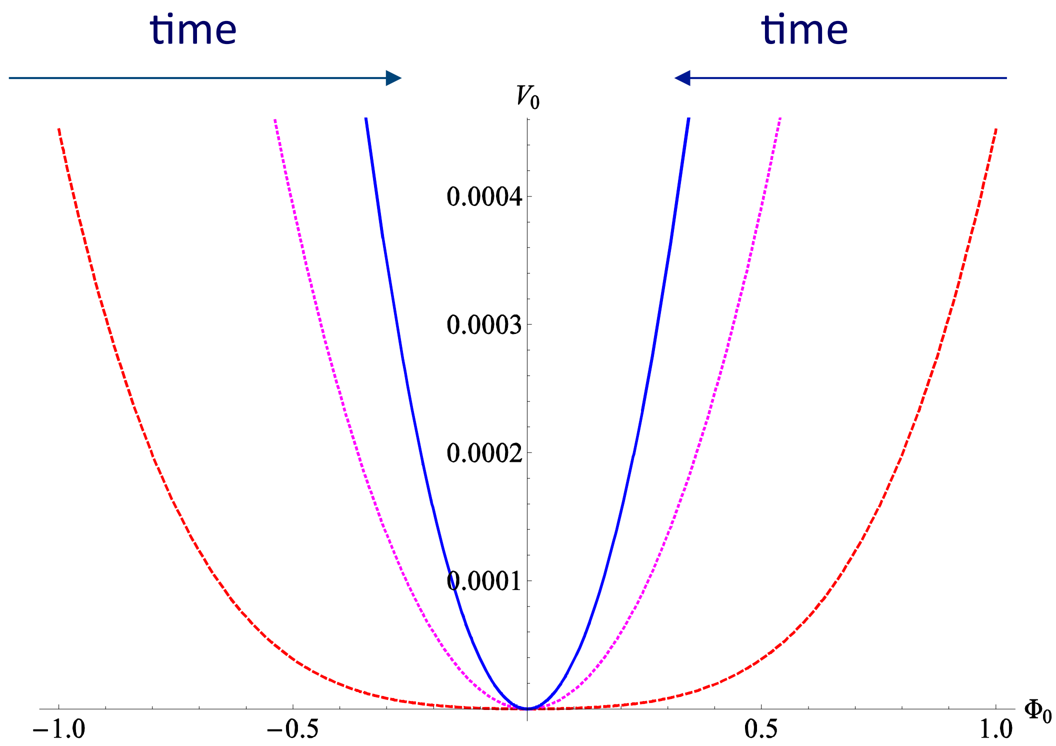


⇒ early times⇐

⇒ late times (13 efolds)⇐

- if $\varepsilon = -(dH/dt)/H^2$ is larger symmetry gets restored earlier
- here $\varepsilon = 0.1$, sym restoration at 9 e-foldings after beginning
- analogous results can be obtained from Starobinsky stochastic formalism

EVOLUTION OF V_{eff} WITH TIME II



- curvature of the effective potential around the origin: strengthens with time
- relevant for preheating: makes production of massive particles more efficient
- suggest breakdown of perturbation theory at late times in inflation
  (also an important – and recognized -- issue in Higgs inflation where ξ~-50000)

SUMMARY AND DISCUSSION, Part I

- Symmetry restoration in inflation can lead e.g. to particle production during inflation, if nonadiabatic
  (fermions: TP+Garbrecht, **PRD [gr-qc/0602011];** scalars; photons?)

