The global de Sitter S-matrix and bulk unitarity

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AEI Hannover – September 12, 2012

The S-matrix

The S-matrix is an invaluable tool for QFT on Minkowski space

- gauge invariant
- invariant under field redefinitions
- admits powerful theorems which reveal structure of Minkowski QFT: Coleman-Mandula, Haag-Lopuszanksi-Sohnius, Weinberg-Witten, ...

At a more mundane level, the S-matrix:

- allows clean comparison of different approaches, choices of gauge, etc.
- is useful for resolving controversies, hastening advances in knowledge

In cosmological setting:

- lack of an S-matrix (or equivalent) has been sorely felt for decades
- remain controversies over the interpretations simple self-interacting theories on fixed backgrounds as well as the more complicated case of gravitational theories

The de Sitter S-matrix

In this talk we introduce the S-matrix for weakly-coupled quantum theories on non-dynamical global de Sitter that may be computed order-by-order in perturbation theory.

For massive scalar fields, we can verify that the S-matrix is:

- unitary
- dS-invariant
- invariant under perturbative field redefinitions
- transforms appropriately under CPT
- reduces to the usual S-matrix in the flat-space limit

We will offer preliminary evidence that a *perturbative* S-matrix – or similar structure – exists for gauge fields and gravity.

Explain why we expect an analogous construction for QFTs on a Poincaré chart.

Bulk unitarity, the S-matrix, and dS/CFT

Further motivation: understand how bulk unitarity constrains the asymptotic behavior of bulk fields.

The S-matrix is unitary map $\mathcal{H} \mapsto \mathcal{H}$; it is an ideal tool for studying the implications of unitarity.

Strategy:

bulk S-matrix $\longleftrightarrow$ asympt. behavior of bulk correlators $\longleftrightarrow$ dS/CFT

Concrete dS/CFT realizations:

- Vasiliev dS_4/CFT_3 [Anninos et. al 2011, Ng Strominger 2011, Anninos et. al 2012]
- dS_5/conformal gravity$_4$ [Maldacenta 2011]
- common feature: Euclidean CFT duals are *not* reflection-positive ("unitary")

Key question: how is bulk unitarity encoded in the Euclidean CFT?

Outline

Outline

Five objections to the dS S-matrix

1. The Minkowski S-matrix is defined using in/out perturbation theory, but in/out perturbation theory in dS suffers from IR divergences. So this definition does not work in de Sitter space. [Polyakov 2007, 2009, Akhmedov 2008, 2009]
2. There is no positive-definite energy-like conserved quantity in (global or Poincaré) de Sitter space. As a result, 1-particle states can decay and all particles are unstable. So there should be no viable asymptotic states. [E.g., Nachtmann 1968, Myhrvold 1983, Boyanovsky 1996, Boyanovsky 2011].
3. The causal structure of global de Sitter space prevents any one observer from interacting with a complete set of ingoing/outgoing states. Therefore that the S-matrix is not experimentally accessible to a single observer and need not necessarily be a well-defined object in a fundamental theory.

Five objections to the dS S-matrix

4. The contracting phase of global de Sitter space tends to blueshift particles to high energies. In a theory with dynamical gravity, many states which are weakly-coupled near past infinity induce large gravitational back-reaction near the minimal-radius sphere. Semi-classically, this should result in gravitational collapse to a cosmological singularity. There is thus no reason to expect weakly-coupled asymptotic states near the future de Sitter boundary.
5. At least in string theory, all known de Sitter vacuua are at best meta-stable. So one expects that mere particle excitations of a de Sitter background cannot provide a complete set of outgoing states.

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5. At least in string theory, all known de Sitter vacuua are at best meta-stable. So one expects that mere particle excitations of a de Sitter background cannot provide a complete set of outgoing states.

Response:

- Issues 4-5 involve dynamical gravity and/or string theory, are non-perturbative in nature, and are not the subject of this talk.

Five objections to the dS S-matrix

3. The causal structure of global de Sitter space, prevents any one observer from interacting with a complete set of ingoing/outgoing states. Therefore the S-matrix is not experimentally accessible to a single observer and need not necessarily be a well-defined object in a fundamental theory.

Response:

- True! But we can nevertheless hope that a de Sitter S-matrix provides a useful theoretical tool, even if not required to exist.

Five objections to the dS S-matrix

There remain the two technical concerns:

1. The Minkowski S-matrix is defined using in/out perturbation theory, but in/out perturbation theory in dS suffers from IR divergences. So this definition does not work in de Sitter space. [Polyakov 2007, 2009, Akhmedov 2008, 2009]

Key technical differences for dS S-matrix:

- We use an appropriate Schwinger-Keldysh perturbation theory rather than "in-out".
- Construct vacuum correlators first, construct particle states at finite time, take $t \to \pm\infty$ limit
- Guarantees particle states are perturbatively connected to vacuum.

