Instability of de Sitter Space & Dynamical Dark Energy

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Recent Review: *Acta. Phys. Pol. B* 41, 2031 (2010)

w. R. Vaulin, *Phys. Rev. D* 74, 064004 (2006)

w. P. Anderson & R. Vaulin, *Phys. Rev. D* 76, 024018 (2007)

Review Article: w. I. Antoniadis & P. O. Mazur, *N. Jour. Phys.* 9, 11 (2007)

w. M. Giannotti, *Phys. Rev. D* 79, 045014 (2009)

w. P. Anderson & C. Molina-Paris, *Phys. Rev. D* 80, 084005 (2009)

w. I. Antoniadis & P. O. Mazur, arXiv:1103.4164, JCAP (2012)

w. P. R. Anderson (2012)

Outline

- Motivation: The Cosmological Constant Problem
- Review of Classical de Sitter Space & Quantum Effects
- The Cosmological Electric Field Problem
- Instability to Pair Creation in Both Situations
- The Relevance of the Trace Anomaly Effective Action
- Conformal Symmetry & Stress Tensor on de Sitter Horizon
- Cosmological Horizon Modes from the Anomaly
- Cosmological Dark Energy as a Dynamical Condensate

The 'Cosmological Constant' Problem

The Classical Einstein's Equations,

$$R_{ab} - \frac{1}{2}R g_{ab} + \Lambda\, g_{ab} = 8\pi G\; T_{ab}$$

relate

- **Curvature of Spacetime,** R_{ab} **to**
- **Energy-Momentum of Matter,** T_{ab}

Metric: g_{ab} **fixes lengths,** $ds^2 = g_{ab}dx^a dx^b$

Flat spacetime: $g_{ab} = \mathrm{diag}(-1, +1, +1, +1)$

Λ **is equivalent to**

Constant Vacuum Energy Everywhere:

$$T_{ab}^{(vac)} = -\frac{\Lambda}{8\pi G}\, g_{ab} \qquad \textbf{or}$$

$$p_\Lambda = -\rho_\Lambda$$

Negative Pressure

Classically, Λ **may be set to zero but ...**

Gravity weighs Everything even Quantum Vacuum Fluctuations:

$$\rho_\Lambda = N \int \frac{d^3\vec{k}}{(2\pi)^3} \frac{\hbar\omega_k}{2} \to \frac{N\hbar c}{16\pi^2} L_{min}^{-4} = -p_\Lambda$$

Quartic Dependence on $L_{min} \to 0$

With any "reasonable" L_{min}, ρ_Λ is HUGE:

The "natural" scale would seem to be

$$\Lambda \simeq \frac{c^3}{\hbar G} = L_{Pl}^{-2} \simeq \left(\frac{1}{10^{-33}\mathrm{cm}}\right)^2$$

The Universe would be curled up then to a radius of 10^{-33} cm. (!)

Since the observable Universe is of order 10^{28} cm (Hubble scale),

$$\Lambda < 10^{-121} \frac{c^3}{\hbar G}$$

our estimate is wrong by some 121 orders of magnitude (!)

Requires <u>both</u> $\hbar$ and G different from zero

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Macroscopic or Microscopic ?
IR or UV ?

- We deal with UV divergences by **Renormalization**, and now understand most (all?) QFT's as *Effective* Theories
- Λ is a free parameter of the **Low Energy Effective Theory**

"Just because something is infinite does not necessarily mean that it is zero." –W. Pauli

- The Standard Model has **Spontaneous Symmetry Breaking**
  When the ground state changes, so does its energy – so we should expect generically $\Lambda > 0$ now
- More Symmetries at Very High Energy (UV) Cannot Help
- This is a problem of fixing the

Quantum Vacuum State of Macroscopic Gravity

Quantum Effects in de Sitter Space

- Particle Creation & Backreaction $$\frac{dH}{dt} = -\frac{4\pi G}{c^2}(\rho + p)$$

  Compare to 'Cosmological Electric Field Problem'

  'Shorting' the vacuum $$\frac{dE}{dt} = -j$$
- Hawking Temperature Instability

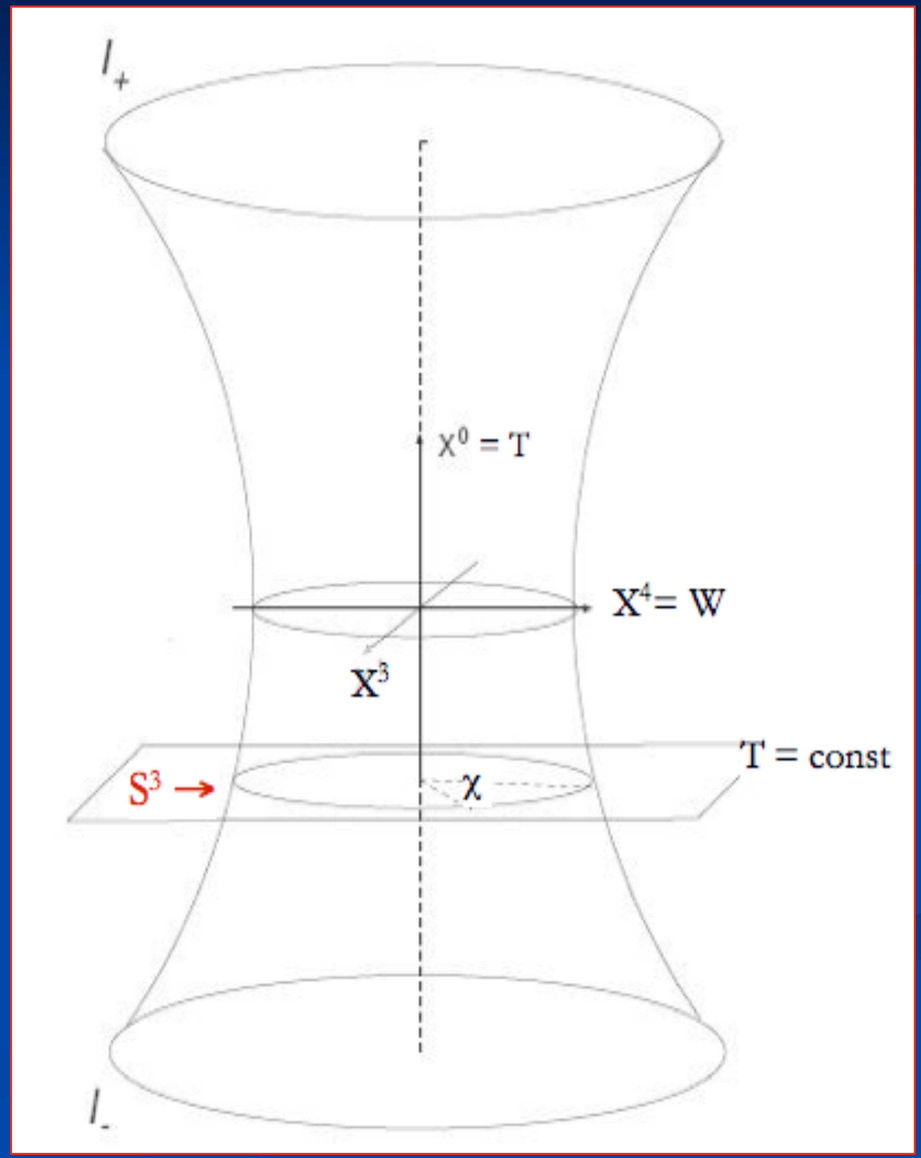
  Compare to Schwarzschild Black Hole $$T_H = \frac{\hbar H}{2\pi k_B} \propto \left(\frac{c^5}{2GH}\right)^{-1} = E_H^{-1}$$

  Negative Heat Capacity
- Graviton Propagator behaves logarithmically

  No Cluster Decomposition, S-Matrix
- Non-trivial Infrared Properties
- Infrared Relevant Operator Missing in Einstein Theory?