- Changes in the potential affect cosmological perturbations

  - amplitude and spectral index of scalar and tensor perturbations
  (e.g.: one has to reexamine models such as Albrecht-Steinhardt new inflation)

- Preheating is more dramatic (recall dependence on ε)

SYMMETRY BREAKING AND GOLDSTONE THEOREM IN DE SITTER SPACE

REAL SCALAR FIELD

- **ACTION**

$$S = \int d^D x \sqrt{-g}\left[-\frac{1}{2}\left(\partial_\mu \phi\right)\left(\partial^\mu \phi\right) - \frac{1}{2} m_0^2 \phi^2 - \frac{\lambda}{4!}\phi^4\right]$$

- **MEAN FIELD (2 loop) EFFECTIVE POTENTIAL & ACTION**

$$V_{\rm MF} = V_0 + \frac{1}{2} m_0^2 \left(\phi^2 + \imath\Delta(x;x)\right) + \frac{\lambda}{4!}\left(\phi^4 + 6\phi^2 \imath\Delta(x;x) + 3[\imath\Delta(x;x)]^2\right) + \frac{\imath}{2}{\rm Tr}\ln[\imath\Delta(x;x)]$$

$$S_{\rm MF} = \int d^D x \sqrt{-g}\left[-\frac{1}{2}\left(\partial_\mu \phi\right)\left(\partial^\mu \phi\right) + \frac{1}{2}[\nabla_\mu \nabla^\mu \imath\Delta(x;x')]_{x'\to x} - V_{\rm MF}\right]$$

SCALAR FIELD PROPAGATOR

- **dS INVARIANT SCALAR FIELD PROPAGATOR**

$$\imath\Delta(x;x') = \frac{H^{D-2}}{(4\pi)^{D/2}} \frac{\Gamma\left(\frac{D-1}{2}+\nu_D\right)\Gamma\left(\frac{D-1}{2}-\nu_D\right)}{\Gamma\left(\frac{D}{2}\right)} \times {}_2F_1\left(\frac{D-1}{2}+\nu_D, \frac{D-1}{2}-\nu_D; \frac{D}{2}; 1-\frac{y(x;x')}{4}\right)$$

$$\nu_D = \left(\frac{D-1}{2}\right)^2 - \frac{m^2}{H^2}, \qquad y(x;x') = a(\eta)a(\eta')H^2\left[-\left(|\eta-\eta'|-\imath\varepsilon\right)^2 + \|\vec{x}-\vec{x}'\|^2\right]$$

- **COINCIDENT SCALAR PROPAGATOR**

$$\imath\Delta(x;x) = \frac{H^{D-2}}{(4\pi)^{D/2}} \frac{\Gamma\left(\frac{D-1}{2}+\nu_D\right)\Gamma\left(\frac{D-1}{2}-\nu_D\right)}{\Gamma\left(\frac{1}{2}+\nu_D\right)\Gamma\left(\frac{1}{2}-\nu_D\right)} \Gamma\left(1-\frac{D}{2}\right) = \imath\Delta(x;x)_{\rm div} + \imath\Delta(x;x)_{\rm fin}$$

$$\imath\Delta(x;x)_{\rm div} = \frac{H^{D-2}\Gamma\left(\frac{D-1}{2}\right)}{4\pi^{(D-1)/2}}\left[\psi(D/2)-\psi(D-1)-\psi(1-D/2)-\gamma_E+\frac{1}{D-1}\right]$$

$$\imath\Delta(x;x)_{\rm fin} = \frac{\Gamma\left(\frac{D-1}{2}\right)}{2\pi^{(D+1)/2}}\frac{H^D}{m^2}$$

RENORMALISATION

● MASS RENORMALISATION

$$m^2 = m_0^2 + \frac{\lambda}{2}\left[\imath\Delta(x;x)\right]_{\rm div} < 0$$

$$\imath\Delta(x;x)_{\rm div} = \frac{H^{D-2}\Gamma\left(\frac{D-1}{2}\right)}{4\pi^{(D-1)/2}}\left[\psi(D/2) - \psi(D-1) - \psi(1-D/2) - \gamma_E + \frac{1}{D-1}\right]$$