Five objections to the dS S-matrix

There remain the two technical concerns:

2. There is no positive-definite energy-like conserved quantity in (global or Poincaré) de Sitter space. As a result, 1-particle states can decay and all particles are unstable. So there should be no viable asymptotic states. [E.g., Nachtmann 1968, Myhrvold 1983, Boyanovsky 1996, Boyanovsky 2011].

Key technical differences for dS S-matrix:

- For heavy fields asympt. particle states will be "unstable," just like "unstable" particle state in Minkowski.
- For very light scalars $M^2\ell^2 = \mathcal{O}(1)$ and gauge fields, "stable" asympt. particle states exist [cf. Bros et al 2006 2008].
- Asympt. particle states do not remain orthogonal $\Rightarrow$ must construct orthonormal bases of initial/final states from initial/final particle states.

Outline

global de Sitter space

D-dimensional de Sitter manifold dS_D:

$$dS_D = \left\{ X \in \mathbb{R}^{D,1} \mid X \cdot X = \ell^2 \right\}.$$

dS isometry group is $SO(D,1)$.

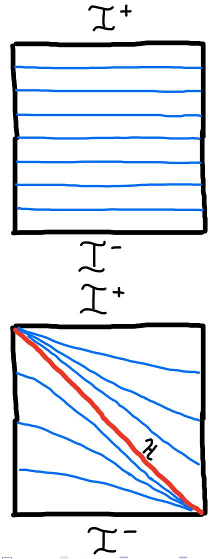
Two global charts

$$\frac{ds^2}{\ell^2} = \left[-\frac{1}{1+\eta^2} d\eta^2 + (1+\eta^2) d\Omega^2_{D-1} \right], \quad \eta \in \mathbb{R}.$$

relation to $g_{tt} = -1$ time: $\eta = \sinh(t/\ell)$

$$\frac{ds^2}{\ell^2} = \tau^2 \left[-\frac{1}{\tau^4} d\tau^2 + d\vec{x}^2 \right], \quad \tau \in \mathbb{R}.$$

relation to conformal time: $\tau = -1/\lambda$



scalar dS QFTs

Fields on de Sitter form representations of $SO(D,1)$.

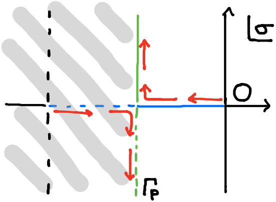
For instance, for a free scalar field $\phi_\sigma(x)$

$$\mathcal{L}_0[\phi_\sigma] = -\frac{1}{2}\nabla_\mu\phi_\sigma\nabla^\mu\phi_\sigma(x) - \frac{M^2}{2}\phi_\sigma^2(x)$$

Define the *weight*:

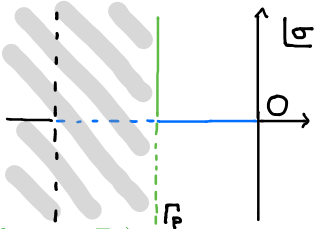
$$M^2(\sigma)\ell^2 = -\sigma(\sigma + D - 1), \quad \Rightarrow \quad \sigma := -\frac{(D-1)}{2} + \left[\frac{(D-1)^2}{4} - M^2\ell^2\right]^{1/2}.$$

Complex σ plane:



scalar dS QFTs

1-particle states form UIRs T^σ of $SO(D,1)$:



1. principal series: (solid green, Γ_p)

$$\frac{(D-1)^2}{4} \leq M^2\ell^2, \quad \Rightarrow \quad \sigma = -\frac{(D-1)}{2} + i\rho, \;\; \rho \in \mathbb{R}, \;\; \rho \geq 0,$$

2. complementary series: (solid blue, negative real line)

$$0 < M^2\ell^2 < \frac{(D-1)^2}{4}, \quad \Rightarrow \quad \sigma \in \left(-\frac{(D-1)}{2}, 0\right),$$

3. discrete series:

$$M^2\ell^2 = -n(n+D-1) \text{ for } n \in \mathbb{N}_0, \quad \Rightarrow \quad \sigma = n.$$

Klein-Gordon modes

A basis of solutions to EOM: $u_{\sigma\vec{L}}(x) = \ell^{(2-D)/2} f_{\sigma L}(\eta) Y_{\vec{L}}(\vec{x})$.

Modes of the same weight form an complete orthonormal set:

$$\begin{aligned}\delta_{\vec{L}_1\vec{L}_2} &= -i \int d\Sigma^{\nu}(x) \left[u_{\sigma\vec{L}_1}(x) \overleftrightarrow{\nabla}_{\nu} u^*_{\sigma\vec{L}_2}(x) \right] \Big|_{\eta=\text{const.}} \\ &= -i\ell^{D-2}(1+\eta^2)^{D/2} \int d\Omega_{D-1}(\vec{x}) \left[u_{\sigma\vec{L}_1}(x) \overleftrightarrow{\partial}_{\eta} u^*_{\sigma\vec{L}_2}(x) \right] \Big|_{\eta=\text{const.}} .\end{aligned}$$

Asymptotics: as $|\eta| \to \infty$,

$$f_{\sigma L}(|\eta| \gg 1) \sim K_{\sigma L}(\eta)^{\sigma} + K_{-(\sigma+D-1)L}(\eta)^{-(\sigma+D-1)}.$$

