Soft Gravitons Screen Couplings in de Sitter Space

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Introduction

- *Time dependent* quantum effects in de Sitter space
- In investigating them, the important issue is whether there is a mechanism to break the dS symmetry
- Propagator for a massless and minimally coupled field breaks the dS symmetry
- In some models with massless and minimally coupled fields, physical quantities acquire time dependences

Scalar field in dS space

$$dS_4: \ ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2 = a^2(\tau)(-d\tau^2 + d\mathbf{x}^2)$$

H: Hubble const.

$a(t) = e^{Ht} = -1/H\tau$

Propagator for a massless and minimally coupled field contains a scale inv. spectrum

B. D. vac.

$$\phi_{\mathbf{p}}(x) = \frac{H\tau}{\sqrt{2p}}(1 - \frac{i}{p\tau})e^{-ip\tau + i\mathbf{p}\cdot\mathbf{x}} \sim \frac{H\tau}{\sqrt{2p}}e^{-ip\tau + i\mathbf{p}\cdot\mathbf{x}}$$

sub-horizon $P \equiv p/a(\tau) \gg H$

$$\sim \frac{H}{\sqrt{2p^3}}e^{+i\mathbf{p}\cdot\mathbf{x}}$$

super-horizon $P \ll H$

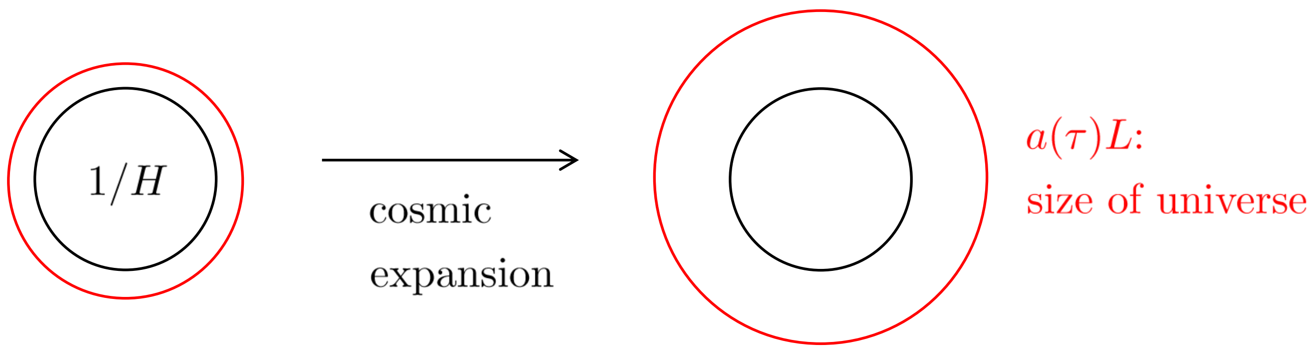
$$\Downarrow$$

$$\langle \varphi^2(x) \rangle \sim \int_{P>H} \frac{d^3P}{(2\pi)^3}\frac{1}{2P} + H^2 \int_{P<H} \frac{d^3P}{(2\pi)^3}\frac{1}{2P^3}$$

IR log divergence

dS symmetry breaking

We introduce an IR cut-off $p_{\min} = 1/L$: $\int_{1/a(\tau)L}^{H} dP$



$$\langle \varphi^2(x) \rangle = (\text{UV const.}) + \frac{H^2}{4\pi^2} \int_{1/a(\tau)L}^{H} \frac{dP}{P}$$

$$= (\text{UV const.}) + \frac{H^2}{4\pi^2} \log HLa(\tau)$$

Not inv. under

$\tau \to C\tau,\ \mathbf{x} \to C\mathbf{x}$

’82 A. Vilenkin, L. H. Ford,
A. D. Linde,
A. A. Starobinsky

dS inv. distance: $y = \frac{-(\tau-\tau')^2 + (\mathbf{x}-\mathbf{x}')^2}{\tau\tau'}$

$$\langle\varphi(x)\varphi(x')\rangle = \frac{H^2}{4\pi^2}\{\frac{1}{y} - \frac{1}{2}\log y + \underbrace{\frac{1}{2}\log H^2L^2a(\tau)a(\tau')}_{SO(1,4)\to E(3)} + 1 - \gamma\}$$