► V_0 also gets renormalised, but not important for us

MEAN FIELD MASS

- **MEAN FIELD MASS** (self-consistent 1 loop correction)

$$m_{\rm MF}^2 = \underbrace{m^2}_{<0} + \frac{\lambda}{2}\left(\phi^2 + \imath\Delta(x;x)_{\rm fin}\right) = -2m^2 - \lambda\imath\Delta(x;x)_{\rm fin}, \text{ if } \phi^2 > 0$$

$$m_{\rm MF}^2 = m^2 + \frac{\lambda}{2}\imath\Delta(x;x)_{\rm fin}, \text{ if } \phi^2 = 0, \qquad \imath\Delta(x;x)_{\rm fin} = \frac{\Gamma\left(\frac{D-1}{2}\right)}{2\pi^{(D+1)/2}}\frac{H^D}{m_{\rm MF}^2}$$

► solving these one gets:

- **BROKEN SYMMETRY CASE:**

$$m_{\rm MF}^2 = -m^2 + \sqrt{m^4 - m_{\rm cr}^4} > 0, \ (\phi^2 > 0), \quad |m^2| > m_{\rm cr}^2 = \sqrt{\frac{\lambda H^D \Gamma\left(\frac{D+1}{2}\right)}{2\pi^{(D+1)/2}}}$$

- **RESTORED SYMMETRY CASE:**

$$m_{\rm MF}^2 = \frac{m^2}{2} + \sqrt{\frac{m^4}{4} + \frac{m_{\rm cr}^4}{2}} > 0, \ (\phi^2 = 0), \quad |m^2| < m_{\rm cr}^2$$

MEAN FIELD ORDER PARAMETER

- **MEAN FIELD MASS** (self-consistent 1 loop correction)

$$\Delta\phi^2 = \frac{3m_{\rm cr}^2}{\lambda} = \sqrt{\frac{9H^D\Gamma\left(\frac{D+1}{2}\right)}{2\lambda\pi^{(D+1)/2}}}$$

► **WHEN $\Delta\phi^2>0$, DOMAIN WALLS FORM** (by the Kibble mech.)

► **NB:** true in adiabatic limit, when spatial and time derivatives can be neglected (when domain thickness <<1/H).
Otherwise: super-Hubble size domain walls restore symmetry.

O(N) MODEL

- **ACTION**

$$S=\int d^Dx\sqrt{-g}\left[-\frac{1}{2}\sum_{a=1}^{N}\left(\partial_\mu\phi_a\right)\left(\partial^\mu\phi_a\right)-\frac{1}{2}m_0^2\sum_{a=1}^{N}\phi_a^2-\frac{\lambda}{4N}\left(\sum_{a=1}^{N}\phi_a^2\right)^2\right]$$

- **MEAN FIELD EFFECTIVE POTENTIAL & ACTION**

$$V_{\rm MF}=\frac{1}{2}m_0^2\sum_{a=1}^{N}\left(\phi_a^2+\iota\Delta_{aa}(x;x)\right)+\frac{\lambda}{4N}\left[\left(\sum_{a=1}^{N}\phi_a^2\right)^2+2\left(\sum_{a=1}^{N}\phi_a^2\right)\sum_{b=1}^{N}\iota\Delta_{bb}(x;x)\right]$$
$$+\frac{\lambda}{4N}\left[4\sum_{a,b=1}^{N}\phi_a\phi_b\iota\Delta_{ab}(x;x)+\left(\sum_{a=1}^{N}\iota\Delta_{aa}(x;x)\right)^2+2\sum_{a,b=1}^{N}\left(\iota\Delta_{aa}(x;x)\right)^2\right]+\frac{\iota}{2}\mathrm{Tr}\ln[\iota\Delta_{aa}(x;x)]$$