- complementary series: two real decays, σ weaker decay
- principal series: two decays $\eta^{-(D-1)/2 \pm i\rho}$, $\rho \in \mathbb{R}$

Klein-Gordon fields

For Klein-Gordon fields quantization is straight-forward. May introduce time-independent ladder operators

$$a^\dagger_{\sigma\vec{L}} := -i \int d\Sigma^\nu(\vec{x}) \left[u_{\sigma\vec{L}}(x) \overleftrightarrow{\nabla}_\nu \phi_\sigma(x) \right] \Big|_{\eta=\text{const}}, \quad a_{\sigma\vec{L}} := \text{h.c.}$$

Canonical commutation relations:

$$\left[a_{\sigma\vec{L}}, a_{\sigma\vec{K}}\right] = 0 = \left[a^\dagger_{\sigma\vec{L}}, a^\dagger_{\sigma\vec{K}}\right], \quad \left[a_{\sigma\vec{L}}, a^\dagger_{\sigma\vec{K}}\right] = \delta_{\vec{L}\vec{K}}.$$

Trivial S-matrix:

$$S = S^\dagger = 1$$

1. asymptotic particle states form orthonormal basis for Fock space
2. asymptotic particle states enjoy particle interpretation on entire manifold
3. can choose final states = initial states
4. no particle production in this basis

The Hartle-Hawking state $|\Omega\rangle$ in interacting theories

This talk will focus on the dense set of states constructed from the Hartle-Hawking state $|\Omega\rangle$ (a.k.a. Bunch-Davies, a.k.a. Euclidean state) of interacting theories.

Many ways to construct local correlators of $|\Omega\rangle$: [Higuchi Marolf IM 2011]

1. on S^{D+1}: construct $SO(D+1)$-invariant state, analytically continue in position space
2. in Poincaré chart: “in-in” Schwinger-Keldysh contour with “Bunch-Davies” boundary conditions
3. in Static chart: thermal Schwinger-Keldysh contour with relativistic KMS boundary condition
4. in global de Sitter: suitable Schwinger-Keldysh contour described in this talk

Not equivalent to constructing a state at global dS Cauchy surface Σ_η, then taking $\eta \to \pm\infty$. [Krotov Polyakov 2011 vs. Marolf IM 2011]

The Hartle-Hawking state $|\Omega\rangle$ in interacting theories

Why focus on this set of states?

Positivity

- Under criteria that is reasonable for scalar QFTs, QFTs on dS_D and S^D are related via the dS version of the Osterwalder-Shrader thm [Schlingemann 2009]
- In $D = 2$ can religiously verify the analytic continuation between Euclidean and Lorentzian correlators as well as causality, regularity, positivity conditions [Frölich $<$ 1985].

Cluster decomposition [Marolf IM 2011, Hollands 2011]

- Correlators of $|\Omega\rangle$ enjoy a version of cluster decomposition associated to large distances (timelike & acausal). If all x_i are taken to large separations from all y_j:

$$\langle\phi(x_1)\phi(x_2)\dots\phi(y_1)\phi(y_2)\dots\rangle_\Omega \to \langle\phi(x_1)\phi(x_2)\dots\rangle_\Omega \, \langle\phi(y_1)\phi(y_2)\dots\rangle_\Omega \, .$$

The Hartle-Hawking state $|\Omega\rangle$ in interacting theories

Why focus on this set of states?

A curved space notion of quantum stability [Marolf IM 2011, Hollands 2011]

- The set of normalized states of the form

$$|\Psi\rangle = \int_{y_1} \dots \int_{y_n} f(y_1, \dots, y_n)\phi(y_1)\dots,\phi(y_n)\,|\Omega\rangle\,,$$

  has $|\Omega\rangle$ as an attractor state for local operators in the asymptotic regions of de Sitter :

$$\langle\phi(x_1)\dots\phi(x_m)\rangle_\Psi \to \langle\phi(x_1)\dots\phi(x_m)\rangle_\Omega\,.$$

- The existence of an attractor state provides a strong notion of stability for QFT in curved spacetime, where a "lowest energy eigenstate" is unavailable.
- The Reeh-Schlieder thm of curved spacetime [Stromaier et al. 2002] proves that this set of states is dense on the Hilbert space containing $|\Omega\rangle$.

Outline

Summary of differences between dS and Minkowski S-matrix

In most respects construction of dS S-matrix is similar to that of Minkowski space.

Three key differences:

1. Time-dependant background ⇒ use Schwinger-Keldysh perturbation theory to construct HH correlators. (Do not use "in-out" pert. thy.)
2. Lack of conserved energy ⇒ particle states do not remain orthogonal. We explicitly construct orthonormal initial/final states.
3. IR divergences in naive LSZ formulation ⇒ add a "projection operator" R_σ in construction of asymptotic states.

May understand S-matrix as a matrix of amplitudes of quantum states constructed explicitly in dS.