De Sitter Spacetime Geometry

- De Sitter Space has Constant Curvature
  $R = 4\Lambda = 12H^2$
- Hyperboloid of Revolution
  $-T^2 + W^2 + X^2 + Y^2 + Z^2 = H^{-2}$
  embedded in flat 5D Minkowski Space
  $ds^2 = -dT^2 + dW^2 + dX^2 + dY^2 + dZ^2$
- Symmetry Group is $O(4,1)$
- *Ten* Generators:
  - 3 Translations
  - 3 Rotations
  - 4 Lorentz Boosts



De Sitter Spacetime Coordinates

- S^3 Hyperboloid Coordinates

$$T = H^{-1} \sinh u$$

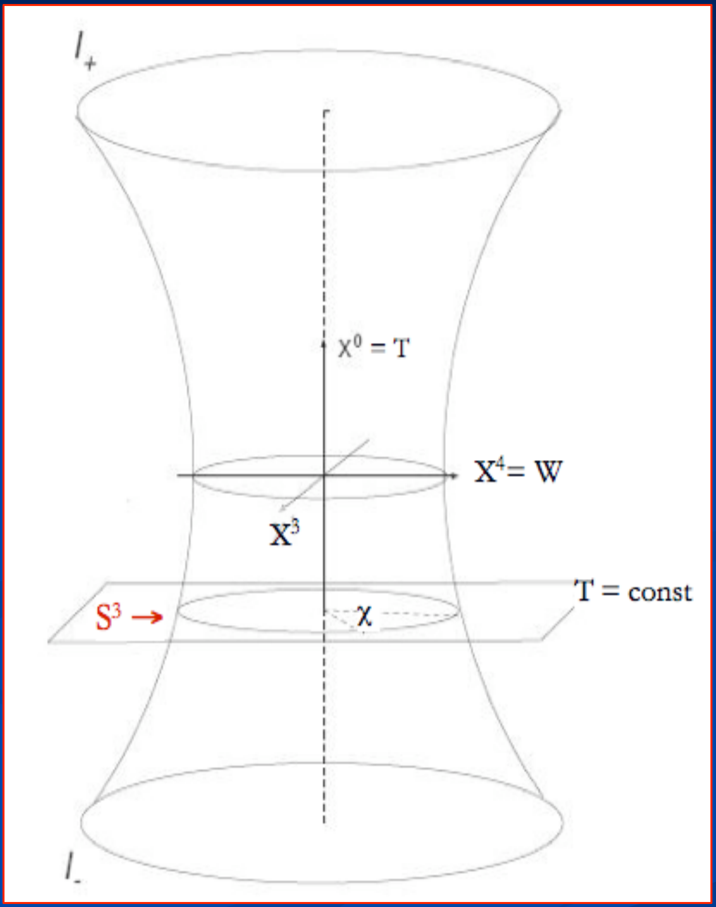
$$W = H^{-1} \cosh u \, \cos \chi$$

$$X^i = H^{-1} \cosh u \, \sin \chi \, \hat{n}^i$$

$$i = 1, 2, 3$$

$$ds^2 = H^{-2} \left[-du^2 + \cosh^2 u \; d\Omega_3^2 \right]$$

$$d\Omega_3^2 \equiv d\chi^2 + \sin^2 \chi \; d\Omega^2$$



- Cover Complete Manifold
- Symmetry Group of S^3 at const. ***u*** is ***O(4)***
- **Six** Generators

De Sitter Spacetime Coordinates

- $\mathbf{R}^3$ Spatially Flat Coordinates

$$T = \frac{1}{2H}\left(a - \frac{1}{a}\right) + \frac{Ha}{2}\,\varrho^2$$

$$W = \frac{1}{2H}\left(a + \frac{1}{a}\right) - \frac{Ha}{2}\,\varrho^2$$

$$X^i = a\,x^i\,,\ \ i = 1, 2, 3$$

$$ds^2 = -d\tau^2 + a^2(\tau)\,d\vec{x}\cdot d\vec{x}$$

$$a(\tau) = e^{H\tau},\ \ \varrho = |\vec{x}|$$

- Covers only the Half Manifold with $T + W > 0$
- **Six** Generators of Rotations & Translations on $\mathbf{R}^3$

De Sitter Spacetime Coordinates

- Static Coordinates

$$T = H^{-1}\sqrt{1 - H^2 r^2}\ \sinh(Ht)$$

$$W = H^{-1}\sqrt{1 - H^2 r^2}\ \cosh(Ht)$$

$$X^i = r\, n^i\,,\ \ i = 1, 2, 3$$

$$ds^2 = -f(r)\, dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$$

$$f(r) \equiv 1 - H^2 r^2$$

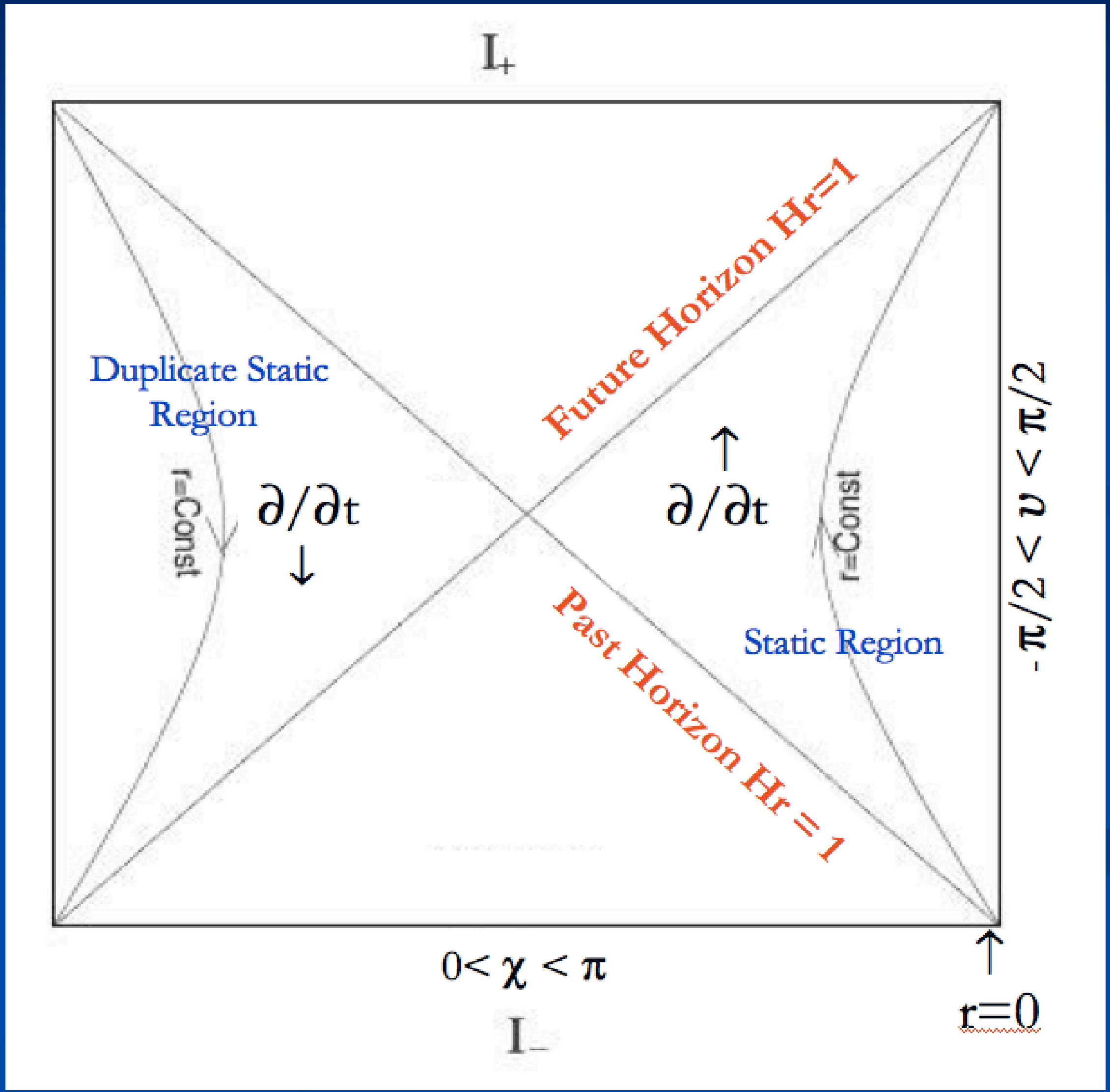
- Covers only the One Quarter Manifold with $T + W > 0$, $W > 0$
- Three Generators of Rotations at fixed on r, t: $O(3)$
- *Horizon* at $r = 1/H$

De Sitter Spacetime
Carter-Penrose Diagram

- Suppress Angular Variables ***θ, ϕ***
- Map hyperboloid to finite time interval
- Light Travels at ***45°***
- Conformal Structure manifest
- Note: Past Infinity I_ is Spacelike
- Static Time Translation

  t ➔ t + Δt

  is a Lorentz boost
- Hamiltonian is Unbounded from below



Cosmological Constant Electric Field Problem

- Sourcefree Maxwell's Eqs. admit a solution of a constant, uniform Electric Field
- All electric fields in Nature are associated with localized sources
- Why do we not observe some very large ***E*** in an arbitrary direction