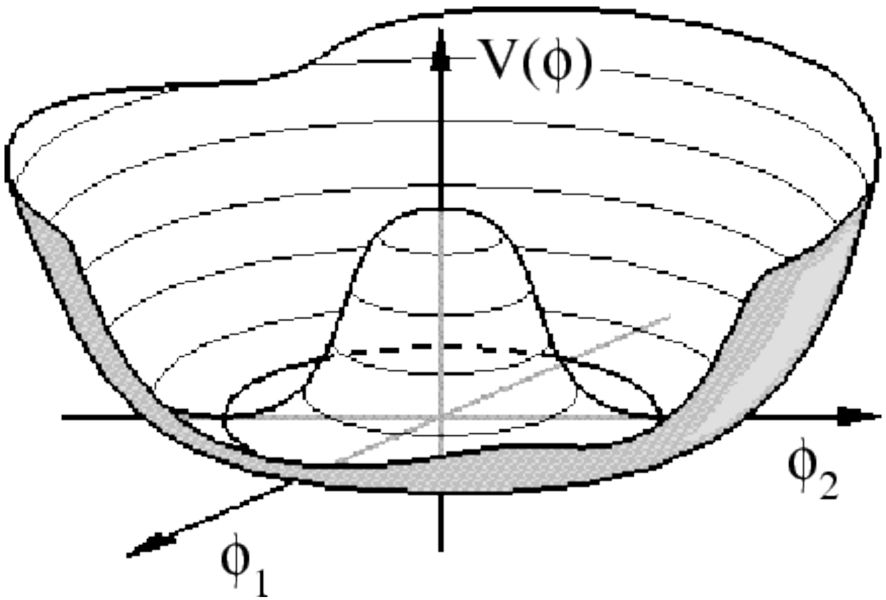
$$S_{\rm MF}=\int d^Dx\sqrt{-g}\left[-\frac{1}{2}\sum_{a=1}^{N}\left(\partial_\mu\phi_a\right)\left(\partial^\mu\phi_b\right)+\frac{1}{2}\sum_{a=1}^{N}[\nabla_\mu\nabla^\mu\iota\Delta_{aa}(x;x')]_{x'\to x}-V_{\rm MF}\right]$$

N=2: MEXICAN HAT POTENTIAL

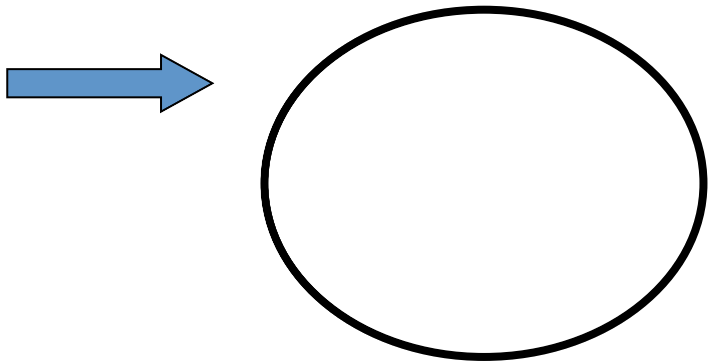
- **FOR TREE LEVEL ACTION**

$$S=\int d^D x\sqrt{-g}\left[-\frac{1}{2}\sum_{a=1}^{N}\left(\partial_\mu\phi_a\right)\left(\partial^\mu\phi_a\right)-\frac{1}{2}m_0^2\sum_{a=1}^{N}\phi_a^2-\frac{\lambda}{4N}\left(\sum_{a=1}^{N}\phi_a^2\right)^2\right]$$

(AND N=2) WE GET A MEXICAN HAT POTENTIAL:



VACUUM MANIFOLD: SPHERE S^1 (1 Goldstone excitation)



RENORMALISATION

• MASS RENORMALISATION

$$m^2 = m_0^2 + \frac{(N+2)\lambda}{N} \frac{H^{D-2}\Gamma\left(\frac{D-1}{2}\right)}{4\pi^{(D-1)/2}} \left[\psi\left(\frac{D}{2}\right) - \psi(D-1) - \psi\left(1-\frac{D}{2}\right) - \gamma_E + \frac{1}{D-1}\right]$$

