Gravitational Effects on Massive Fermions during Inflation

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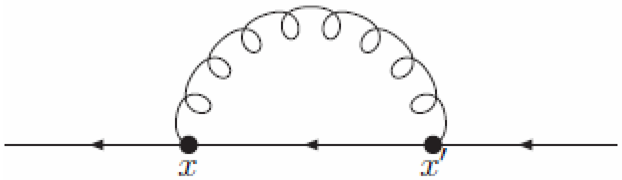
Conventions for the Background

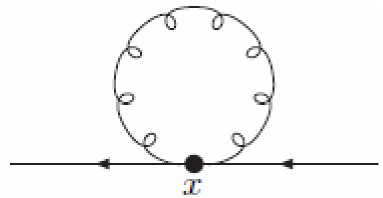
- Homogeneous, isotropic and spatial flat:
  $$ds^2 = -dt^2 + a^2(t) d\vec{x} \cdot d\vec{x}$$
- Physical distance: $a(t) d\vec{x}$
- Physical momentum: $\frac{\vec{k}}{a(t)}$
- Max. accelerated: $H(t)=H=\frac{\dot{a}}{a}$ ➔ $a(t) \sim e^{Ht}$
- $GH^2<10^{-10}$ ➔ $H<10^{14}$ Gev
  - QG still perturbative, not negligible
  - Particles almost effectively massless
- Locally de Sitter Background

What We Look for

- 1-loop fermion self energy from QG



from two 3-point vertices



from 4-point vertices.

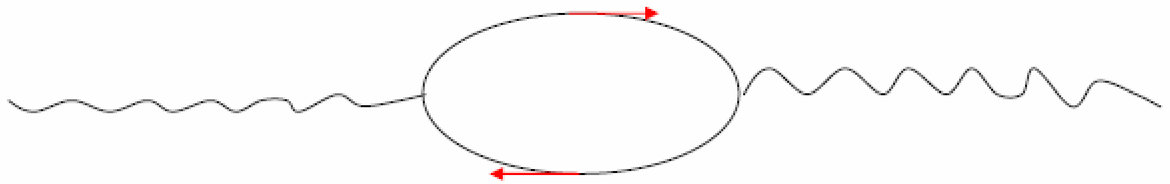
counterterms.

- Use dimensional regularization and renormalize it with BPHZ
- Quantum correct Dirac eqn.

$$\sqrt{-g}\, i\not{D}_{ij}\psi_j(x) - \int d^4x' \left[{}_i\Sigma_j\right](x;x')\,\psi_j(x') = 0$$

Strength of Quantum Loops

- Classical response to virtual particles
- TWO FACTORS gives density of virtuals
  1. Time virtuals persist (controlled by Energy-Time Uncertainty Principle)
  2. Rate at which they emerge (controlled by kinetic term in Lagrangian)



E–Time Unc. Principle

- $\Delta t \Delta E > 1$ to resolve ΔE,
  - Hence $\Delta t \Delta E < 1$ to NOT resolve
- Flat Space: Virtual pair has $\Delta E = 2(m^2+k^2)^{1/2}$
  - Hence can last $\Delta t < ½\,(m^2+k^2)^{-1/2}$
  - Eg: Vacuum polarization
    - Most for $e^{\pm}$ because smallest m
    - Smallest k's live longest → EM stronger at shorter distance (less polarization)
- FRW: $E(t) = [m^2+k^2/a^2(t)]^{1/2}$
  - $\Delta t \Delta E$ ➔ $\int_t^{t+\Delta t} dt'\, E(t') < 1$
  - $m=0$ ➔ $\int_t^{t+\Delta t} dt'/a(t') < 1$
  - Any $m=0$ virtual with $k<Ha(t)$ live forever

Killer Symmetry Suppresses Emergence Rate

- Conformal invariance

  $\mathcal{L} \xrightarrow{g'_{\mu\nu}=\Omega^2 g_{\mu\nu}} \mathcal{L}' \xrightarrow{\text{field redefinition}} \mathcal{L}$