$$\frac{\partial \vec{E}}{\partial t} = 0$$

$$\vec{\nabla} \cdot \vec{E} = 0$$

$$\vec{E} = E\,\hat{z}$$

Answer: 'Vacuum' in electric field is Unstable to Particle Creation

Constant Uniform Electric Field

- Expect Pair Creation in Time Dependent Gauge

$$\vec{A} = -Et\,\hat{z}$$

- Background is Time Independent; Static Gauge Exists

$$A_0 = Ez$$

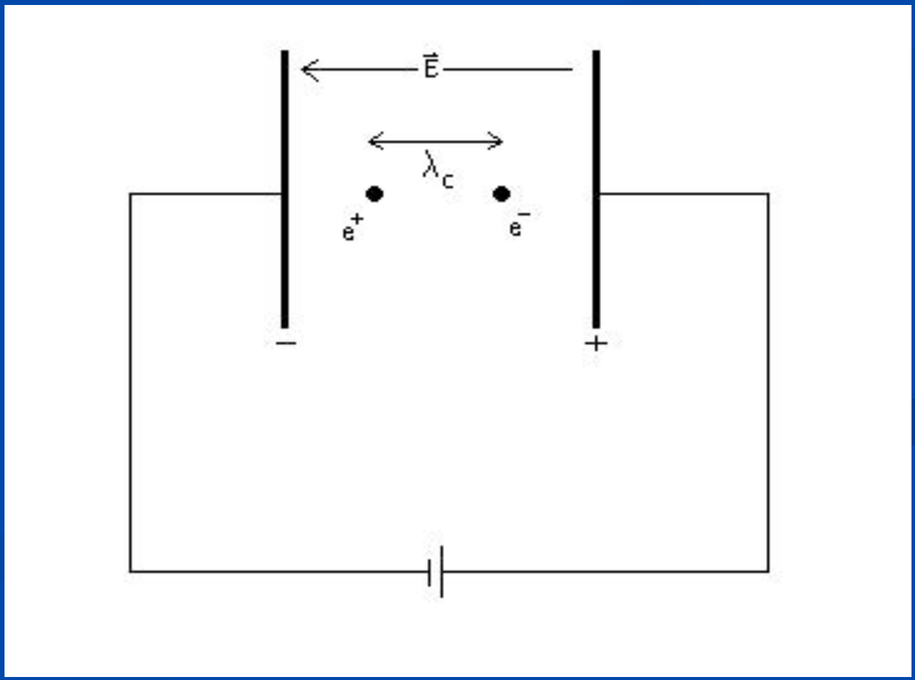
- But Hamiltonian is Unbounded from below

$$H = \sqrt{\vec{p}^2 + m^2} - eA_0$$

- Classical Pair separated by ***d*** has Energy ***eEd > 2mc²***
- Vacuum has fluctuating pairs at scale $d \sim \hbar/mc$
- Expect Significant Real Spontaneous Pair Creation for

$$eE > 2m^2c^3/\hbar$$

- Vacuum Instability (Klein
- Paradox) Even in 'free' theory



Spontaneous Pair Creation in a Constant Uniform Electric Field

- Decay Rate/Volume of ***E*** Vacuum (Schwinger 1951)

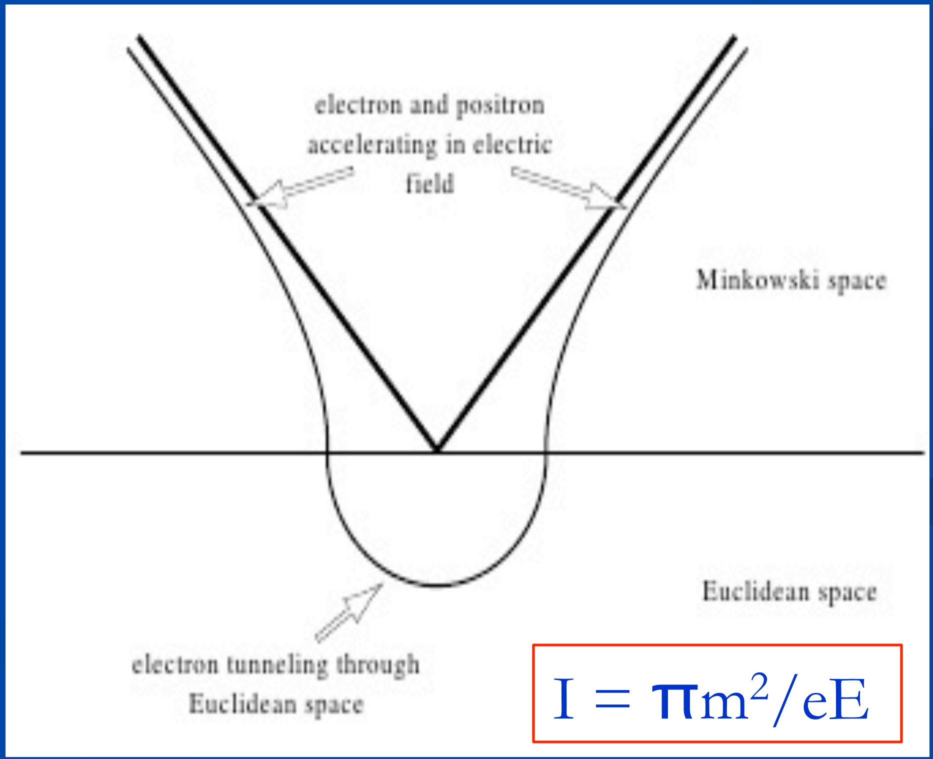
$$\Gamma = \frac{(eE)^2}{(2\pi)^3\hbar^2}\sum_{n=1}^{\infty}\frac{(\mp)^{n+1}}{n^2}\exp\left(-\frac{n\pi m^2}{\hbar eE}\right)$$

- for bosons/+ for fermions

- Non-Perturbative Dependence on ***eE***
- Semi-classical Euclidean to Lorentzian Tunneling Picture
- Verified by Adiabatic Switching $T \to \infty$

$$A_z = -ET\tanh(t/T)$$

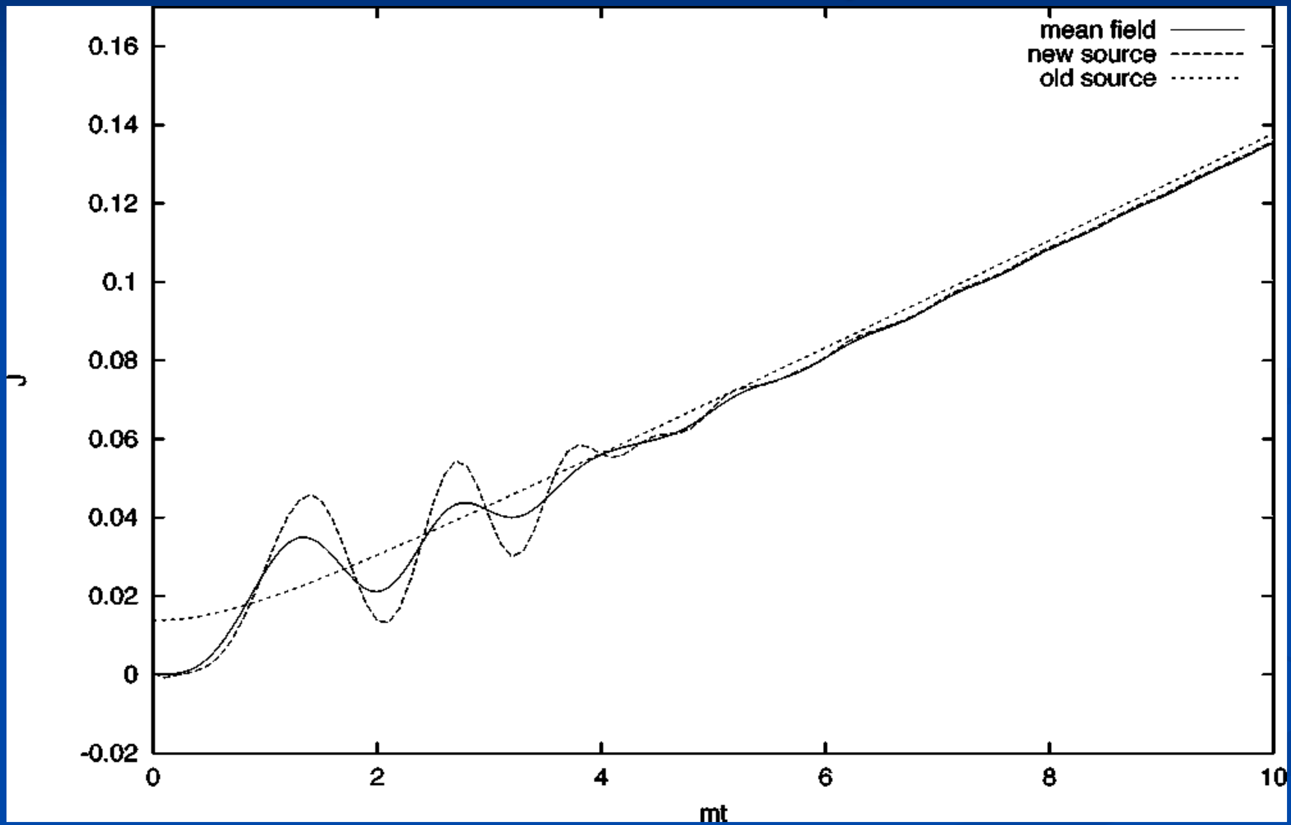
$$\vec{E} = E\hat{z}\,\mathrm{sech}^2(t/T)$$



$I = \pi m^2/eE$

Particles Created in Uniform Electric Field

- Created Particles are accelerated & produce a Current ***J***
- Electric Current ***J*** Grows Linearly in Fixed ***E*** Field



PRD 58 125015 (1998)