May also define S-matrix by residues of poles in a suitable complex weight (σ) plane.

Asymptotic particle states

Consider a field $\phi_\sigma(x)$ with:

1. Bare mass $M^2(\sigma) > 0$
2. mass gap (determined by the Lehmann-Källen weight)

Properties of initial (final) states $|\psi\rangle_{i/f}$ satisfied as $\eta \to -\infty$ $(+\infty)$:

1. normalizable: $|{}_{i/f}\langle a|b\rangle_{i/f}| < \infty$
2. definite particle content labelled by dS UIRs

$$|a\rangle_{i/f} := |n_1, n_2, \ldots, n_k\rangle_{i/f}\,, \quad n = (\sigma, \vec{L})$$

3. states transform as direct products of UIRs under dS group

$$U(g)|n_1, n_2, \ldots, n_k\rangle_{i/f} = |gn_1, gn_2, \ldots, gn_k\rangle_{i/f}, \quad gn = (m^2, \vec{L}')$$

4. desire flat-space limit ⇒ initial/final vacuua are Hartle-Hawking state $|\Omega\rangle$

Asymptotic particle states

Construction

LSZ prescription with addition of a "projection operator" R_σ

$$|n_1, n_2, \ldots, n_k\rangle_{i/f} = \lim_{\eta \to \mp\infty} a^\dagger_{n_1}(\eta) a^\dagger_{n_2}(\eta) \ldots a^\dagger_{n_k}(\eta) |\Omega\rangle,$$

$$a^\dagger_n(\eta) = -i \int d\Sigma^\nu(\vec{x}) \left[u_n(x) \overleftrightarrow{\nabla}_\nu R_\sigma \phi_\sigma(x) \right] \Big|_\eta ,$$

Projection operator R_σ

R_σ ensures that ${}_{i/f}\langle a|b\rangle_{i/f}$ is free of power-law IR divergences.

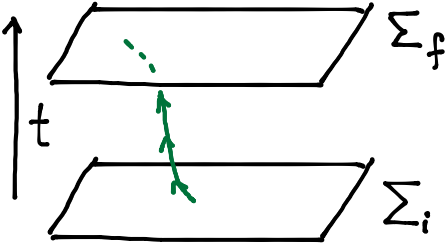
- selects the "mass pole" part of $\phi_\sigma(x)$
- free fields: $R_\sigma \phi_\sigma(x) = \phi_\sigma(x)$
- theories of heavy fields R_σ can generally be ignored

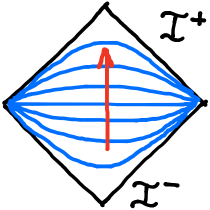
Preserves the logarithmic IR divergences expected in perturbation theory (which encode perturbative renormalization, anomalies, . . .).

Schwinger-Keldysh perturbation theory

Correlation functions of the HH state may be constructed explicitly in Lorentzian dS using an appropriate Schwinger-Keldysh contour.

In Minkowski "in-out" construction:



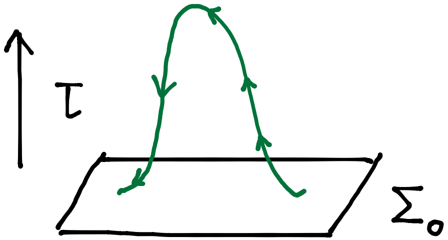


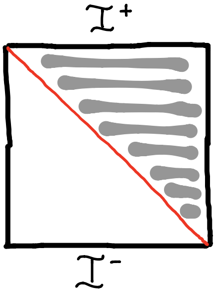
In diagrammatic expansion there is one type of vertex.

Schwinger-Keldysh perturbation theory

Correlation functions of the HH state may be constructed explicitly in Lorentzian dS using an appropriate Schwinger-Keldysh contour.

In Poincaré "in-in" construction:



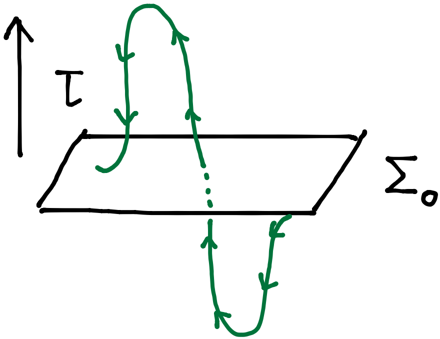


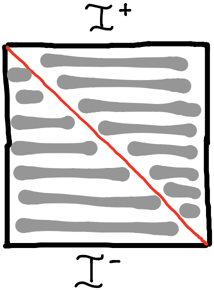
In diagrammatic expansion there are two types of vertices.

Schwinger-Keldysh perturbation theory

Correlation functions of the HH state may be constructed explicitly in Lorentzian dS using an appropriate Schwinger-Keldysh contour.