  - EM in 4D: $\mathcal{L}' = -1/4\, F_{\mu\rho} F_{\nu\sigma} g^{\mu\nu} g^{\rho\sigma} g^{1/2} \Omega^{D-4}$, $F'_{\mu\rho}=F_{\mu\rho}$
- $-dt^2+a^2dx^2 = a^2(-d\eta^2+dx^2)$ ➔ $a\,d\eta=dt$
- Conformal invariance ➔ same locally (in conformal coordinates) as flat
  - Hence $dN/d\eta = \Gamma_{flat}$
  - Hence $dN/dt = \Gamma_{flat}/a(t)$
- Any m=0, conformal virtuals that emerge Do live forever, but few emerge

Maximum IR Enhancements

- Requirements:
  - PERSISTENCE TIME → m=0 + inflation
  - EMERGENCE RATE → no conformal invariance
- Realized by
  - Massless, minimally coupled scalars
  - Gravitons
- Eg: Massless Fermions (conf.) + gravitons
  - Field strength ~ $\#GH^2 \ln[a(t)]$, at 1-loop
  - Perturbation breaks down at $\ln[a(t)] \sim 1/GH^2$

Two Competing Effects

- Now: Light Fermions (no conf.) + gravitons
  - Suppress: how fermions propagate
    - $u(t,k) \sim f(\omega/H)\, e^{i\omega t}$, $\omega = [m^2+k^2/a^2(t)]^{1/2}$
    - u oscillates → interactions at different times cancel
  - Enhance: how they interact with gravity
    - New (mass) interaction does not fall off
- Expected for $[{}_i\Sigma_j](x;x')$ from $m h \bar{\psi} \psi$
  - a(t)ln[a(t)] relative to classical Kinetic term
  - ln[a(t)] relative to classical mass term
- Expected dividing line: fermions mass ~ H
  - m_{cl} big wrt H → small enhancement in Ψ
  - m_{cl} small (cl. kinetic term dominated) → big in Ψ

Feynman Rules for Massive Dirac

- Time translation broken in de Sitter
  → Feynman rules simpler in position space
- Eg: $\mathcal{L}_{\rm Dirac} \equiv \overline{\psi} e^{\mu}{}_{b} \gamma^{b} i \mathcal{D}_{\mu} \psi \sqrt{-g} - m \overline{\psi} \psi \sqrt{-g}$ $\quad \mathcal{D}_{\mu} \equiv \partial_{\mu} + \frac{i}{2} A_{\mu c d} J^{c d}$
  - Re-scale metric & fermions: $g_{\mu\nu} = a^2 \, \widetilde{g}_{\mu\nu}$ $\quad \Psi \equiv a^{\frac{D-1}{2}} \psi$
  - Impose symmetric gauge: $e_{\beta b} = e_{b \beta}$
  - Solve vierbein:

$$\widetilde{e}[\widetilde{g}]_{\beta b} \equiv \left(\sqrt{\widetilde{g}\eta^{-1}}\right)_{\beta}{}^{\gamma} \eta_{\gamma b} = \eta_{\beta b} + \frac{1}{2}\kappa h_{\beta b} - \frac{1}{8}\kappa^2 h_{\beta}{}^{\gamma} h_{\gamma b} + \dots \qquad \widetilde{g}_{\mu\nu} \equiv \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

- Perturbed massive Dirac
  - $\mathcal{L}_{\rm Dirac} = \overline{\Psi}\Big[i\partial\!\!\!/ - am\Big]\Psi + \frac{\kappa}{2}\overline{\Psi}\Big[h i\partial\!\!\!/ - h^{\mu\nu}\gamma_{\mu} i \partial_{\nu} - h_{\mu\rho,\sigma}\gamma^{\mu} J^{\rho\sigma} - amh\Big]\Psi$

    $+\kappa^2$ [10 terms] $+\dots$

- PS: 1-loop $[{}_i\Sigma_j](x;x')$: only multiplication of vertices, propagators and derivatives