- Slope can be understood from 2D Chiral Anomaly (for m=0)
- Secular growth ➔ Backreaction must be taken into account

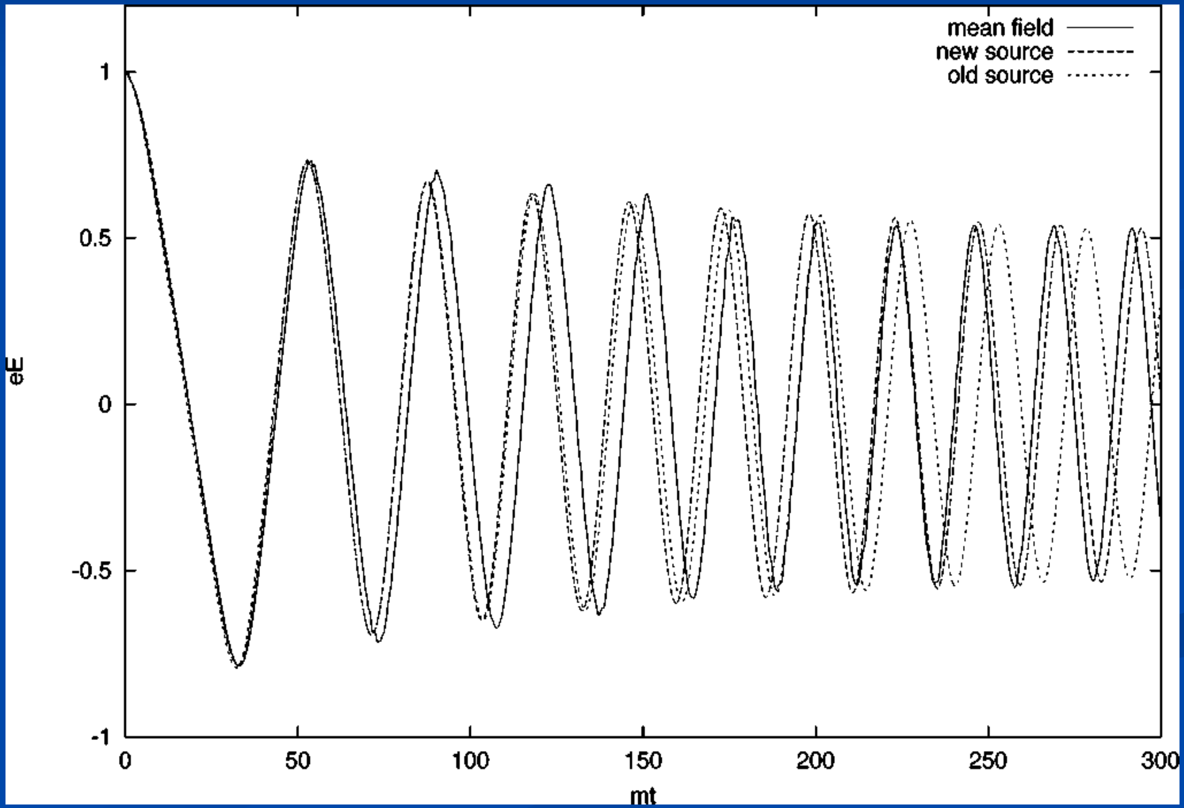
Backreaction in Uniform Electric Field

- Can be taken into account in Semi-Classical Mean Field (Large N) Approximation (collisionless)

$$\frac{\partial \vec{E}}{\partial t} = \langle \vec{j} \rangle$$

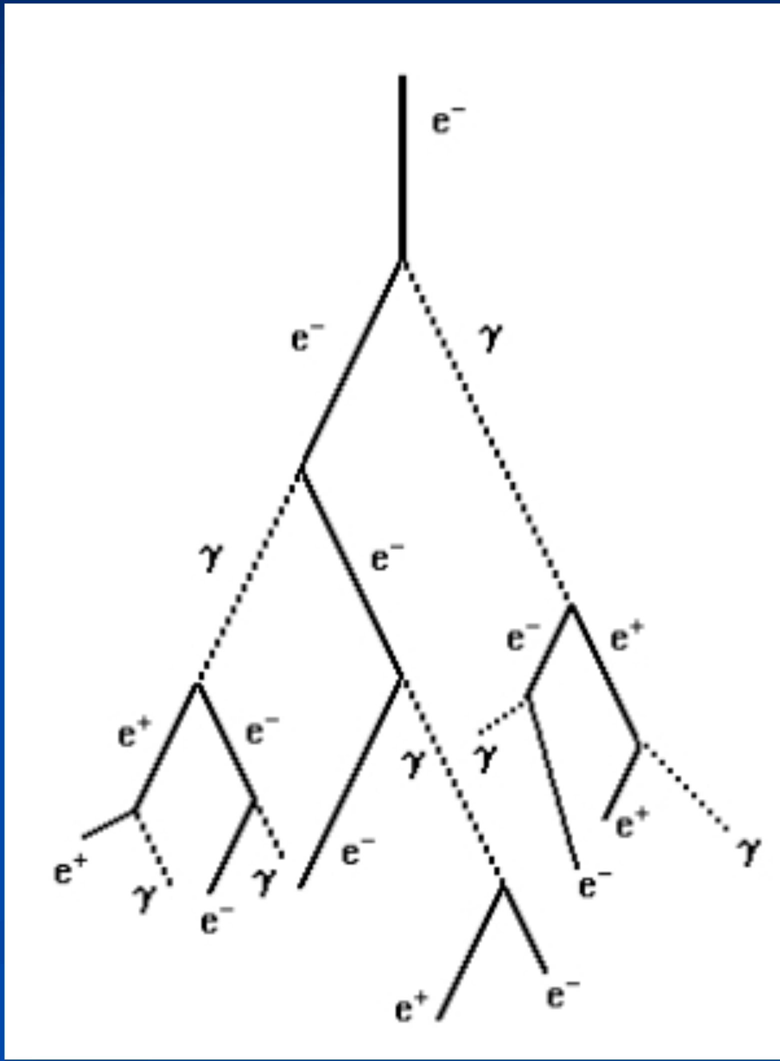
- Longitudinal Mode of Electric Field constrained in sourcefree Maxwell eqs. becomes dynamical

- Plasma Oscillations



Induced Pair Creation Cascade

- Spontaneous Pair Creation always accompanied/followed by Induced Creation
- Cascade Time Scale between emissions is short/classical
  $$\Delta t \simeq 2mc/eE$$
- Particle Density grows until interactions become important
- Mean Field Approximation is no longer adequate



Decay of Cosmological 'Constant' Electric Field

- Mean Free Path between collisions $\lambda = (n\sigma)^{-1}$ determined by Particle Density n and Scattering Cross Section $\sigma \sim \alpha^2/m^2$

- Current becomes proportional to λ (Ohm's Law)

$$J = e\,n\,v_{max} \sim en\,\frac{eE}{m}\frac{\lambda}{c} = \sigma_C E$$

$$\sigma_C \sim \frac{e^2 n\lambda}{mc} \sim \frac{mc^2}{\hbar\alpha} \to \frac{k_B T}{\hbar\alpha} \times \text{logs}$$

- Irreversible Process

$$\frac{\partial E}{\partial t} = -J = -\sigma_C E$$

- Electrical Conductivity σ_C of the Relativistic Plasma determines the Relaxation Time σ_C^{-1} to zero field vacuum

Spontaneous Pair Creation in de Sitter Spacetime

- Like E field expect particle creation in time dependent gauges
- In static gauges the Hamiltonian is unbounded from below
- Classical Pair must be separated by $d \sim \hbar/mc \gtrsim 1/H$
- Decay Rate/Volume of ***de Sitter*** Vacuum

$$\Gamma = \frac{8H^4}{\pi^2} \ln[\coth(\pi\gamma)] \to \frac{16H^4}{\pi^2} \exp\left(-\frac{2\pi m}{\hbar H}\right)$$

$$\gamma \equiv \sqrt{\frac{M^2}{H^2} - \frac{9}{4}}$$

PRD 31, 754 (1985)

- Semi-classical Euclidean ***S^4*** to Lorentzian Tunneling Picture
- Verified by Adiabatic Switching from ***$R \times S^3$***

$I = 2\pi m/H$

Bunch-Davies de Sitter Invariant State

- State of maximum O(4,1) symmetry for massive fields
- Can be obtained by Euclidean Continuation from ***S^4***
- Stress Tensor $\langle T^a_{\ b}\rangle = p\,\delta^a_{\ b} = -\rho\,\delta^a_{\ b} = const.$

"Nothing happens"

- Actually even in ***E*** field case an analogous state exists -in any time reversal invariant background possible to find a state which is ***T*** invariant—doesn't mean it's the ground state
- Corresponds to exact balancing of pair creation vs. time reversed pair annihilation events
- Imaginary Part of Effective Action $\Gamma = 0$ for such a state
- So was Schwinger wrong?
- No --this state is unstable, not the vacuum ground state