In global dS construction:

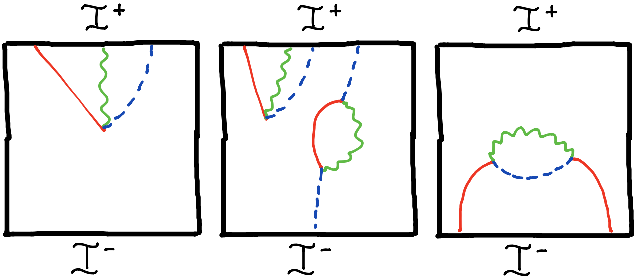




In diagrammatic expansion there are three types of vertices.

Orthonormalization

There exist non-vanishing contributions to particle states in same basis ${}_i\langle a|b\rangle_i$. E.g.,



Give to each initial particle state $|a\rangle_i$ an order $I(a)$, letting the vacuum $|\Omega\rangle$ have the lowest order. Orthonormal initial basis $\{|A\rangle_i\}$ may be constructed as follows:

$$|B\rangle_i = \frac{|b\rangle_i - \sum_{I(A)<I(b)} |A\rangle_i \, {}_i\langle A|b\rangle_i}{\left[{}_i\langle b|b\rangle_i - \sum_{I(A)<I(b)} |{}_i\langle A|b\rangle_i|^2 \right]^{1/2}}, \quad I(B) = I(b).$$

The S-matrix

S-matrix:

$$S := \{{}_f\langle A|B\rangle_i\}$$

Properties

1. The vacuum-to-vacuum amplitude is unity.
2. Covariance under the dS group:

$$_f\langle A|B\rangle_i = {}_f\langle A|1\,|B\rangle_i = {}_f\langle A|U^{-1}(g)U(g)\,|B\rangle_i = {}_f\langle gA|gB\rangle_i\,.$$

3. Behavior under CPT: $\Theta S = S^{-1}\Theta$
4. Invariance under perturbative field-redefinitions:

$$\phi_\sigma(x) \to \phi_\sigma(x) + g\mathscr{O}(x), \quad |g| \ll 1.$$

5. Unitarity: $S^\dagger S = 1$ and $SS^\dagger = 1$. Equivalently, for $S = 1 + i\mathcal{T}$ have the Optical theorem

$$2\mathrm{Im}\,\mathcal{T} = \mathcal{T}^\dagger\mathcal{T}$$

Outline

Model theory

Consider a theory of three massive scalars $\phi_i(x)$, $i = 1, 2, 3$, on dS_D:

$$
\begin{aligned}
\mathcal{L}[\vec{\phi}] &= \sum_{i=1}^{3} \mathcal{L}_0[\phi_i] + \mathcal{L}_{\text{int}}[\vec{\phi}] + \mathcal{L}_{\text{c.t.}}[\vec{\phi}], \\
\mathcal{L}_{\text{int}}[\vec{\phi}] &= g\phi_3\phi_2\phi_1(x), \\
\mathcal{L}_{\text{c.t.}}[\vec{\phi}] &= \sum_{i=1}^{3} \left[-\frac{(Z_{\phi_i} - 1)}{2} \nabla_\mu \phi_i \nabla^\mu \phi_i(x) - \frac{(Z_{M_i} - 1) M_i^2}{2} \phi_i^2(x) \right] + \mathcal{O}(g^3).
\end{aligned}
$$

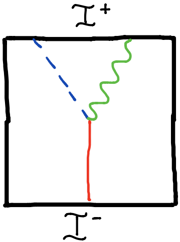
[Marolf IM 2010, Bros et. al 2006, 2008, Jatkar et. al 2011]

$\mathcal{O}(g)$ transition amplitude

At $\mathcal{O}(g)$ only tree-level amplitudes:

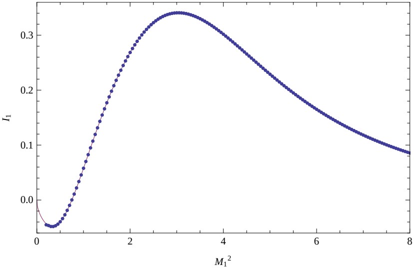
When R_σ projectors may be ignored:

$$_f\langle N_3 N_2 | N_1 \rangle_i^{(1)} = {}_f\langle n_3 n_2 | n_1 \rangle_i^{(1)} = ig \int_y u_3^* u_2^* u_1(y)$$



- happens to agree with naive use of LSZ
- Im as req. by Optical theorem
- non-vanishing except possibly for discrete configurations

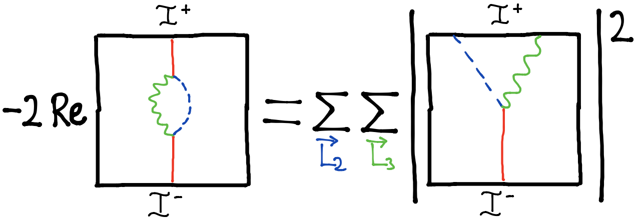
Plot: (amplitude/ig) as a function of M_1^2 with $M_{2,3}^2 = 2,\ 1.25$ in $D = 3$.



Amplitude peaked "off-shell" at $\sigma_1 = \sigma_2 + \sigma_3$, $M^2(\sigma_2 + \sigma_3) \in \mathbb{C}$.