Henceforth,
$L = 1/H$

If massless, the IR divergence takes place also in D-dimension
If massive, the IR divergence does not take place

$$\int \frac{d^{D-1}p}{p^{2\nu}}, \quad \nu = \sqrt{\frac{(D-1)^2}{4} - \frac{m^2}{H^2}}$$

Dirac, Gauge fields

Since the actions of Dirac and gauge fields are conformal invariant,

$$\mathcal{L}_D = i\sqrt{-g}\bar{\psi}e^{\mu}{}_{a}\gamma^a D_\mu\psi = i\bar{\tilde{\psi}}\gamma^\mu\partial_\mu\tilde{\psi}$$

$$\mathcal{L}_G = -\frac{1}{4}\sqrt{-g}g^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$g_{\mu\nu} = a^2\eta_{\mu\nu}, \quad e^{\mu}{}_{a} = a^{-1}\delta^{\mu}{}_{a}, \quad \tilde{\psi} = a^{-\frac{3}{2}}\psi$$

the corresponding wave function is

$$\phi_{\mathbf{p}}(x) = \frac{1}{\sqrt{2p}}e^{-ip\tau + i\mathbf{p}\cdot\mathbf{x}}$$

These fields do not induce the dS symmetry breaking

Why such IR effects are important?

e.g. $V(\varphi) = \frac{\lambda}{4!}\varphi^4$,

$$\langle V(\varphi)\rangle \simeq H^4\{\frac{\lambda}{2^7\pi^4}\log^2 a(\tau) - \frac{\lambda^2}{2^9\cdot 3\pi^6}\log^4 a(\tau) + \cdots\}$$

$$\to H^4 \cdot \underline{\frac{3}{32\pi^2}} \quad \text{at } t \to \infty$$

dS inv. is retained

Not suppressed by λ

Even if $\lambda \ll 1$, perturbation is broken after $\lambda \log^2 a(\tau) \sim 1$

In models with $V(\varphi)$, we can evaluate the IR effects nonperturbatively by resummation of the leading logarithms

'94 A. A. Starobinsky, J. Yokoyama
'05 N. C. Tsamis, R. P. Woodard

Yukawa '06 S. P. Miao, R. P. Woodard

sQED '07 T. Prokopec, N. C. Tsamis, R. P. Woodard

The resummation formula for a general model with derivative interactions has not been known

We should evaluate differetiated field components exactly

e.g. dS inv. $\times$ dS broken $=$ dS broken

$$\langle \partial_\mu \varphi(x) \partial_\nu \varphi(x) \rangle \times \langle \varphi(x)\varphi(x) \rangle \simeq -\frac{3H^4}{32\pi^2} g_{\mu\nu} \times \frac{H^2}{4\pi^2} \log a(\tau)$$

$$\langle \varphi(x)\varphi(x) \rangle \simeq \frac{H^2}{4\pi^2} \log a(\tau)$$ Stochastic

$$\langle \partial_\mu \varphi(x) \partial_\nu \varphi(x) \rangle = -\frac{3H^4}{32\pi^2} g_{\mu\nu}$$ Exact

- In a general scalar field theory, we need to fine-tune the mass term to obtain such IR effects

  Nonlinear sigma model consists of massless and minimally coupled scalar fields for its global symmetry

  There, the leading IR effects to the cosmological constant cancel out each other at all orders