Perturbations of Bunch-Davies State

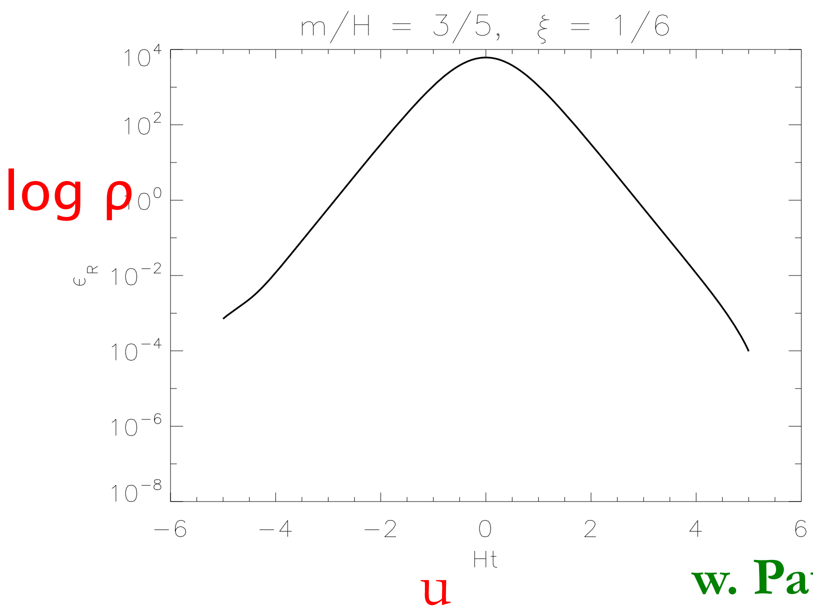
- Can consider arbitrary O(4) invariant states
- The stress tensor of these states falls off at least as fast as a^{-3} as $a \to \infty$ for massive fields in expanding de Sitter universe PRD 62, 124019 (2000); 72, 043515 (2005)
- Conversely an arbitrarily small perturbation of the energy density grows at least as fast as a^{-3} in a contracting de Sitter universe—BD is infinitely fine tuned at $\tau = -\infty$
- If these arbitrarily small perturbations are at arbitrarily high k they are blueshifted (IR → UV) during the contraction to give arbitrarily large physical energy densities

$$GmH\frac{k^3}{a^3} \ll 1\,, a \to \infty \qquad \text{but}$$

$$GmHk^3 \gg 1\,, a \sim 1$$

Instability of Bunch-Davies State

- Generic effect in adiabatic states is in fact a^{-4} since created particles rapidly become relativistic
- This a^{-4} blueshifted energy density is obtained from the 4D trace anomaly (compare J linear growth in E)
- Sensitivity to Initial Conditions
- Quantum Backreaction must be taken into account



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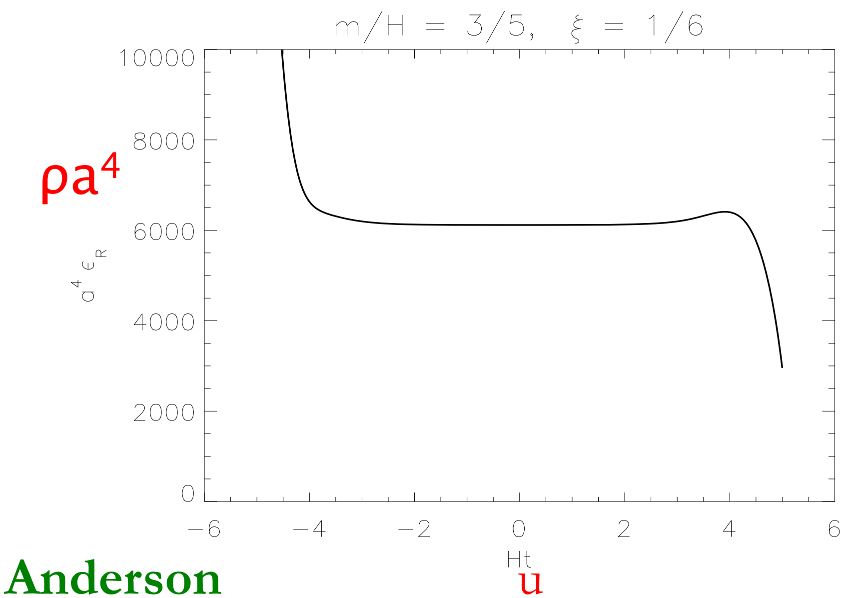


Relevance of the Trace Anomaly

- Expansion of Effective Action in *Local* Invariants assumes Decoupling of Short Distance from Long Distance Modes
- **But Relativistic Particle Creation is Non-Local**
- *Massless* **Modes do not decouple**
- **Special Non-local Additions to Local EFT**
- *IR* **Sensitivity to UV degrees of freedom**
- **QFT Conformal Behavior, Breaking & Bulk Viscosity (analog of conductivity) determined by Anomaly**
- **Blueshift on Horizons → behavior conformal there**
- **Additional Scalar Degree(s) of Freedom in EFT of Gravity allow & predict Dynamics of Λ**

2D Gravity

$$S_{cl}[g] = \int d^2x \sqrt{g}(\gamma R - 2\lambda)$$

has no local degrees of freedom in 2D, since

$$g_{ab} = \exp(2\sigma)\bar{g}_{ab} \to \exp(2\sigma)\eta_{ab}$$

(all metrics conformally flat) and

$$\sqrt{g}R = \sqrt{\bar{g}}\bar{R} - 2\sqrt{\bar{g}}\,\bar{\Box}\,\sigma$$

gives a total derivative in S_{cl}.

Quantum Trace or Conformal Anomaly

$$\langle T_a{}^a \rangle = -\frac{c_m}{24\pi} R$$

$c_m = N_S + N_F$ for massless scalars or fermions.

Linearity in σ in the variational eq.

$$\frac{\delta \Gamma_{WZ}}{\delta \sigma} = \sqrt{g}\langle T_a{}^a \rangle$$

determines the Wess-Zumino Action by inspection:

2D Anomaly Action

- Integrating the linear anomaly gives

$$\Gamma_{WZ} = (c/24\pi) \int d^2x \sqrt{-g}\, (-\sigma\Box\sigma + R\sigma)$$

- This is local but non-covariant. Note kinetic term for
- By solving for σ the WZ action can be also written

$$\Gamma_{WZ} = S_{anom}[g] - S_{anom}[\bar{g}]$$

- Polyakov form of the action is covariant but non-local

$$S_{anom}[g] = -\frac{c}{96\pi} \int d^2x \sqrt{-g_x} \int d^2y \sqrt{-g_y}\, R_x\, (\Box^{-1})_{xy}\, R_y$$

- A covariant and local form requires an auxiliary dynamical field φ

$$S_{anom}[g;\varphi] = -\frac{c}{96\pi} \int d^2x \sqrt{-g}\, \{(\nabla\varphi)^2 - 2R\varphi\}$$

$$-\Box\varphi = R$$

2D Anomaly Stress Tensor

- The stress-energy tensor of the 2D anomaly action is

$$T_{ab}^{(anom)}[g;\varphi] \equiv -\frac{2}{\sqrt{-g}}\frac{\delta S_{anom}[g;\varphi]}{\delta g^{ab}} =$$
$$\frac{c}{24\pi}\left[\nabla_a\nabla_b\varphi - g_{ab}\Box\varphi + \frac{1}{2}\nabla_a\varphi\nabla_b\varphi - \frac{g_{ab}}{4}\nabla_c\varphi\nabla^c\varphi\right]$$

- **General soln. to $\Box\varphi = -R = f''$ with $\varphi(r^*)$ easily found in static de Sitter (Schwarzschild) : $ds^2 = f(r^*)(-dt^2 + dr^{*2})$**

$$\varphi = 2qHr^* + 2pHt + \ln f$$

$$T_t^{\ t} = \frac{cH^2}{24\pi}\left\{-\frac{1}{f}\left(p^2+q^2-1\right)+1\right\}$$
$$T_{r^*}^{\ r^*} = \frac{cH^2}{24\pi}\left\{\frac{1}{f}\left(p^2+q^2-1\right)+1\right\}$$
$$T_t^{\ r^*} = \frac{cH^2}{12\pi}\frac{pq}{f}$$

- Quantum stress tensor fully determined from the anomaly
- Generally divergent at $f=1-H^2r^2=0$
- Finite if $p = 0, q = \pm 1$ (BD)
- q is a kind of topological charge associated with Noether current

Ward Identity and Massless Poles

Effects of Anomaly may be seen in flat space amplitudes

Conservation of T_{ab} Ward Identity in 2D implies

$$\Pi_{abcd}(k) = (\eta_{ab}k^2 - k_a k_b)(\eta_{cd}k^2 - k_c k_d)\,\Pi(k^2)$$

Anomalous Trace Ward Identity in 2D implies

$$k^2\,\Pi(k^2) \neq 0$$ **at $k^2 = 0$ massless pole**

Effects of 2D Trace Anomaly

- Modification of Classical Theory required by Quantum Fluctuations & Covariant Conservation of $T^a{}_b$
- Metric conformal factor $e^{2\sigma}$ (was constrained) becomes **dynamical** & itself fluctuates freely
- Gravitational 'Dressing' of critical exponents: **long distance** macroscopic physics
- Non-perturbative/non-classical conformal fixed point of 2D gravity: Running of Λ
- Additional non-local **Infrared** Relevant Operator in S_{EFT}
- 'New' **Massless Scalar** Degree of Freedom in effective theory of low energy gravity