$\mathcal{O}(g^2)$ transition amplitudes

Optical theorem requires



$$ {}_f\langle N_1|N_1\rangle_i^{(2)} = {}_f\langle n_1|n_1\rangle_i^{(2)} - {}_i\langle n_1|n_1\rangle_i^{(2)} = \sum_{\vec{L}_2}\sum_{\vec{L}_2}\left|{}_f\langle n_3 n_2|n_1\rangle_i^{(1)}\right|^2 $$

- contains 18 1-loop Schwinger-Keldysh diagrams
- Re part independent of UV counterterms

For generic configurations the 1-1 scattering amplitude contains an imaginary part. ⇒ In Minkowski space, this indicates unstable particles.

Outline

Asymptotic behavior of 2-pt. functions

For $|\eta_1\eta_2| \gg 1$, a free field 2-pt. function has asympt. behaviors:

$$\begin{aligned} W_\sigma(x_1, x_2) &:= \langle 0|\, \phi_\sigma(x_1)\phi_\sigma(x_2)\, |0\rangle \\ &\sim c_\sigma(\eta_1\eta_2)^\sigma(1 - \vec{x}_1 \cdot \vec{x}_2)^\sigma \\ &\quad + c_{-(\sigma+D-1)}(\eta_1\eta_2)^{-(\sigma+D-1)}(1 - \vec{x}_1 \cdot \vec{x}_2)^{-(\sigma+D-1)} \end{aligned}$$

Define fast/slow decays $\Delta_\pm$:

$$\Delta_-^{(0)} := -(\sigma + D - 1), \quad \Delta_+^{(0)} := \sigma.$$

In interacting theory:

$$\begin{aligned} \langle \phi_\sigma(x_1)\phi_\sigma(x_2)\rangle_\Omega \sim\ & c_{\Delta_+}(\eta_1\eta_2)^{\Delta_+}(1 - \vec{x}_1 \cdot \vec{x}_2)^{\Delta_+} \\ & + c_{\Delta_-}(\eta_1\eta_2)^{\Delta_-}(1 - \vec{x}_1 \cdot \vec{x}_2)^{\Delta_-} + \dots \end{aligned}$$

Expect

- new decay channels due to interactions, “intermediate states”
- $\Delta_\pm$ receives quantum corrections $\Delta_\pm = \sum_{n=0} \Delta_\pm^{(n)}$

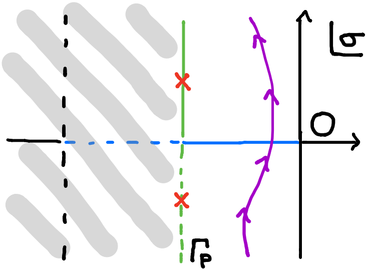
Lehmann-Källen form of the 2-pt. function

Write as contour integral in complex σ plane: [Marolf & IM 2010, Hollands 2011]

$$\langle \phi_\sigma(x_1)\phi_\sigma(x_2)\rangle = \int_\mu \rho(\mu) W_\mu(x_1, x_2).$$

E.g., for a free theory in the principal series:

$$\langle 0|\, \phi_\sigma(x_1)\phi_\sigma(x_2)\, |0\rangle = \int_\mu \frac{(2\mu + D - 1)}{(\mu - \sigma)(\mu + \sigma + D - 1)} W_\mu(x_1, x_2) = W_\sigma(x_1, x_2).$$



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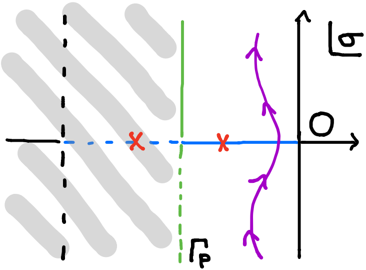
$$\langle \phi_\sigma(x_1)\phi_\sigma(x_2)\rangle = \int_\mu \rho(\mu) W_\mu(x_1, x_2).$$

E.g., for a free theory in the complementary series:

$$\langle 0|\, \phi_\sigma(x_1)\phi_\sigma(x_2)\, |0\rangle = \int_\mu \frac{(2\mu + D - 1)}{(\mu - \sigma)(\mu + \sigma + D - 1)} W_\mu(x_1, x_2) = W_\sigma(x_1, x_2).$$



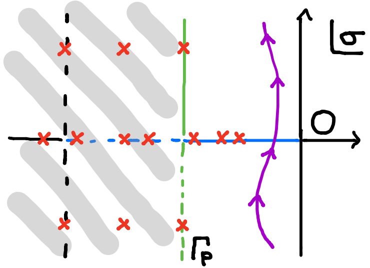
Lehmann-Källen form of the 2-pt. function

Write as contour integral in complex σ plane: [Marolf & IM 2010, Hollands 2011]

$$\langle \phi_\sigma(x_1)\phi_\sigma(x_2)\rangle = \int_\mu \rho(\mu) W_\mu(x_1, x_2).$$