Feynman Rules for Massive Dirac

- From the solutions of Candelas and Raine ➔

$$iS[m](x;x')_{C.R.} = (aa')^{-\frac{D-1}{2}} iS[m](x;x')$$

$$iS[m](x;x') = \frac{\Gamma(\frac{D}{2}-1)}{4\pi^{\frac{D}{2}}}\Big[i\partial\!\!\!/ + a\,m\Big]\frac{1}{\Delta x^{D-2}} + \frac{(H^2aa')^{\frac{D}{2}-1}}{(4\pi)^{\frac{D}{2}}}\frac{\Gamma(\frac{D}{2}-1)\Gamma(2-\frac{D}{2})(i\frac{m}{H})}{\Gamma(1+i\frac{m}{H})\Gamma(1-i\frac{m}{H})}$$
$$\Big[i\partial\!\!\!/ + (\frac{D}{2}-1)iHa\gamma^0 + a\,m\Big] \times \sum_{n=0}^{\infty}\Big\{\frac{\Gamma(n+\frac{D}{2}-1+i\frac{m}{H})\Gamma(n+\frac{D}{2}-1-i\frac{m}{H})}{\Gamma(n+\frac{D}{2}-1)\Gamma(n+1)}$$
$$\Big[\frac{i\frac{m}{H}}{(n+\frac{D}{2}-1)}+\gamma^0\Big]\Big(\frac{y}{4}\Big)^n - \frac{\Gamma(n+1+i\frac{m}{H})\Gamma(n+1-i\frac{m}{H})}{\Gamma(n+1)\Gamma(n+3-\frac{D}{2})}\Big[\frac{i\frac{m}{H}}{(n+1)}+\gamma^0\Big]\Big(\frac{y}{4}\Big)^{n+2-\frac{D}{2}}\Big\}$$

  - Here $y(x;x') = aa'H^2\Big(\|\vec{x}-\vec{x}'\|^2 - (|\eta-\eta'|-i\delta)^2\Big)$

1. The infinite series $\nRightarrow$ any elementary funs.
2. Finite from the infinite series even in D=4
   - In D=4, the two series tend to cancel
   - But they both are multiplied by Γ(D/2 –1)

Fermion & Graviton Prop.

- m≪H:

$$i\big[{}_iS_j\big](x;x') = \frac{\Gamma(\frac{D}{2}-1)}{4\pi^{\frac{D}{2}}}\big[i\partial\!\!\!/ + ma\big]\frac{1}{\Delta x^{D-2}} - \Big(\frac{m}{H}\Big)\frac{(H^2aa')^{\frac{D}{2}-1}}{(4\pi)^{\frac{D}{2}}}\frac{\Gamma(\frac{D}{2}-1)\Gamma(3-\frac{D}{2})}{(2-\frac{D}{2})}\Big[\partial\!\!\!/\gamma^0 + \Big(\frac{D}{2}-1\Big)Ha\Big]$$
$$\sum_{n=0}^{\infty}\left\{\frac{\Gamma(n+\frac{D}{2}-1)}{\Gamma(n+1)}\Big(\frac{y}{4}\Big)^n - \frac{\Gamma(n+1)}{\Gamma(n+3-\frac{D}{2})}\Big(\frac{y}{4}\Big)^{n+2-\frac{D}{2}}\right\} + \mathcal{O}([\frac{m}{H}]^2)$$

- Graviton prop:

$$i\big[{}_{\mu\nu}\Delta_{\rho\sigma}\big](x;x') = \sum_{I=A,B,C}\big[{}_{\mu\nu}T^I_{\rho\sigma}\big]i\Delta_I(x;x')$$

i.e. $$\big[{}_{\mu\nu}T^A_{\rho\sigma}\big] = 2\overline{\eta}_{\mu(\rho}\overline{\eta}_{\sigma)\nu} - \frac{2}{D-3}\overline{\eta}_{\mu\nu}\overline{\eta}_{\rho\sigma}$$

$$i\Delta_A(x;x') = i\Delta_{\rm cf}(x;x') + \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}}\frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})}\left\{\frac{D}{D-4}\frac{\Gamma^2(\frac{D}{2})}{\Gamma(D-1)}\Big(\frac{4}{y}\Big)^{\frac{D}{2}-2} - \pi\cot\Big(\frac{\pi}{2}D\Big) + \ln(aa')\right\}$$
$$+\frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}}\sum_{n=1}^{\infty}\left\{\frac{1}{n}\frac{\Gamma(n+D-1)}{\Gamma(n+\frac{D}{2})}\Big(\frac{y}{4}\Big)^n - \frac{1}{n-\frac{D}{2}+2}\frac{\Gamma(n+\frac{D}{2}+1)}{\Gamma(n+2)}\Big(\frac{y}{4}\Big)^{n-\frac{D}{2}+2}\right\}$$

$$i\Delta_{\rm cf}(x;x') = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}}\Gamma\Big(\frac{D}{2}-1\Big)\Big(\frac{4}{y}\Big)^{\frac{D}{2}-1}$$