Quantum Trace Anomaly in Flat Space

QED in an External EM Field A_μ

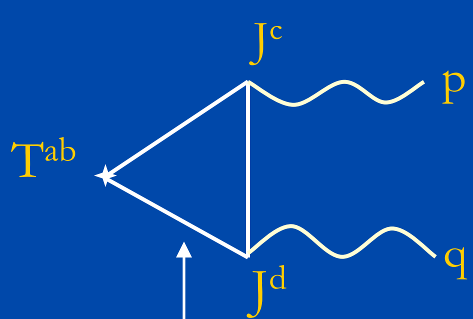
$$\langle T^\mu_\mu \rangle = \frac{e^2}{24\pi^2} F^{\mu\nu} F_{\mu\nu}$$

Triangle One-Loop Amplitude as in Chiral Case

$$\Gamma^{abcd}(p,q) = (k^2 g^{ab} - k^a k^b)(g^{cd}\, p\cdot q - q^c p^d)\, F_1(k^2) + (\text{traceless terms})$$

In the limit of massless fermions, $F_1(k^2)$ must have a massless pole:

$k = p + q$



$$F_1(k^2) = \frac{e^2}{18\pi^2 k^2}$$

$$\rho_T(s) \to \frac{e^2}{6\pi^2}\,\delta(s)$$

Corresponding Imag. Part Spectral Fn. has a δ fn
This is a new massless scalar degree of freedom in the two-particle correlated spin-0 state

<TJJ> Triangle Amplitude in QED

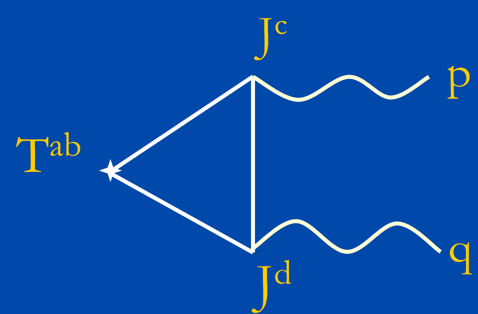
Determining the Amplitude by Symmetries and Its Finite Parts

M. Giannotti & E. M. *Phys. Rev. D* **79, 045014 (2009)**

$$\Gamma^{abcd}(p,q) = \int d^4x \int d^4y\, e^{ip\cdot x + iq\cdot y}\ \frac{\delta^2 \langle T^{ab}(0)\rangle_A}{\delta A_c(x)\delta A_d(y)}\bigg|_{A=0}$$

Γ^{abcd}: Mass Dimension 2 Use low energy symmetries:

$k = p + q$



1. By Lorentz invariance, can be expanded in a complete set of 13 tensors $t_i^{abcd}(p,q)$, $i = 1, \ldots 13$:

$$\Gamma^{abcd}(p,q) = \Sigma_i\, F_i\, t_i^{abcd}(p,q)$$

2. By current conservation: $p_c t_i^{abcd}(p,q) = 0 = q_d t_i^{abcd}(p,q)$

All (but one) of these 13 tensors are dimension ≥ 4, so $\dim(F_i) \leq -2$ so these scalar $F_i(k^2; p^2, q^2)$ are completely **UV Convergent**

<TJJ> Triangle Amplitude in QED

Ward Identities

3. By stress tensor conservation Ward Identity: $\partial_b \langle T^{ab} \rangle_A = eF^{ab} \langle J_b \rangle_A$

$$k_b \Gamma^{abcd}(p,q) = (g^{ac} p_b - \delta^c_b p^a)\Pi^{bd}(q) + (g^{ad} q_b - \delta^d_b q^a)\Pi^{bc}(p)$$

4. Bose exchange symmetry: $\Gamma^{abcd}(p,q) = \Gamma^{abdc}(q,p)$

Finally all 13 scalar functions $F_i(k^2; p^2, q^2)$ can be found in terms of

finite (IR) Feynman parameter integrals and the polarization,

$$\Pi^{ab}(p) = (p^2 g^{ab} - p^a p^b)\, \Pi(p^2)$$

$$\Gamma^{abcd}(p,q) = (k^2 g^{ab} - k^a k^b)(g^{cd}\, p\bullet q - q^c p^d)\, F_1(k^2; p^2, q^2) + \ldots$$

(12 other terms, 11 traceless, and 1 with zero trace when m=0)

Result:

$$F_1(k^2;p^2,q^2) = \frac{e^2}{18\pi^2 k^2}\left\{1 - 3m^2 \int_0^1 dx \int_0^{1-x} dy \frac{(1-4xy)}{D}\right\}$$

with $D = (p^2 x + q^2 y)(1-x-y) + xy\, k^2 + m^2$

UV Regularization Independent

<TJJ> Triangle Amplitude in QED

Spectral Representation and Finite Sum Rule

$$F_1(k^2;p^2,q^2) = \frac{1}{3k^2}\int_0^\infty \frac{ds}{k^2+s-i\epsilon}\left[(k^2+s)\rho_T - m^2\rho_m\right]$$

Numerator & Denominator cancel here

Im $F_1(k^2 = -s)$: Non-anomalous,vanishes when m=0

$$\rho_T(s;p^2,q^2) = \frac{e^2}{2\pi^2}\int_0^1 dx\int_0^{1-x} dy\ (1-4xy)\ \delta\left(s - \frac{(p^2x+q^2y)(1-x-y)+m^2}{xy}\right)$$

$$\int_0^\infty ds\,\rho_T(s;p^2,q^2) = \frac{e^2}{6\pi^2}$$

obeys a finite sum rule independent of p^2, q^2, m^2

and as p^2, q^2, m^2 ➔ 0^+

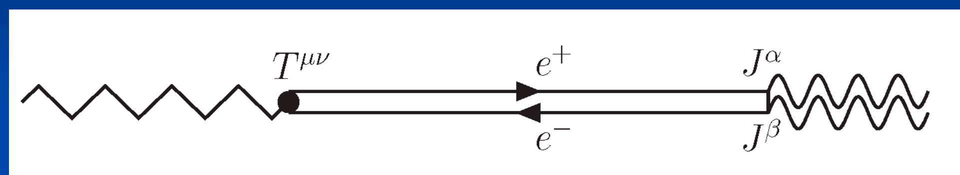
$$\rho_T(s) \to \frac{e^2}{6\pi^2}\,\delta(s)$$

$$F_1(k^2) \to \frac{e^2}{18\pi^2k^2}$$

Massless scalar intermediate two-particle state analogous to the pion in chiral limit of QCD

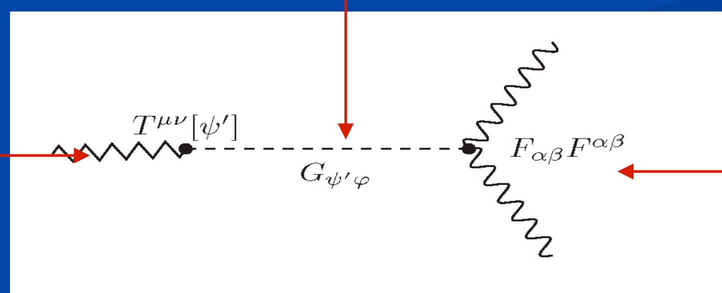
Massless Anomaly Pole

For $p^2 = q^2 = 0$ (both photons on shell) and $m_e = 0$ the pole at $k^2 = 0$ describes a *massless* $e^+ e^-$ pair moving at $v=c$ collinearly, with opposite helicities in a total spin-0 state



a massless scalar 0^+ state (Cooper pair) which couples to gravity

$h_{\mu\nu}(\partial^\mu \partial^\nu - \eta^{\mu\nu}\Box)\psi'$



Effective vertex

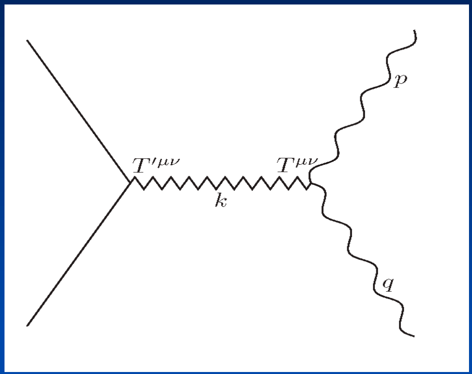
$\varphi F^{\alpha\beta} F_{\alpha\beta}$