• MASS MATRIX

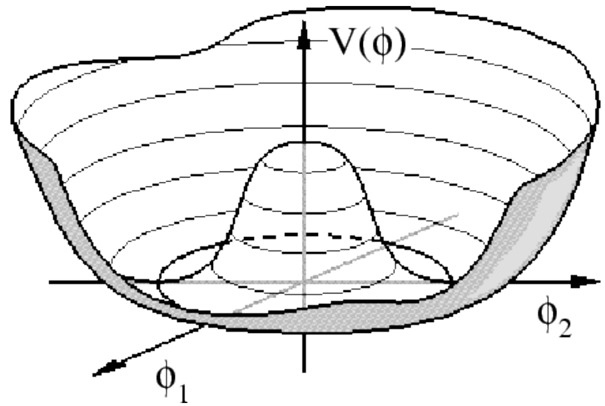
$$M_{ab}^2 = \left[m^2 + \frac{\lambda}{N}\sum_{c=1}^{N}\left(\phi_c^2 + \iota\Delta_{cc}(x;x)_{\text{fin}}\right)\right]\delta_{ab} + \frac{2\lambda}{N}\left(\phi_a\phi_b + \iota\Delta_{ab}(x;x)_{\text{fin}}\right)$$

► this can be diagonalised..

SELF CONSISTENT MEAN FIELD APPROX

► upon rotation to the diagonal frame: $\vec{\phi} \to R\vec{\phi} = \begin{pmatrix} \phi_1 \\ 0 \\ .. \\ 0 \end{pmatrix}, \phi_1^2 > 0$ one gets:

- **MASSIVE `HIGGS-LIKE' EXCITATION:**



$$M_1^2 = -2m^2 - \frac{\lambda H^D \Gamma\left(\frac{D+1}{2}\right)}{N\pi^{(D+1)/2}} \left[\frac{N-1}{M_g^2} + \frac{3}{M_1^2} \right]$$

- **WOULD-BE GOLDSTONES** (become massive!):

$$M_g^2 = \frac{\lambda H^D \Gamma\left(\frac{D+1}{2}\right)}{N\pi^{(D+1)/2}} \left[\frac{1}{M_g^2} - \frac{1}{M_1^2} \right]$$

☺ admits a small coupling expansion (in powers of $\sqrt{\lambda}$, as in TFT)

► when these are solved, one gets..

EXPANSION AROUND λ=0

- **DE SITTER RESUMMED ONE LOOP MASS (broken case):**

$$\mu_1^2 = 1-(N-1)\sqrt{\lambda_D} - \frac{N+5}{2}\lambda_D + O(\lambda_D^{3/2}), \quad \mu_1^2 = \frac{M_1^2}{-2m^2}$$

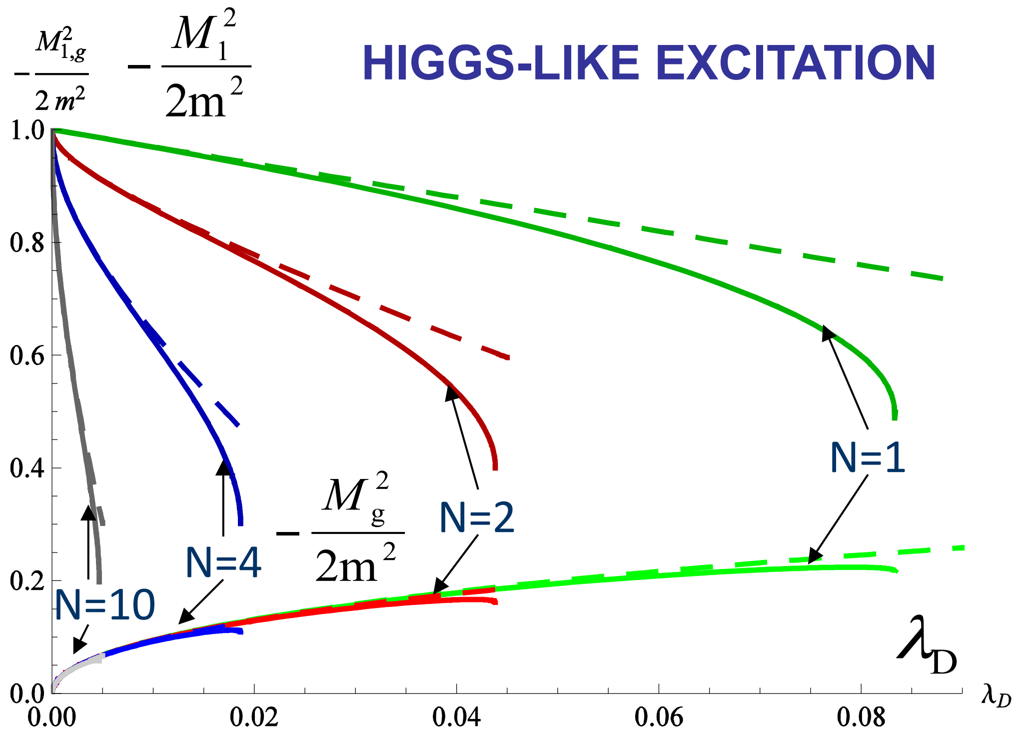
$$\mu_g^2 = \sqrt{\lambda_D} - \frac{1}{2}\lambda_D + O(\lambda_D^{3/2}), \quad \mu_g^2 = \frac{M_g^2}{-2m^2}, \quad \lambda_D = \frac{\lambda H^D \Gamma((D+1)/2)}{N\pi^{(D+1)/2}(-2m^2)^2}$$