E.g., at 1-loop:

$$\langle \phi_\sigma(x_1)\phi_\sigma(x_2)\rangle^{\text{1-loop}} = \int_\mu \frac{(2\mu + D - 1)\Pi(\mu)}{(\mu-\sigma)^2(\mu+\sigma+D-1)^2} W_\mu(x_1, x_2).$$



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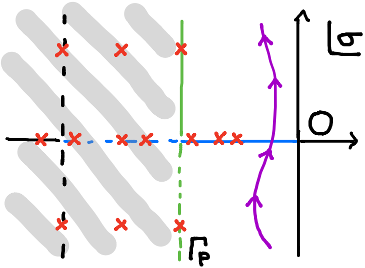
E.g., after 1PI sum:



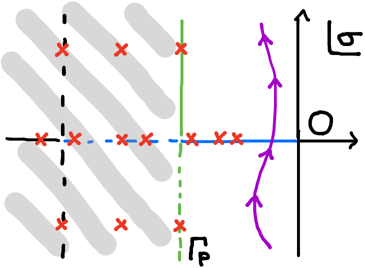
$$\langle \phi_\sigma(x_1)\phi_\sigma(x_2)\rangle^{\text{1-loop}}_{\text{1PI}} = \int_\mu \frac{(2\mu + D - 1)}{(\mu - \sigma)(\mu + \sigma + D - 1) - \Pi(\mu)} W_\mu(x_1, x_2).$$

Asymptotics again

May obtain asymptotic behavior ($|\eta_1\eta_2| \gg 1$) from Lehmann-Källen representation. Separate decays:

$$W_\sigma(x_1,x_2) = H_\sigma(x_1,x_2) + H_{-(\sigma+D-1)}(x_1,x_2), \quad |\eta_1\eta_2| \gg 1,$$
$$H_\sigma(x_1,x_2) = c_\sigma(\eta_1\eta_2)^\sigma(1-\vec{x}_1\cdot\vec{x}_2)^\sigma\left[1+\mathcal{O}((\eta_1\eta_2)^{-4})\right],$$

- For $H_{-(\mu+D-1)}$ may close integration contour to RHS $\Rightarrow 0$.
- For H_μ may deform integration contour to LHS $\Rightarrow$ obtain asymptotic expansion from residues of poles.



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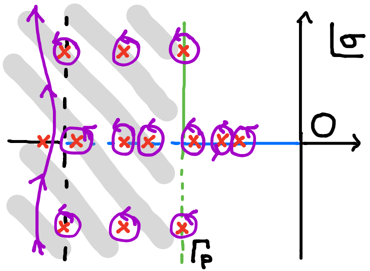

Mass poles

Focusing only on the "mass poles" at $\mu = \sigma$ and $\mu = -(\sigma + D - 1)$:

$$\langle \phi_\sigma(x_1)\phi_\sigma(x_2)\rangle^{\text{1-loop}} = \frac{1}{(2\sigma + D - 1)}\Big[\Pi(\sigma)\partial_\sigma H_\sigma(x_1, x_2) + \Pi'(\sigma)H_\sigma(x_1, x_2) \\ -\Pi(-(\sigma + D - 1))\partial_\sigma H_{-(\sigma+D-1)}(x_1, x_2) \\ +\Pi'(-(\sigma + D - 1))H_{-(\sigma+D-1)}(x_1, x_2)\Big],$$

determine pert. correction to weights $\Delta_\pm$; equiv., shift in mass poles [Marolf IM 2010, Jatkar et al 2011, LeBlond in prep]

$$\Delta_+ = \sigma + \frac{\Pi(\sigma)}{(2\sigma + D - 1)}, \quad \Delta_- = -(\sigma + D - 1) - \frac{\Pi(-(\sigma + D - 1))}{(2\sigma + D - 1)},$$

The self-energy $\Pi(\mu)$ depends on UV counterterms (a.k.a. renormalization scheme), so let us seek a renormalization independent statement.

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- first two terms constitute mass and field renormalization.
- $\Pi(\mu)$, $\Pi'(\mu)$ depend on mass and field renormalization counterterms
- coefficients of third & forth independent of UV counterterms
- these terms represent an additional, distinct renormalization of the fast decay Δ_-

$$\Delta_+ + \Delta_- = -(D-1) + \frac{\Pi(\sigma) - \Pi(-(\sigma+D-1))}{(2\sigma+D-1)}.$$

Puzzle

In all cases analysed, the perturbative correction to asymptotic behavior satisfies

$$\Delta_+ + \Delta_- = -(D-1) + \frac{\Pi(\sigma) - \Pi(-(\sigma + D - 1))}{(2\sigma + D - 1)} \leq -(D-1)$$

1. massive scalars, computed many ways [Marolf IM 2010, Jatkar et al 2011]
2. massive fermions [LeBlond in prep]
3. Yang-Mills coupled to scalar

Since this appears generic it should be a consequence of some basic ingredient in the QFT. What is this ingredient?