  ’11, ’12 H. K., Y. Kitazawa

- Dirac, Gauge fields do not induce the dS symmetry breaking

Gravitational field in dS space

- Gravitational field contains massless and minimally coupled modes without the fine-tuning
- Gravitational effects seem to be suppressed by $GH^2 \ll 1$, but there are associated enhancement factors: $(GH^2 \log a(\tau))^n$
- Such gravitational IR effects have not investigated enough due to some difficulties: gauge invariance, derivative interaction

Semiclassical approach

'10 S. B. Giddings, M. S. Sloth

$$g_{\mu\nu} = \Omega^2(x)\tilde{g}_{\mu\nu},\ \Omega(x) = a(\tau)e^{\kappa w(x)},$$
$$\det \tilde{g}_{\mu\nu} = -1,\ \tilde{g}_{\mu\nu} = (e^{\kappa h(x)})_{\mu\nu}$$

$$\kappa^2 = 16\pi G$$

GF term: $\mathcal{L}_{\rm GF} = -\frac{1}{2}a^2 F_\mu F^\mu,$

$$F_\mu = \partial_\rho h_\mu{}^\rho - 2\partial_\mu w + 2h_\mu{}^\rho \partial_\rho \log a + 4w\partial_\mu \log a$$

'94 N. C. Tsamis, R. P. Woodard

Conformally coupled modes

$$\langle h^{0i}(x)h^{0j}(x')\rangle = -\delta^{ij}\langle\phi(x)\phi(x')\rangle,$$
$$\langle Y(x)Y(x')\rangle = \langle\phi(x)\phi(x')\rangle,$$
$$\langle b^0(x)\bar{b}^0(x')\rangle = -\langle\phi(x)\phi(x')\rangle$$

$$\langle\phi(x)\phi(x')\rangle = \frac{H^2}{4\pi^2}\frac{1}{y}$$

Minimally coupled modes

$$\langle X(x)X(x')\rangle = -\langle\varphi(x)\varphi(x')\rangle,$$
$$\langle\tilde{h}^i{}_j(x)\tilde{h}^k{}_l(x')\rangle = (\delta^{ik}\delta_{jl} + \delta^i{}_l\delta_j{}^k - \frac{2}{3}\delta^i{}_j\delta^k{}_l)\langle\varphi(x)\varphi(x')\rangle,$$
$$\langle b^i(x)\bar{b}^j(x')\rangle = \delta^{ij}\langle\varphi(x)\varphi(x')\rangle$$

$$\langle\varphi(x)\varphi(x')\rangle = \frac{H^2}{4\pi^2}\{\frac{1}{y} - \frac{1}{2}\log y + \frac{1}{2}\underline{\log a(\tau)a(\tau')} + 1 - \gamma\}$$

$\tilde{h}^i{}_j$:traceless, $X \equiv 2\sqrt{3}w - h^{00}/\sqrt{3},\ Y \equiv h^{00} - 2w$

Soft gravitational effects on local dynamics of matter fields

'12 H. K., Y. Kitazawa

- Although we can not observe the super-horizon modes directly since the commutator of them is zero
- it is possible that virtual gravitons at the super-horizon scale affect the local dynamics of matter fields at the sub-horizon scale
- Our investigation is up to the one-loop level: $\log a(\tau) \gg 1,$