  - Infinite series only contributes when it times 1/(D-4)

Counterterms

- S.S.D. = 3 at 1-loop in D=4
- Dimensional regularization respects diffeo.
- Graviton prop. Gauge fixing: de Sitter inv.
  - → invariant one can found by dimensional analysis
  - allow non-invariant one But ∞ possibilities
  - restrain # of non-(dS) invariant counterterms
    - Residual de Sitter symmetry
      - Spatial translation, rotation & dilatation
    - Even # of γ's at order m
    - Poincare invariance at H → 0 limit
- All counterterms at order m for 1-loop

$$\alpha_1 \kappa^2 \frac{m}{a} \bar{\partial}^2 \ , \ \alpha_4 \kappa^2 H^2 m a \ , \qquad \alpha_2 \kappa^2 m H \partial_0 \ , \ \alpha_3 \kappa^2 m H \gamma^0 \bar{\not{\partial}}$$

Counterterms Analysis

- Invariant one: $\Delta\mathcal{L}_{\rm inv} = \lambda_1\kappa^2 m\overline{\psi}(i\not{D})^2\psi\sqrt{-g} + \lambda_2\kappa^2 mR\overline{\psi}\psi\sqrt{-g}$

  ➔ $\lambda_1\kappa^2\Big(\frac{m}{a}\partial^2 + mH\gamma^0\not{\partial}\Big) = \lambda_1\kappa^2 m\not{\partial}a^{-1}\not{\partial}$ $\quad\quad$ $\lambda_2 D(D-1)\kappa^2 H^2 ma$

- Non-invariant: $\Delta\mathcal{L}_{\rm non} = \kappa^2 H^2 ma\overline{\Psi}\mathcal{S}\big((Ha)^{-1}\gamma^0\partial_0, (Ha)^{-1}\gamma^i\partial_i\big)\Psi$
  - ◦Spatial translation, rotation → $(\gamma^i\partial_i)^2 = -\nabla^2$
  - ◦Dilatation (η=cη, x^i=cx^i) → $a^{-1}\partial_\mu$
  - ◦Even # of γ's → $\kappa^2 ma\Big\{\beta_1(a^{-1}\gamma^0\partial_0)^2 + \beta_2\big[(a^{-1}\gamma^0\partial_0)(a^{-1}\gamma^i\partial_i)\big] + \beta_3(a^{-1}\gamma^i\partial_i)^2$
  $+H\gamma^0\Big(\beta_4(a^{-1}\gamma^0\partial_0) + \beta_5(a^{-1}\gamma^i\partial_i)\Big) + H^2\beta_6\Big\}$
  - ◦H → 0 ➔ $\kappa^2 ma\Big\{(a^{-1}\gamma^0\partial_0)^2 + 2\big[(a^{-1}\gamma^0\partial_0)(a^{-1}\gamma^i\partial_i)\big] + (a^{-1}\gamma^i\partial_i)^2\Big\} = \kappa^2 m\not{\partial}a^{-1}\not{\partial}$
  - ◦Non-invariant ones left:
    - $\kappa^2 mH\gamma^0\partial_0$, $\kappa^2 mH\gamma^i\partial_i$ not independent of $\kappa^2 mH\gamma^0\not{\partial}$

    ➔ $\alpha_1\kappa^2\frac{m}{a}\partial^2$, $\alpha_4\kappa^2 H^2 ma$, $\quad$ $\alpha_2\kappa^2 mH\partial_0$, $\alpha_3\kappa^2 mH\gamma^0\overline{\not{\partial}}$

Gauge Issue of GR. Prop.