- **THERMAL RESUMMED ONE LOOP MASS (m²>0):**

$$m(T) = \sqrt{m^2 + \frac{\lambda T^2}{24} + \frac{\lambda^2 T^2}{(16\pi)^2}} - \frac{\lambda T}{16\pi} \Rightarrow m^2(T) \approx \frac{\lambda T^2}{24}\left(1 - \frac{\sqrt{3\lambda}}{2\sqrt{2}\pi}\right) \quad (m^2 > 0)$$

► also non-perturbative, but stronger effects in de Sitter.

O(N) MODEL IN MEAN FIELD APPROXIMATION



$$\lambda_D = \frac{\lambda H^D \Gamma\left(\frac{D+1}{2}\right)}{N\pi^{(D+1)/2}(-2m^2)^2}$$

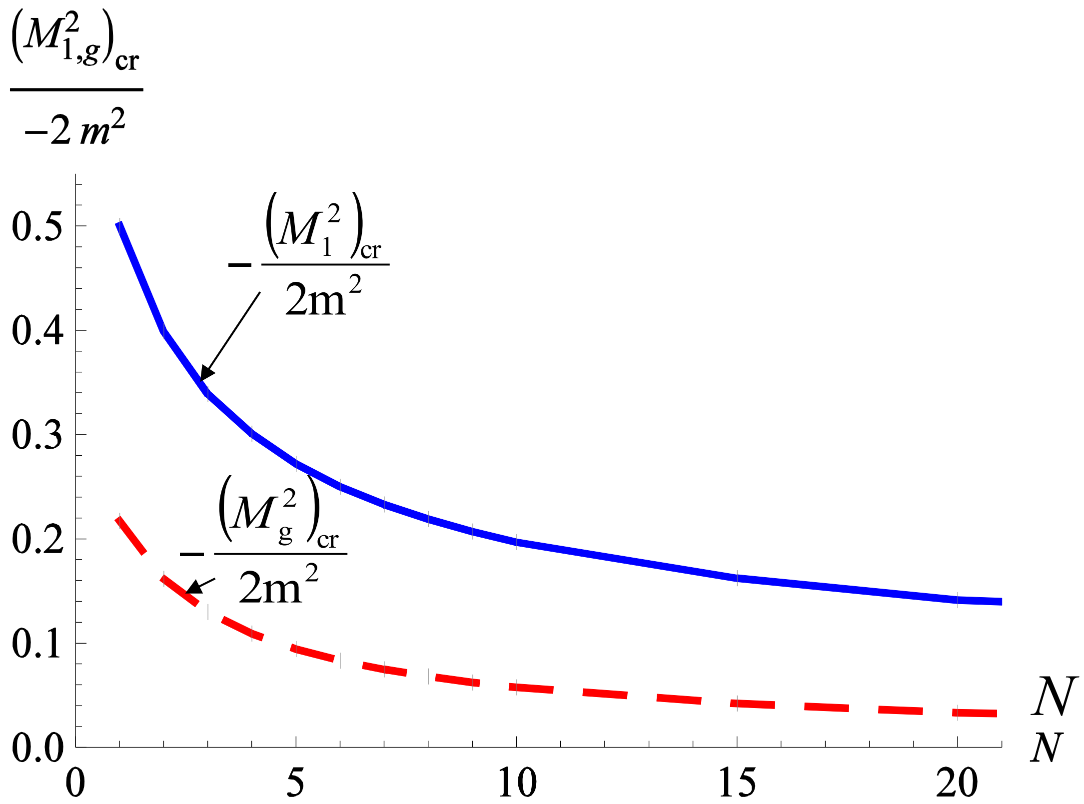
WOULD-BE GOLDSTONES

NOTE: ∃ END POINTS $(\lambda_D)_{cr}$ WHERE NO MORE SOLUTIONS MASSES ARE FINITE.