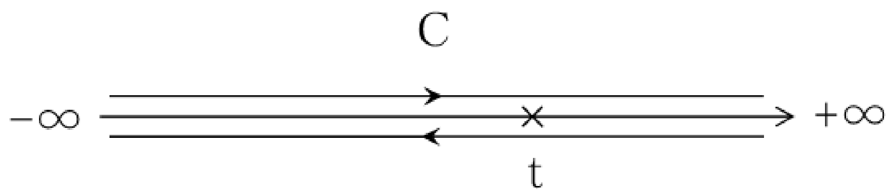
$$\kappa^2 H^2 \log a(\tau) \ll 1$$

Investigations of soft graviton effects on matter fields

	S. B. Giddings, M. S. Sloth	R. P. Woodard, E. O. Kahya, S. P. Miao		H. K., Y. Kitazawa
Parametrization	$ds^2 = -dt^2 + a^2(e^{\kappa h})_{ij}dx^i dx^j$,	$g_{\mu\nu} = a^2(\eta_{\mu\nu} + \kappa h_{\mu\nu})$		$g_{\mu\nu} = (ae^{\kappa w})^2(e^{\kappa h})_{\mu\nu}$,
Gauge	$h_i{}^i = 0, \quad \partial_j h_i{}^j = 0$	$F_\mu = \partial_\rho h_\mu{}^\rho - \frac{1}{2}\partial_\mu h_\rho{}^\rho + 2h_\mu{}^\rho \partial_\rho \log a$		$h_\mu{}^\mu = 0$ do.
External momentum	$P \ll H$		$P \ll H$ ( )	$P \gg H$
Matter fields	m.m.c. scalar	m.m.c. scalar	massless Dirac (massive)	m.c.c. fields with dimensionless couplings

Effective equation of motion

In investigating interactions on a time dependent background like dS space, we need to adopt the Schwinger-Keldysh formalism



'61 J. S. Schwinger,
'64 L. V. Keldysh,

$$\int_C dt = \int_{-\infty}^{\infty} dt_+ - \int_{-\infty}^{\infty} dt_-$$

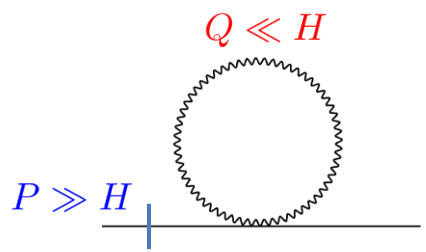
$\varphi \to \hat{\varphi} + \varphi, \quad \hat{\varphi}$: classical field, φ: quantum fluctuation

Effective e.o.m.:

$$\Box\hat{\varphi}(x) + \int \sqrt{-g} d^4x' \left[\Sigma_{++}(x,x') - \Sigma_{+-}(x,x')\right]\hat{\varphi}(x') = 0$$

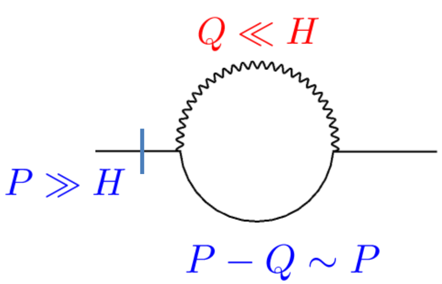
How the soft graviton contributes to the local dynamics?

$$\mathcal{L}_s = -\frac{1}{2}\sqrt{-g}\big[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{R}{6}\phi^2\big] \;\rightarrow -\frac{1}{2}\big[\tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{\tilde{R}}{6}\phi^2\big] \qquad \begin{array}{l} g_{\mu\nu} = \Omega^2\tilde{g}_{\mu\nu}, \\ \Omega\phi \rightarrow \phi \end{array}$$



$$\Sigma^{\text{4-pt}}_{++}(x,x') = \delta^{(4)}(x-x') \times \kappa^2 H^2\{c_1 a^2 H^2 + c_2 a H\partial + (c_3\log a + c_4)\partial\partial\}$$

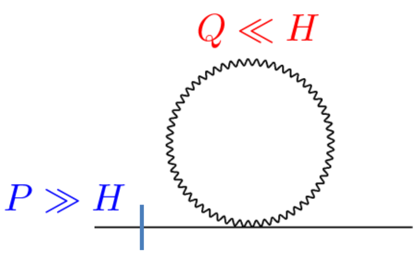
$$P \gg H \;\Rightarrow\; \log a\partial \gg \partial\log a \sim Ha$$