- Faddeev–Popov in flat R^4 $\nRightarrow$ general geometry
- Covariant gauge fixing term to $\mathscr{L}$ ➔
  1. Divergent response for a point source (GR)
  2. On-shell singularity for $M^2[x;x']$ of SQED in the de Sitter invariant analogue of Feynman gauge
- linearization instability
    - Gauge theory, background isometries, spatial T^n
  - Fix:
    - Non de Sitter invariant gauge
    - imposing exact gauge condition
  - The same ln[a(t)] behavior in the exact de Donder gauge

Massive ${}_i\Sigma_j[x;x']$ at 1–loop

$$-i\left[\Sigma^{\rm ren}\right](x;x') = \frac{i\kappa^2}{16\pi^2}\Big\{\Big[3\ln a + \frac{1}{8}\Big]\frac{m}{a}\partial^2 + \Big[\frac{97}{16}\ln a - \frac{39}{16}\Big] mH\partial_0 + \Big[\frac{9}{16}\ln a + \frac{5}{16}\Big] mH\gamma^0\bar{\not{\partial}}$$

$$+\Big[\frac{95}{8}\ln a - \frac{29}{32} - \frac{85}{16}\psi(1) + \frac{5}{8}\ln\frac{H^2}{4\mu^2}\Big] H^2 ma\Big\}\delta^4(x-x') + \frac{\kappa^2}{64\pi^4}\Big\{\Big[\frac{3}{2}\frac{m}{a'}\partial^2 + \Big(\frac{7}{8}\frac{a}{a'} - \frac{27}{32}\Big)mH\partial_0$$

$$+\Big(\frac{9}{16}\frac{a}{a'} - \frac{9}{32}\Big)mH\gamma^0\bar{\not{\partial}} + H^2 m\Big(\frac{215}{32}a + \frac{9}{32}a'\Big)\Big]\partial^2 + H^2 ma\Big[\nabla^2 + 6Ha\partial_0 + 4Ha\gamma^0\bar{\not{\partial}}\Big]\Big\}\Big[\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\Big]$$

$$+\frac{\kappa^2H^2}{64\pi^4}\Big\{\frac{9}{16}ma'\partial_0^2 + \Big[\frac{-53}{16}ma + \frac{3}{16}ma'\Big]\gamma^0\partial_0\bar{\not{\partial}} + \Big[\frac{-49}{16} + \ln\frac{H^2}{4\mu^2}\Big]ma\nabla^2 + \Big[\frac{9}{2} + 6\ln\frac{H^2}{4\mu^2}\Big]Hma^2\partial_0$$

$$+\Big[\frac{35}{8} - \frac{11}{8}\ln\frac{y}{4} + 4\ln\frac{H^2}{4\mu^2}\Big]Hma^2\gamma^0\bar{\not{\partial}} + \Big[\frac{-5}{8} - \frac{3}{8}\ln\frac{y}{4}\Big]Hmaa'\gamma^0\bar{\not{\partial}}\Big\}\frac{1}{\Delta x^2}$$

$$+\frac{\kappa^2 mH}{64\pi^4}\Big\{8\frac{\gamma^0\gamma^k\Delta x_k}{\Delta x^6}\Big[\frac{(-Y^2+7Y)}{(1-Y)^3} + \frac{(Y^3+6Y^2-Y)}{(1-Y)^4}\ln(Y)\Big]$$

Here Y stands for $\frac{y}{4}$

$$+\Big[5\frac{Ha\gamma^0\Delta\eta\gamma^k\Delta x_k}{\Delta x^6} + \frac{Ha'\gamma^0\Delta\eta\gamma^k\Delta x_k}{\Delta x^6} - 3\frac{Ha\Delta\eta^2}{\Delta x^6} - 3\frac{Ha'\Delta\eta^2}{\Delta x^6}\Big]\Big[\frac{-3Y^3+26Y^2+Y}{2(1-Y)^3} + \frac{3Y^3+22Y^2-3Y}{2(1-Y)^4}\ln(Y)\Big]$$

$$+\Big[\frac{Ha}{\Delta x^4}+\frac{Ha'}{\Delta x^4}\Big]\Big[\frac{-12(Y^2+2Y)}{(1-Y)^3} + \frac{-3(Y^3+8Y^2+3Y)}{(1-Y)^4}\ln(Y)\Big]$$