Effect on asymptotics (1PI summed correlators)

Renormalization of weights:

$$\Delta_+^{(2)} + \Delta_-^{(2)} = \left[\frac{\Pi(\sigma) - \Pi(-(\sigma + D - 1))}{(2\sigma + D - 1)}\right] \leq 0, \quad \Rightarrow \Delta_- + \Delta_+ \leq -(D-1).$$

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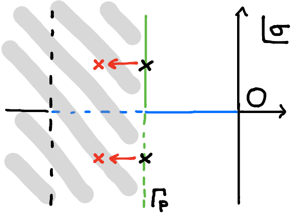
Principal series fields:

- renormalized $\Delta_\pm$ do not correspond to UIRs
- renormalized masses (self-energy) have imaginary part



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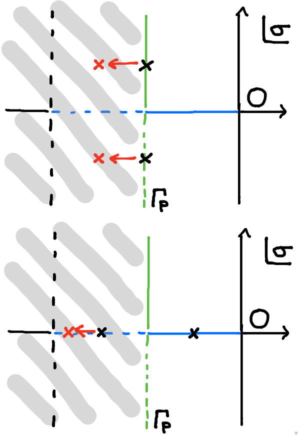


Complementary series fields:

- renormalized Δ_+ remains in complementary series
- renormalized mass $M^2(\Delta_+)$ real

Back to S: $\mathcal{O}(g^2)$ transition amplitudes

Can relate $1 \to 1$ scattering amplitude to self-energy $\Pi(\mu)$ (or Lehmann-Källen weight): [Marolf IM 2010, Bros et. al 2006, 2008, Jatkar et. al 2011]

$$\begin{aligned} -2\mathrm{Re}\,{}_f\langle N_1|N_1\rangle_i^{(2)} &= \int_{\overline{x}}\int_x u_1^*(\overline{x})u_1(x)(\Box_x - M_1^2)(\Box_{\overline{x}} - M_1^2)\,\langle\phi_\sigma(\overline{x})\phi_\sigma(x)\rangle^{(2)} \\ &= -\left[\frac{\Pi(\sigma) - \Pi(-(\sigma+D-1))}{(2\sigma+D-1)}\right](2\log H + \text{finite}) \\ &\quad -\left[\frac{\Pi'(\sigma) + \Pi'(-(\sigma+D-1))}{(2\sigma+D-1)}\right](\text{finite}), \end{aligned}$$

with spacetime integrals regulated $|\overline{\eta}|,\ |\eta| < H$.

Coefficients are those of the renormalization-independent corrections to asymptotics.

Optical theorem requires:

$$\left[\frac{\Pi(\sigma) - \Pi(-(\sigma+D-1))}{(2\sigma+D-1)}\right] \leq 0$$

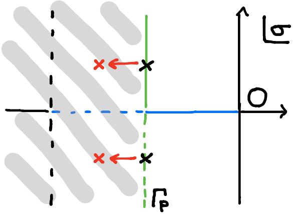
Consequences of unitarity

Renormalization of weights:

$$\Delta_+^{(2)} + \Delta_-^{(2)} = \left[\frac{\Pi(\sigma) - \Pi(-(\sigma + D - 1))}{(2\sigma + D - 1)}\right] \leq 0, \quad \Rightarrow \Delta_- + \Delta_+ \leq -(D-1).$$

Principal series fields:

- renormalized $\Delta_\pm$ do not correspond to UIRs
- renormalized masses (self-energy) have imaginary part
- "unstable" asymptotic particle states



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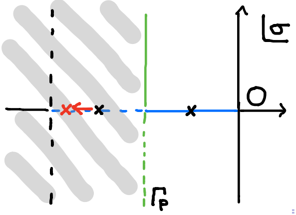

Outline

Review

The global dS S-matrix for massive fields at weak coupling:

- unitary
- dS-invariant
- invariant under perturbative field redefinitions
- transforms appropriately under CPT
- reduces to the usual S-matrix in the flat-space limit

Construction has many subtle features:

- requires Schwinger-Keldysh correlators in Lorentz signature
- asymptotic states must be re-orthonormalized
- extract only mass pole part – involved for light fields (R_σ)

A detailed scalar model

- verified all properties to $\mathcal{O}(g^2)$ which includes both tree and loop interactions

Review

Resolved a puzzle in renormalized asympt.

- district renormalization of fast- and slow- decays is a renormalization scheme-independent consequence of bulk unitarity
- generic for any matter coupled to massive fields

Conjecture 1:

Distinct renormalization of fast- and slow- decays is generic, occurs in gauge fields and gravity.

Conjecture 2:

There should exist an S-matrix for the Poincaré chart.

- from global perspective this is just a change of basis: initial states described at $\mathscr{I}^-$ $\rightarrow$ initial states described on a cosmological horizon.
- construction explicitly in Poincaré could be delicate (UV subtleties)

Review

dS/CFT

- hope: these results useful for understanding dS/CFT interpretation of bulk states connected to the HH state
- advocate the strategy:

$$\text{bulk S-matrix} \longleftrightarrow \begin{array}{c}\text{asympt. behavior of} \\ \text{bulk correlators}\end{array} \longleftrightarrow \text{dS/CFT}$$