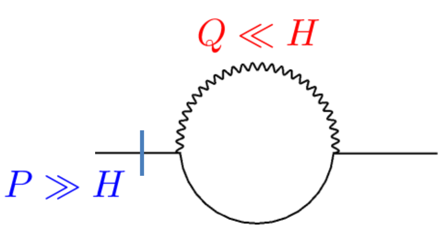
$$\Sigma^{\text{3-pt}}_{++}(x,x') - \Sigma^{\text{3-pt}}_{+-}(x,x') \rightarrow \delta^{(4)}(x-x') \times \log a(\tau)\partial\partial$$

$$\langle\partial_\mu\phi_+(x)\partial'_\nu\phi_+(x')\rangle \rightarrow i\delta_\mu{}^0\delta_\nu{}^0\delta^{(4)}(x-x')$$

Free scalar field theory



$$\simeq \frac{\kappa^2 H^2}{4\pi^2} \log a(\tau)\{\frac{3}{8}\partial_0^2 + \frac{13}{8}\partial_i^2\}\hat{\phi}$$



$$\simeq \frac{\kappa^2 H^2}{4\pi^2} \log a(\tau)\{-\frac{3}{4}\partial_0^2 - \frac{5}{4}\partial_i^2\}\hat{\phi}$$

$$\{1 + \frac{3\kappa^2 H^2}{32\pi^2} \log a(\tau)\}\partial^2\hat{\phi} \simeq 0$$

Lorentz inv.

Normalizable by

$$\phi \to Z_\phi \phi, \quad Z_\phi \simeq 1 - \frac{3\kappa^2 H^2}{64\pi^2} \log a(\tau)$$

$$P \gg H \Rightarrow \log a\partial \gg \partial \log a$$

Free Dirac field theory

$$\mathcal{L}_D = i\sqrt{-g}\bar{\psi}e^{\mu}{}_{a}\gamma^a D_\mu \psi \ \rightarrow i\bar{\psi}\tilde{e}^{\mu}{}_{a}\gamma^a \tilde{D}_\mu \psi$$

$$e^{\mu}{}_{a} = \Omega^{-1}\tilde{e}^{\mu}{}_{a},$$
$$\Omega^{\frac{3}{2}}\psi \rightarrow \psi$$

$$\{1 + \frac{3\kappa^2 H^2}{128\pi^2}\log a(\tau)\} \times i\gamma^\mu \partial_\mu \hat{\psi} \simeq 0$$

Lorentz inv.

$$\psi \rightarrow Z_\psi \psi, \quad Z_\psi \simeq 1 - \frac{3\kappa^2 H^2}{256\pi^2}\log a(\tau)$$

Soft graviton effects on the kinetic terms can be absorbed by the wave function renormalizations

On the other hand, soft gravitons contribute to interacting field theories

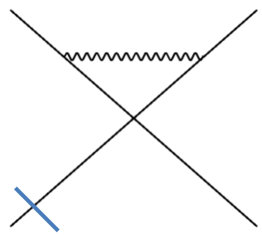
ϕ^4 interaction

$$\delta\mathcal{L}_4 = -\frac{\lambda_4}{4!}\phi^4 \;\to -\frac{\lambda_4}{4!}Z_\phi^4\phi^4$$

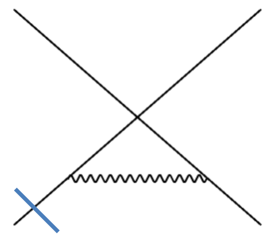
$\sqrt{-g}$ is absorbed
by $\Omega\phi \to \phi$

Non-linear term in e.o.m.:

$$-\frac{\lambda_4}{6}Z_\phi^4\hat{\phi}^3 \quad +3$$



$$+3$$



$$\simeq -\frac{\lambda_4}{6}\left\{1 - \frac{21\kappa^2H^2}{16\pi^2}\log a(\tau)\right\}\hat{\phi}^3$$

$$(\lambda_4)_{\rm eff} \simeq \lambda_4\left\{1 - \frac{21\kappa^2H^2}{16\pi^2}\log a(\tau)\right\}$$