$$+\frac{H^2a^2\gamma^0\gamma^k\Delta x_k}{\Delta x^4}\Big[\frac{-3(Y^2-6Y+29)}{4(1-Y)^3} + \frac{-3(3Y^4-12Y^3+23Y^2+10Y)}{4(1-Y)^4}\ln(Y)\Big]$$

$$+\frac{H^2aa'\gamma^0\gamma^k\Delta x_k}{\Delta x^4}\Big[\frac{(5Y^3-14Y^2-15Y)}{4(1-Y)^3} + \frac{(-3Y^4+14Y^3-35Y^2)}{4(1-Y)^4}\ln(Y)\Big]\Big\} \; +$$

	$\ln \frac{H^2 \Delta x^2}{4}$	1
$\frac{\Delta\eta^3}{\Delta x^6}$	$-20 f_1(Y) - 20 f_2(Y) \ln Y$	$-5 g_1(Y) - 5 g_2(Y) \ln Y$
$\frac{\gamma^0 \Delta\eta^2 \gamma^k \Delta x_k}{\Delta x^6}$	$12 f_1(Y) + 12 f_2(Y) \ln Y$	$3 g_1(Y) + 3 g_2(Y) \ln Y$
$\frac{\Delta\eta}{\Delta x^4}$	$2 f_3(Y) + 2 f_4(Y) \ln Y$	$g_3(Y) + g_4(Y) \ln Y$
$\frac{\gamma^0 \gamma^k \Delta x_k}{\Delta x^4}$	$-6 f_5(Y) - 6 f_6(Y) \ln Y$	$-g_5(Y) - g_6(Y) \ln Y$
$\frac{H a \Delta\eta^2}{\Delta x^4}$	$6 f_7(Y) + 6 f_8(Y) \ln Y$	$5 g_7(Y) + 5 g_8(Y) \ln Y$
$\frac{H a' \Delta\eta^2}{\Delta x^4}$	$-4 f_9(Y) - 4 f_{10}(Y) \ln Y$	$-2 g_9(Y) - 2 g_{10}(Y) \ln Y$
$\frac{H a \gamma^0 \Delta\eta \gamma^k \Delta x_k}{\Delta x^4}$	$-4 f_{11}(Y) - 4 f_{12}(Y) \ln Y$	$-2 g_{11}(Y) - 2 g_{12}(Y) \ln Y$
$\frac{H a' \gamma^0 \Delta\eta \gamma^k \Delta x_k}{\Delta x^4}$	$4 f_9(Y) + 4 f_{10}(Y) \ln Y$	$2 g_9(Y) + 2 g_{10}(Y) \ln Y$
$\frac{H a}{\Delta x^2}$	$9 f_{13}(Y) + 9 f_{14}(Y) \ln Y$	$g_{13}(Y) + g_{14}(Y) \ln Y$
$\frac{H a'}{\Delta x^2}$	$-2 f_{15}(Y) - 2 f_{16}(Y) \ln Y$	$-g_{15}(Y) - g_{16}(Y) \ln Y$
$\frac{H^2 a^2 \Delta\eta}{\Delta x^2}$	$-6 f_{17}(Y) - 6 f_{18}(Y) \ln Y$	$-3 g_{17}(Y) - 3 g_{18}(Y) \ln Y$
$\frac{H^2 a^2 \gamma^0 \gamma^k \Delta x_k}{\Delta x^2}$	$4 f_{17}(Y) + 4 f_{18}(Y) \ln Y$	$2 g_{19}(Y) + 2 g_{20}(Y) \ln Y$
$\frac{H^2 a a' \gamma^0 \gamma^k \Delta x_k}{\Delta x^2}$	$-2 f_{19}(Y) - 2 f_{20}(Y) \ln Y$	$-g_{21}(Y) - g_{22}(Y) \ln Y$
$H^3 a^2 a'$	$-3 f_{21}(Y) - 3 f_{22}(Y) \ln Y$	$-\frac{3}{2} g_{19}(Y) - \frac{3}{2} g_{20}(Y) \ln Y$