Effective Action

special case of general form

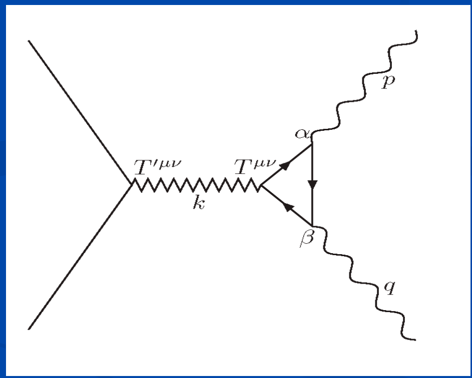
$$\int d^4x \sqrt{-g}\left\{-\psi'\Box\varphi - \frac{R}{3}\psi' - \frac{e^2}{48\pi^2}\varphi F^{\alpha\beta}F_{\alpha\beta}\right\}$$

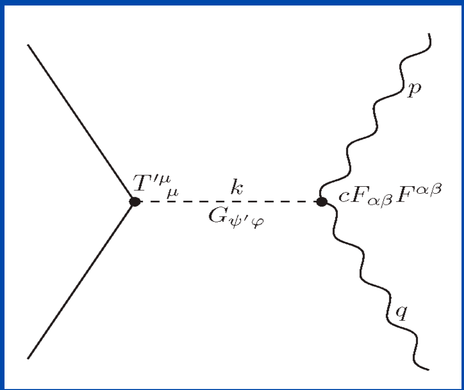
Scalar Pole in Gravitational Scattering



- In Einstein's Theory only transverse, tracefree polarized waves (spin-2) are emitted/absorbed and propagate between sources $T'^{\mu\nu}$ and $T^{\mu\nu}$
- The scalar parts give only non-progagating constrained interaction (like Coulomb field in E&M)

- But for $m_e = 0$ there is a scalar pole in the **<TJJ>** triangle amplitude coupling to photons
- This scalar wave propagates in gravitational scattering between sources $T'^{\mu\nu}$ and $T^{\mu\nu}$





- Couples to trace $T'^{\mu}{}_{\mu}$
- **<TTT>** triangle of massless photons has similar pole
- At least one new scalar degree of freedom in EFT

4D Anomalous Effective Action

Conformal Parametization

$$\rightarrow \qquad g_{ab} = \exp(2\sigma)\,\bar{g}_{ab}$$

Since $\sqrt{g}\,F_4 = \sqrt{\bar{g}}\,\bar{F}_4$

is independent of σ, and

$$\sqrt{g}\,(E_4 - \tfrac{2}{3}\Box R) = \sqrt{\bar{g}}\,(\bar{E}_4 - \tfrac{2}{3}\bar{\Box}\bar{R}) + 4\sqrt{\bar{g}}\bar{\Delta}_4\sigma$$

is linear in σ, the variational eq.,

$$\frac{\delta\Gamma_{WZ}}{\delta\sigma} = \sqrt{g}\langle T_a{}^a\rangle = b\,\sqrt{g}F_4 + b'\sqrt{g}\,(E_4 - \tfrac{2}{3}\Box R)$$

determines the Wess-Zumino Action by inspection:

$$\Gamma_{WZ} = 2b'\int d^4x\sqrt{\bar{g}}\,\sigma\bar{\Delta}_4\sigma$$

$$+\int d^4x\sqrt{\bar{g}}\left[b\bar{F}_4 + b'\left(\bar{E}_4 - \frac{2}{3}\bar{\Box}\bar{R}\right)\right]\sigma\ ,$$

$$\Delta_4 \equiv \Box^2 + 2R^{ab}\nabla_a\nabla_b - \tfrac{2}{3}R\Box + \tfrac{1}{3}(\nabla^a R)\nabla_a$$

$F_4 = F = C_{abcd}C^{abcd}$; $E_4 = E = R_{abcd}R^{abcd} - 4R_{ab}R^{ab} + R^2$

The Additional Term in the Action

$$S_{anom} = \frac{Q^2}{(4\pi)^2} \int d^4x \sqrt{-g} \int d^4x' \sqrt{-g'} \times$$

$$\left(E - \frac{2}{3}\Box R\right) (\Delta_4)^{-1}_{xx'} \left(E - \frac{2}{3}\Box R\right)' ;$$

$$\Delta_4 \equiv \Box^2 + 2R^{ab}\nabla_a\nabla_b + \frac{1}{3}(\nabla^a R)\nabla_a - \frac{2}{3}R\Box$$

$$E \equiv R_{abcd}R^{abcd} - 4R_{ab}R^{ab} + R^2$$

This is a long distance modification of General Relativity

It allows the scalar part of the metric σ to fluctuate

(In the Einstein theory σ is constrained.)

The σ fluctuations lead to anomalous scaling dimensions for low energy dimensionful parameters:

$$\beta_p = 4 - p + \frac{\beta_p^2}{2Q^2}$$

Only free parameter is:

$$Q^2 = \frac{1}{180}(N_s + 11N_f + 62N_v - 28) + Q^2_{grav}$$

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Effective Action of 4D Gravity

By inverting the eq. for σ the local W-Z action may be expressed as a difference of fully covariant but non-local actions,

$$\Gamma_{WZ}[\bar{g};\sigma] = S_{anom}[g] - S_{anom}[\bar{g}] \quad \text{and}$$

$$S_{eff}[g] = \frac{1}{16\pi G}\int d^4x\sqrt{g}\,(R - 2\Lambda) + S_{anom}[g]$$

Consequences of Conformal Anomaly

- S_{anom} determined by general principles of covariance and QM, independently of any Planck scale physics.
- Additional term is relevant at large distances (scales as a marginally relevant operator under $\sigma \to \sigma + \sigma_0$).
- Conformal factor, constrained in classical Einstein theory, contains new dynamical degrees of freedom.
- New conformally invariant phase of gravity in 4D
- Running of G^{-1} and Λ to zero in this new phase.
- Possible imprint on CMBR Spectrum and Statistics.

I. Antoniadis, P. O. Mazur, E. M., Phys. Rev. D 55 (1997) 4756, 4770;
Phys. Rev. Lett. 79 (1997) 14

Effective Action for the Trace Anomaly
Local Auxiliary Field Form

$$S_{anom} = \frac{b}{2}\int d^4x\sqrt{-g}\left[-2\varphi\Delta_4\psi + F\,\varphi + \left(E_4 - \frac{2}{3}\Box R\right)\psi\right]$$
$$+\frac{b'}{2}\int d^4x\sqrt{-g}\left[-\varphi\Delta_4\varphi + \left(E_4 - \frac{2}{3}\Box R\right)\varphi\right]$$

- New Conformal Anomaly Degree(s) of Freedom
- Variation of the action with respect to -- the anomaly scalar fields -- leads to the equations of motion,

$$\Delta_4\varphi = \frac{1}{2}\left(E_4 - \frac{2}{3}\Box R\right) \qquad \Delta_4\psi = \frac{1}{2}F$$

$$\Delta_4 = \Box^2 + 2R^{ab}\nabla_a\nabla_b - \frac{2}{3}R\Box + \frac{1}{3}(\nabla^a R)\nabla_a$$

IR Relevant Term in the Action

The effective action for the trace anomaly scales logarithmically with distance and therefore should be included in the low energy macroscopic EFT description of gravity—

Not given in powers of Local Curvature

This is a non-trivial modification of classical General Relativity required by quantum effects in the Std. Model

$$S_{Gravity}[g,\varphi,\psi] = S_{H-E}[g] + S_{Anom}[g,\varphi,\psi]$$

Fluctuations of new scalar degrees of freedom allow Λ_{eff} to vary dynamically, and can generate a Quantum Conformal Phase of 4D Gravity where $\Lambda_{\text{eff}} \to 0$

Stress Tensor of the Anomaly

Variation of the Effective Action with respect to the metric gives stress-energy tensor

$$T_{\mu\nu}(g_{\mu\nu}, \varphi, \psi) = -\frac{2}{\sqrt{-g}} \frac{\delta S_{anom}}{\delta g_{\mu\nu}}$$

- Quantum Vacuum Polarization in Terms of (Semi-) Classical Auxiliary potentials
- φ,ψ Depend upon the global topology of spacetimes and its boundaries, horizons

Conformal Symmetry Near Horizons

- An horizon is a characteristic null surface,
  conformal to flat space light cone
- Fields become effectively massless there
- Conformal Anomaly becomes the dominant
  term in the effective action
- Stress Tensor(s) from S_{anom} determines $<T_{ab}>$
- Stress Tensor is generally singular there
- Singular behavior has invariant meaning in terms of new spacetime scalar degrees of freedom on horizon