$(\lambda_4)_{\rm eff}$ decreases with cosmic evolution

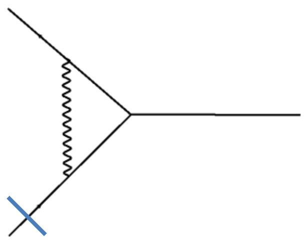
Yukawa interaction

$$\delta\mathcal{L}_Y = -\lambda_Y \phi\bar{\psi}\psi \ \rightarrow -\lambda_Y Z_\phi Z_\psi^2 \phi\bar{\psi}\psi$$

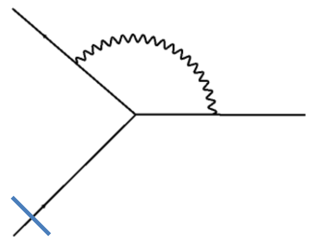
$\sqrt{-g}$ is absorbed by
$\Omega\phi \rightarrow \phi$, $\Omega^{\frac{3}{2}}\psi \rightarrow \psi$

Non-linear term in e.o.m.:

$-\lambda_Y Z_\phi Z_\psi^2 \hat{\phi}\hat{\psi} \ +$



$+2$



$\simeq -\lambda_Y \{1 - \frac{39\kappa^2 H^2}{128\pi^2}\log a(\tau)\}\hat{\phi}\hat{\psi}$

$\Downarrow$

$$(\lambda_Y)_{\rm eff} \simeq \lambda_Y \{1 - \frac{39\kappa^2 H^2}{128\pi^2}\log a(\tau)\}$$

$(\lambda_Y)_{\rm eff}$ decreases with cosmic evolution

Gauge interactions

Effective e.o.m.:

$$\frac{1}{g^2}\{1+\frac{3\kappa^2H^2}{8\pi^2}\log a(\tau)\}(\hat{D}_\mu \hat{F}^{\mu\nu})^a \qquad gA^a_\mu \to A^a_\mu$$

$$+\{1+\frac{3\kappa^2H^2}{128\pi^2}\log a(\tau)\}\hat{\bar{\psi}}\gamma^\nu t^a \hat{\psi} \simeq 0$$

Nomalized by $\psi \to Z_\psi \psi$

$$\Downarrow$$

$$g_{\rm eff} \simeq g\{1-\frac{3\kappa^2H^2}{16\pi^2}\log a(\tau)\}$$

$g_{\rm eff}$ decreases with cosmic evolution

The behavior is independent of a gauge group

Gauge dependence

We introduce the gauge parmeter β as

$$\mathcal{L}_{GF} = -\frac{1}{2}a^2 F_\mu F^\mu,$$

$$F_\mu = \beta\partial_\rho h_\mu{}^\rho - 2\beta\partial_\mu w + \frac{2}{\beta}h_\mu{}^\rho\partial_\rho \log a + \frac{4}{\beta}w\partial_\mu \log a$$

Original gauge at $\beta = 1$

For a continuous β ($|\beta^2 - 1| \ll 1$), soft graviton effects do not spoil Lorentz invariance and can be absorbed by the wave function renormalizations

$$Z_\phi \simeq 1 - \big(1 - (\beta^2 - 1)\big)\frac{3\kappa^2 H^2}{64\pi^2}\log a(\tau),$$

$$Z_\psi \simeq 1 - \big(1 - (\beta^2 - 1)\big)\frac{3\kappa^2 H^2}{256\pi^2}\log a(\tau)$$