Table 32: The total result for $i\delta\Delta_A \times i[S]_{n\geq 0}$. The factor $\frac{i\kappa^2 H^2}{2^6 \pi^4} \frac{mHaa'}{2}$ multiplies all contributions. Here $Y = \frac{y}{4}$; $\ln \frac{H^2 \Delta x^2}{4}$ and 1 are the multiplicative factors for the each individual column. The various functions $f_i(Y)$ and $g_i(Y)$ are presented in Table 33

$f_i(Y)$		$g_i(Y)$	
$f_1(Y)$	$\frac{Y(2Y^2+5Y-1)}{(1-Y)^3}+2$	$g_1(Y)$	$\frac{-Y(Y^2-12Y-1)}{(1-Y)^3}-1$
$f_2(Y)$	$\frac{6Y^3}{(1-Y)^4}$	$g_2(Y)$	$\frac{Y(3Y^2+10Y-1)}{(1-Y)^4}$
$f_3(Y)$	$\frac{Y(7Y^2-86Y+91)}{(1-Y)^3}+7$	$g_3(Y)$	$\frac{Y(49Y^2-282Y+257)}{2(1-Y)^3}+\frac{49}{2}$
$f_4(Y)$	$\frac{-6Y(5Y^2+Y-8)}{(1-Y)^4}$	$g_4(Y)$	$\frac{Y(-45Y^2-82Y+151)}{2(1-Y)^4}$
$f_5(Y)$	$\frac{Y(5Y^2-22Y+29)}{(1-Y)^3}+5$	$g_5(Y)$	$\frac{Y(37Y^2-194Y+229)}{2(1-Y)^3}+\frac{37}{2}$
$f_6(Y)$	$\frac{2Y(-Y^2-Y+8)}{(1-Y)^4}$	$g_6(Y)$	$\frac{Y(-29Y^2-26Y+127)}{2(1-Y)^4}$
$f_7(Y)$	$\frac{Y(3Y^2-7Y+34)}{(1-Y)^3}+3$	$g_7(Y)$	$\frac{Y(13Y^2-48Y+71)}{2(1-Y)^3}+\frac{13}{2}$
$f_8(Y)$	$\frac{2Y(7Y+8)}{(1-Y)^4}$	$g_8(Y)$	$\frac{Y(Y^3-4Y^2+Y+38)}{2(1-Y)^4}-\frac{1}{2}$
$f_9(Y)$	$\frac{Y(-Y^2+5Y+2)}{(1-Y)^3}-1$	$g_9(Y)$	$\frac{Y(Y+5)}{(1-Y)^3}$
$f_{10}(Y)$	$\frac{6Y^2}{(1-Y)^4}$	$g_{10}(Y)$	$\frac{2Y(2Y+1)}{(1-Y)^4}$
$f_{11}(Y)$	$\frac{Y(7Y^2-23Y+46)}{(1-Y)^3}+7$	$g_{11}(Y)$	$\frac{Y(12Y^2-43Y+61)}{(1-Y)^3}+12$
$f_{12}(Y)$	$\frac{6Y(Y+4)}{(1-Y)^4}$	$g_{12}(Y)$	$\frac{-2Y(2Y-17)}{(1-Y)^4}$
$f_{13}(Y)$	$\frac{Y(3Y^2-10Y+19)}{(1-Y)^3}+3$	$g_{13}(Y)$	$\frac{Y(79Y^2-270Y+407)}{4(1-Y)^3}+\frac{79}{4}$
$f_{14}(Y)$	$\frac{2Y(Y+5)}{(1-Y)^4}$	$g_{14}(Y)$	$\frac{Y(-5Y^3+20Y^2-29Y+230)}{4(1-Y)^4}+\frac{5}{4}$
$f_{15}(Y)$	$\frac{Y(Y^2-2Y+13)}{(1-Y)^3}+1$	$g_{15}(Y)$	$\frac{Y(3Y^2-4Y+13)}{(1-Y)^3}+3$
$f_{16}(Y)$	$\frac{6Y(Y+1)}{(1-Y)^4}$	$g_{16}(Y)$	$\frac{Y(-3Y^3+12Y^2-7Y+10)}{(1-Y)^4}+3$
$f_{17}(Y)$	$\frac{Y(2Y^2-7Y+11)}{(1-Y)^3}+2$	$g_{17}(Y)$	$\frac{Y(13Y^2-38Y+37)}{2(1-Y)^3}+\frac{13}{2}$
$f_{18}(Y)$	$\frac{6Y}{(1-Y)^4}$	$g_{18}(Y)$	$\frac{3Y(-Y^3+4Y^2-6Y+5)}{(1-Y)^4}+3$
$f_{19}(Y)$	$\frac{Y(Y^2-3Y+8)}{(1-Y)^3}+1$	$g_{19}(Y)$	$\frac{Y(5Y^2-15Y+16)}{(1-Y)^3}+5$
$f_{20}(Y)$	$\frac{2Y(Y+2)}{(1-Y)^4}$	$g_{20}(Y)$	$\frac{-2Y(Y^3-4Y^2+6Y-6)}{(1-Y)^4}+2$
$f_{21}(Y)$	$\frac{Y(5Y^2-15Y+16)}{(1-Y)^3}+5$	$g_{21}(Y)$	$\frac{-3Y^2(Y-3)}{(1-Y)^3}-3$
$f_{22}(Y)$	$\frac{-2Y(Y^3-4Y^2+6Y-6)}{(1-Y)^4}+2$	$g_{22}(Y)$	$\frac{2Y(Y^3-4Y^2+8Y-2)}{(1-Y)^4}-2$