$$(\lambda_D)_{cr} \approx \frac{1.6}{(N+2)^{1.9}}$$

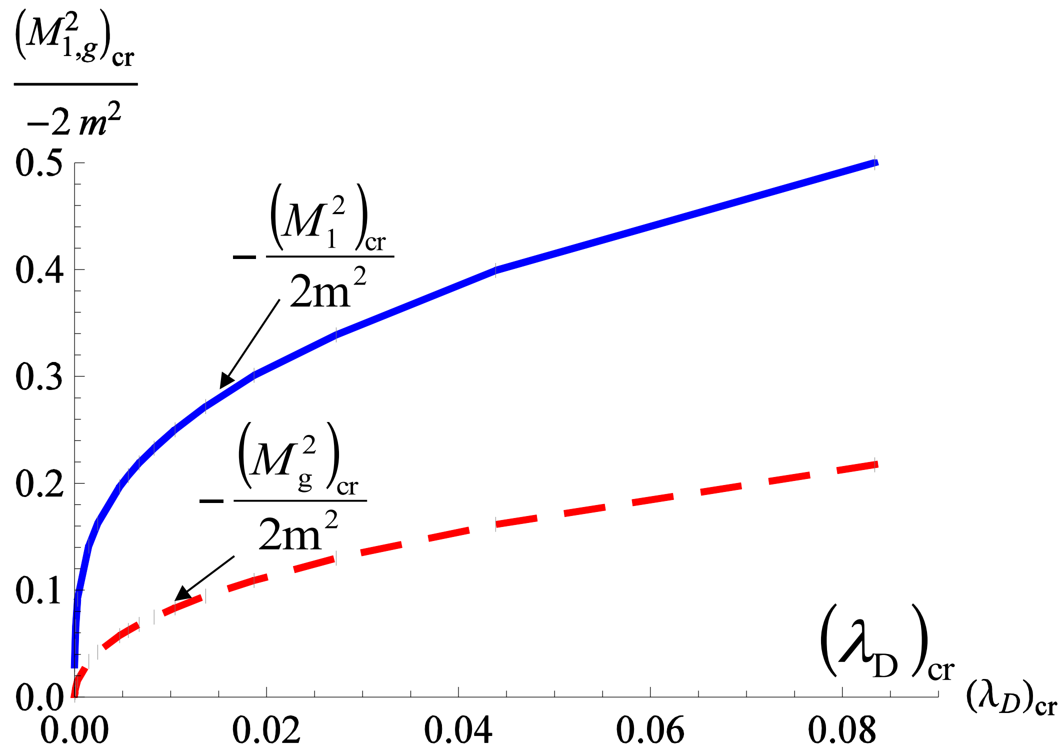
HIGS-LIKE & PSEUDO-GOLDSTONE MASSES

••• AS A FUNCTION OF N



HIGS-LIKE & PSEUDO-GOLDSTONE MASSES II

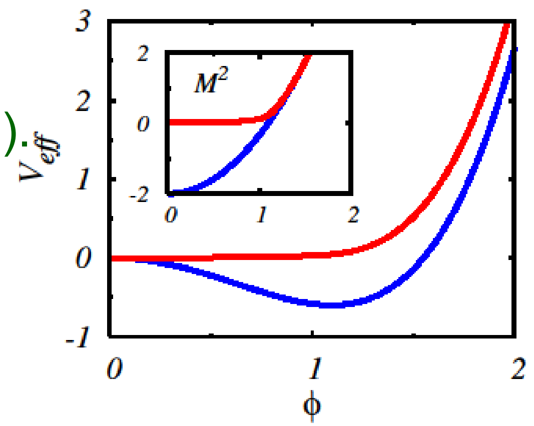
- AS A FUNCTION OF $\left(\lambda_D\right)_{cr} \approx \frac{3}{2(N+2)^2}$



PHASE TRANSITION STRENGTH

- **MEAN FIELD MASS** (self-consistent 1 loop correction)

$$\Delta\phi^2 = \frac{N(M_1^2)_{\rm cr}}{\lambda} \propto \frac{|m|N^{1/2}}{\lambda^{3/4}} > 0$$



► NB: $\Delta\phi^2/N \to 0$ as $N \to \infty$ (agrees with Serreau 2011).

► NB2: For N=1, $\Delta\phi^2 \propto 1/\sqrt{\lambda}$.

► NB3: In symmetric phase, where $\Delta\phi^2=0$:

$$M_1^2 = M_g^2 = \frac{m^2}{2} + \sqrt{\frac{m^4}{4} + \frac{(N+2)m_{\rm cr}^4}{6}}$$

☺ all masses are degenerate, and when N=1 it agrees with the N=1 result.

˘THESE RESULTS VALID IN ADIABATIC REGIME.
WHEN DEFECT SCALE ≥1/H, THEN SYMMETRY RESTORED BY DEFECTS!

DISCUSSION, Part IIA

- WE STUDIED SYMMETRY BREAKING IN A SCALAR O(N) MODEL IN DE SITTER SPACE IN MEAN FIELD APPROXIMATION

- WE FOUND THAT FOR REN. MASS $m^2 < -m_{cr}^2$ SYM GETS BROKEN