Although each coupling is gauge dependent,

$$(\lambda_4)_{\rm eff} \simeq \lambda_4\big\{1-\big(1-(\beta^2-1)\big)\frac{21\kappa^2H^2}{16\pi^2}\log a(\tau)\big\} = \lambda_4 f(\tau)^{\frac{21}{4}},$$

$$(\lambda_Y)_{\rm eff} \simeq \lambda_Y\big\{1-\big(1-(\beta^2-1)\big)\frac{39\kappa^2H^2}{128\pi^2}\log a(\tau)\big\} = \lambda_Y f(\tau)^{\frac{39}{32}},$$

$$g_{\rm eff} \simeq g\big\{1-\big(1-(\beta^2-1)\big)\frac{3\kappa^2H^2}{16\pi^2}\log a(\tau)\big\} = g f(\tau)^{\frac{3}{4}}$$

$$f(\tau) = 1-\big(1-(\beta^2-1)\big)\frac{\kappa^2H^2}{4\pi^2}\log a(\tau)$$

the relative scaling exponents are gauge invariant

$$(\lambda_Y)_{\rm eff}/\lambda_Y = \big\{(\lambda_4)_{\rm eff}/\lambda_4\big\}^{\frac{13}{56}},\ g_{\rm eff}/g = \big\{(\lambda_4)_{\rm eff}/\lambda_4\big\}^{\frac{1}{7}}$$

We interpret this result as follows

- Each coupling is gauge dependent because there is no unique way to specify the time as it depends on an observer
- A sensible strategy is to pick a certain coupling and use its time evolution as a physical time

Scheme dependence

Soft graviton effects depend on the parametrization of the metric and the Lorentz inv. is broken in a general case

$$g_{\mu\nu} = a^2 e^{2\kappa w} (e^{\kappa h})_{\mu\nu} \qquad , h_\mu{}^\mu = 0 \qquad \text{: Ours}$$

$$= a^2 (\eta_{\mu\nu} + 2\kappa\Phi\eta_{\mu\nu} + \kappa\Psi_{\mu\nu}) \quad , \Psi_\mu{}^\mu = 0 \qquad \text{: Woodards'}$$

The difference between them starts at the non-linear level

$$\kappa w = \kappa\Phi + \mathcal{O}(\kappa^2\Phi^2, \kappa^2\Psi^2),$$

$$\kappa h_{\mu\nu} = \kappa\Psi_{\mu\nu} + \mathcal{O}(\kappa^2\Phi\Psi, \kappa^2\Psi\Psi),$$

and so it contributes only to the tadpole diagrams at 1-loop

Such effects can be eliminated by shifting the background metric

$$h_{\mu\nu} \to h_{\mu\nu} + v_{\mu\nu}$$

The choice of the matter field redefinition contributes to soft graviton effects

$$\varphi_{\alpha_s}(x) = a \exp(\frac{2-\alpha_s}{2}\kappa w)\varphi(x), \quad \psi_{\alpha_D}(x) = a^{\frac{3}{2}} \exp(\frac{3-\alpha_D}{2}\kappa w)\psi(x)$$

Ours: $\alpha_s = \alpha_D = 0$

Woodards': $\alpha_s = 2, \quad \alpha_D = 3$

The Lorentz inv. is broken in a general choice and it can be compensated by shifting the background metric only when $\alpha_s = \alpha_D$

A sensible setting may be that dimensionless couplings are scale independent at the calassical level: $\alpha_s = \alpha_D = 0$

$$-e^{2\alpha_s\kappa w}\frac{\lambda_4}{4!}\varphi_{\alpha_s}^4, \quad -e^{(\frac{1}{2}\alpha_s+\alpha_D)\kappa w}\lambda_Y\varphi_{\alpha_s}\bar{\psi}_{\alpha_D}\psi_{\alpha_D}$$

Summary

- We have investigated soft gravitational effects on local dynamics of matter fields at the sub-horizon scale
- The IR effects on the kinetic terms do not spoil Lorentz invariance and can be absorbed by wave function renormalizations
- Soft gravitons dynamically screen dimensionless couplings whose relative scaling exponents are gauge invariant

Future works

- Lorentz, Gauge invariance are preserved at higher loop levels, against large deformations of a gauge parameter?
- Against large deformations of a gauge parameter, the screening of couplings do not turn to the enhancement?
- In addition to pure matter and pure gravity contributions, the mixed contribution from matter and gravity makes the cosmological constant time dependent