Conformal Symmetry & de Sitter Horizon

De Sitter space in static coordinates

$$ds^2 = -(1 - H^2 r^2) dt^2 + \frac{dr^2}{1 - H^2 r^2} + r^2 d\Omega^2$$

$$= f(r)\, ds^2_{opt}\,, \qquad f(r) \equiv 1 - H^2 r^2$$

Conformal to 'optical' metric

$$ds^2_{opt} = -dt^2 + d\chi^2 + \sinh^2\chi\, d\Omega^2$$

Lobachewsky (Euclidean AdS) Space

$$Hr = \tanh\chi \qquad d\ell^2_{Ł}$$

AdS/CFT: expect conformal behavior

on the horizon boundary $\mathbb{S}^2$ at $r = r_H = 1/H$

Conformal Scaling & de Sitter Horizon

Lobachewsky/EAdS$_3$ in Poincare coordinates

$$\bar{x} \equiv \frac{r\, n_x}{1 - H r n_z}$$

$$\bar{y} \equiv \frac{r\, n_y}{1 - H r n_z}$$

$$\bar{z} \equiv \frac{r_H\, f^{\frac{1}{2}}}{1 - H r n_z}$$

$\mathbb{S}^2$ at r=r$_H$ mapped to $\mathbb{R}^2$ at $\bar{z} = \infty$

$$d\ell_{Ł}^2 = \frac{1}{H^2 \bar{z}^2} \left(d\bar{z}^2 + d\bar{x}^2 + d\bar{y}^2 \right)$$

Distance $d_{Ł}(x, x') = \frac{\left[(\bar{z} - \bar{z}')^2 + (\bar{x} - \bar{x}')^2 + (\bar{y} - \bar{y}')^2\right]}{4 \bar{z} \bar{z}'}$

$$\to \frac{1 - \hat{n}\cdot\hat{n}'}{2\sqrt{f f'}}, \qquad r \to r_H$$

Conformal field: weight w $\quad \Phi_w(r) \propto \bar{z}^w \propto [f(r)]^{\frac{w}{2}}$

$$\langle \Phi_w(r, \hat{n}) \Phi_w(r', \hat{n}') \rangle \sim [d_{Ł}(x, x')]^{-w} \to (f f')^{\frac{w}{2}} (1 - \hat{n}\cdot\hat{n}')^{-w}$$

Anomaly Stress Tensor in de Sitter Space

- General soln. for φ as fn. of static r and linear in t is

$$\varphi(r,t)\Big|_{dS} = c_0 + 2Hpt + \ln\left(1 - H^2r^2\right) + \frac{q}{2}\ln\left(\frac{1-Hr}{1+Hr}\right) + \frac{2c_H - 2 - q}{2Hr}\ln\left(\frac{1-Hr}{1+Hr}\right)$$

- Bunch-Davies state has p = 1, q = 0, $c_H = 1$

$$T_{ab}\Big|_{BD,dS} = 6b' H^4 g_{ab} = -\frac{H^4}{960\pi^2}\, g_{ab}\left(N_s + 11N_f + 62N_v\right)$$

- This is the soln. for conformal map to flat spacetime

$$ds^2 = e^{\varphi_{BD}}\,(ds^2)_{flat}$$

- Otherwise T_{ab} is generally divergent at the static horizon $r=H^{-1}$ behaving like $(1-H^2r^2)^{-2}$ **PRD 74, 064004 (2006)**

Linear Response in de Sitter Space

w. P. R. Anderson & C. Molina-Paris, *Phys. Rev. D* 80, 084005 (2009)

- Variation of the state of QFT away from Bunch-Davies produces variation of <$T^a{}_b$> in de Sitter of the kind

$$\langle T^a{}_b \rangle_R \to \frac{\pi^2}{90}\,\frac{k_{_B}^4}{(\hbar c)^3}\,\frac{(T^4 - T_{_H}^4)}{(1 - H^2 r^2)^2}\,\mathrm{diag}\,(-3, 1, 1, 1)$$

- These variations on/near the horizon are described by the scalar degree(s) of freedom of the anomaly

$$\delta\rho = \frac{2H^2 b'}{3}\,\frac{\vec{\nabla}^2}{a^2} u$$

$$u_{\vec{k}} \equiv \left(\frac{d^2}{dt^2} + H\frac{d}{dt} + k^2 e^{-2Ht}\right)\varphi$$

$$\left(\frac{d^2}{dt^2} + 5H\frac{d}{dt} + 6H^2 + k^2 e^{-2Ht}\right) u_{\vec{k}} = 0$$

Linear Response in de Sitter Space

- Variation of $<T^a{}_b>$ in de Sitter space contains contributions from S_{anom} of scalar anomaly fields **φ, ψ**
- Same as varying the state of underlying QFT from Bunch-Davies
- Additional massless scalar degrees of freedom in cosmology
- The relevant scalar modes satisfy second order wave eqs. ("Inflaton without inflaton")
- Couple weakly to the metric with strength $G_N H^2 << 1$
- But grow significant close to the de Sitter horizon $r_H = H^{-1}$

$$G_N \delta\langle T^t_t\rangle \sim \frac{G_N H^4}{(1 - H^2 r^2)^2}$$

Large One-Loop Effect
but Spatially Inhomogeneous

- Becomes of order of classical $R^t{}_t = 3H^2$ at a macroscopic proper distance from r_H $\quad \ell \sim \sqrt{r_H L_{Pl}}$ (not zero, so not infinite $T^a{}_b$)
- Should be quantized

Cosmological Horizon Modes

$$\delta\left(R^a_{\ b} - \frac{R}{2}\,\delta^a_{\ b} + \Lambda\,\delta^a_{\ b}\right) = 8\pi G\,\delta\langle T^a_{\ b}\rangle$$

$T^a_{\ b}$ is Stress Tensor of Anomaly Action

Two Gauge Invariant Gravitational Potentials:

$$\bar{g}_{\tau\tau} + h_{\tau\tau} = -(1 + 2\Upsilon_{\mathcal{A}})$$ in FRW coordinates

$$\bar{g}_{ij} + h_{ij} = a^2(\tau)\,(1 + 2\Upsilon_{\mathcal{C}})\,\delta_{ij} + h^{\perp}_{ij}$$

Solns. of Linearized Einstein eqs. are:

$$\Upsilon_{\mathcal{A}} + \Upsilon_{\mathcal{C}} = -\frac{16\pi G H^2 b'}{3}\left[\frac{c_1}{f} + \frac{c_2}{H r f}\right]$$ in static coordinates

$$\Upsilon_{\mathcal{C}} - \Upsilon_{\mathcal{A}} = 8\pi G H^2 b'\left[\frac{c_1}{Hr}\ln\left(\frac{1 - Hr}{1 + Hr}\right) + \frac{c_2}{Hr}\ln f\right]$$

Correct log scaling for scale invariant Harrison spectrum
Fluctuations of Anomaly Fields can generate CMB w/o inflaton

3-pt. Correlator on Horizon Sphere

- **Conformal Identities Applied on Horizon Sphere give**

$$G_3(\hat{n}_1, \hat{n}_2, \hat{n}_3; w) = \frac{a_3(w)}{[(1-\hat{n}_1\cdot\hat{n}_2)(1-\hat{n}_2\cdot\hat{n}_3)(1-\hat{n}_1\cdot\hat{n}_3)]^{\frac{w}{2}}}$$

- **As $w \to 0$ appropriate for HZ scale invariance this becomes**

$$G_3(\hat{n}_1, \hat{n}_2, \hat{n}_3; 0) = C_3\left[\ln(1-\hat{n}_1\cdot\hat{n}_2) + \ln(1-\hat{n}_2\cdot\hat{n}_3) + \ln(1-\hat{n}_1\cdot\hat{n}_3)\right] + c$$

which is completely separable and different again from both the prediction of slow roll inflation and the Shape Fn. found from Conformal Invariance of flat R^3 sections embedded in deS space

- If this non-Gaussian Bispectrum G_3 is observed in the CMB it implies the temperature fluctuations arise at or near the deS horizon S^2, related to the cosmological dark energy today, and
- Our Universe is then not globally spatially homogeneous and isotropic---Physics at the Horizon

Λ as Vacuum Energy of a Gravitational Bose-Einstein Condensate

- **The Conformal Factor of the metric $g_{ab} = e^{2\sigma}\bar{g}_{ab}$ is frozen by the classical Einstein's Eqs. $R = 4\Lambda$**
- **But the trace anomaly of massless quantum fields forces the scalar 'condensate' $\langle e^{2\sigma}\rangle$ to fluctuate**
- **This generates a well-defined additional term in the low energy action, and**
- **Describes a New Conformally Invariant Phase, Infrared Fixed Point of Gravity**
- **The quantum phase transition to this phase is characterized by 'melting' of the scalar condensate $\langle e^{2\sigma}\rangle$**

Λ_{eff} Dynamical, generated by SSB of Global Conformal Invariance $\sigma \rightarrow \sigma + \sigma_0$

Summary

De Sitter Space is *unstable* to Quantum Pair Creation and Fluctuations about the **O(4,1)** Invariant State

Many Analogies with Electric Fields in QED

The Conformal/Trace Anomaly encapsulates these non-local effects & gives Relevant Perturbations to de Sitter Space

There are *new scalar degree(s) of freedom* in this *extended EFT of Gravity* required by the Conformal Anomaly

They are important at *macroscopic* Distances & near the Cosmological de Sitter Event Horizon

$\Lambda_{\mathbf{eff}}$ is <u>not</u> a constant as a result but a *dynamical condensate*

The observed dark energy of our Universe may be a *macroscopic finite size effect* whose residual small value depends on the cosmological horizon boundary—non FRW globally