BOTH HIGGS-LIKE AND GOLDSTONE LIKE EXCITATION GET A MASS

••• A NOVEL MASS GENERATION MECHANISM:

PSEUDO-GOLDSTONES ACQUIRE MASS FROM ENHANCED IR FLUCTS

••• JUMP IN ORDER PARAMETER: $\Delta\phi^2 \propto \frac{|m|\, N^{1/2}}{\lambda^{3/4}}$

DISCUSSION, Part IIB

- COMPARISON WITH STOCHASTIC INFLATION RESULTS:

Starobinsky, Yokoyama, PRD50 (1994) 6357 [astro-ph/9407016]
Arvin Rajaraman (2010); Martin Beneke (in preparation).

♣When mass is adjusted m= 0 (no symmetry breaking)

$$\frac{m^2_{\rm stoch}}{\rm m^2_{MF}} = \frac{2\sqrt{3}\,\Gamma(3/4)}{\Gamma(1/4)} \approx 1.17$$

••• But, when $m=(\lambda/2)\langle\phi^2\rangle$, then $m^2_{\rm stoch} = \rm m^2_{MF}$

In order to give a more compete answer to phase transitions during inflation, one needs to go beyond mean field (2 loops 2PI).
Stochastic formalism (and Euclidean formalism of the 0 mode) resum all loops, suggesting that stochastic formalism gives the correct answer (in progress).

<u>RELEVANCE:</u> preheating, inflationary dynamics, cosmological perturbations, fermion production & baryogenesis/leptogenesis, **etc.**