Table 33: The coefficient functions for the table 32

Why QG Results are Reliable

- QG not renomalizable
  - No physical principle fixes the finite part

$$i\kappa^2\left\{\Delta\alpha_1\frac{m}{a}\partial^2+\Delta\alpha_2 mH\partial_0+\Delta\alpha_3 mH\gamma^0\bar{\partial}+\Delta\alpha_4 H^2 ma\right\}\delta^4(x-x')$$

  - But loops of massless ➔ non-analytical contri. Can't be affected by local counterterms
  - Nonlocal terms dominated over $\Delta\alpha$ at late time

$$\frac{i\kappa^2}{16\pi^2}\left\{3\ln a\frac{m}{a}\partial^2+\frac{97\ln a}{16}mH\partial_0+\frac{9\ln a}{16}mH\gamma^0\bar{\partial}+\frac{95\ln a}{8}H^2 ma\right\}\delta^4(x-x')$$

- Low energy effective theory
  - Fermi theory versus Standard Model etc.
  - The UV completion of QG cannot
    - Add new massless particles
    - Change the behavior of long range forces

} Same IR behavior

Physical Pictures & Consequence

- Inflation creates the sea of IR gravitons & mmc, we study how particles get affected
  - Massless fermions due to IR MMC ➔ growing mass
  - Massless fermions due to IR gravitons ➔ field strength growing with time
  - mmc due to IR gravitons ➔ no significant change
    - IR gravitons only couple to mmc through red-shift K.E.
    - But fermions has extra SPIN in addition (0803.2377)
  - Expected: light fermions propagating through this also causes field strength to grow with time
    - Spin & mass interaction term both do not red shift-just like inflaton power spectrum

Physical Pictures & Consequence

- Also change the energy of Universe
- Fermion production during inflation
  - B-mode of polarization
  - Possibility for the inflationary baryogenesis
- perturbation eventually breaks down ➔ need re-summation technique
  - Starobinsky's formalism for a scalar model
    - Actives: produce IR Log
    - Passives: propagate through IR Log
    - Each and only scalars produces IR Log (no derivative)
    - Truncate scalars & set in D=4 (UV div. at leading log)

Non-Pertuabative technique for general models

- actives + passives (∂'s ~~passives~~)
  - integrate out passives & evaluate the effective action with const. actives (effective [potential)
  - UV div. at leading Log ➔ turn D on
    - Eg: Yukawa, SQED
- ∂'s actives + passives or ∂'s passives
  - No systematic way
  - Massless Dirac + GR (we have the exact result)
    - Can't either ignore ∂'s actives or integrate out fields
    - Keep dimensional regulation on
  - Potential: nonlinear sigma model (Kitazawa, Kitamoto)