Quantum effects to Scalars during Inflation

Emre Kahya

University of Jena / İstanbul Technical University

EO Kahya, VK Onemli, RP Woodard, Phys.Lett.B694 2010,101

EO Kahya, SP Miao, RP Woodard, J Math Phys 53, 2012, 022304

What am I doing?

Calculating the quantum effects to the scalar fields during Inflation.

Correcting the mode function involves two steps:

1) One loop self-mass $M^2(x;x')$ of a massless scalars in a locally de Sitter background geometry.

$$L_{SQED} = -(\partial_\mu - i e_0 A_\mu)\phi^* (\partial_\nu + i e_0 A_\nu)\phi g^{\mu\nu}\sqrt{-g} - \zeta_0 \phi^* \phi R\sqrt{-g} - \frac{1}{4} F_{\mu\nu} F_{\rho\sigma} g^{\mu\rho} g^{\nu\sigma}\sqrt{-g}$$

$$L_{MMCS} = -\frac{1}{2}\partial_\mu\phi\partial_\nu\phi g^{\mu\nu}\sqrt{-g} + \frac{1}{16\pi G}(R-(D-2)\Lambda)\sqrt{-g}$$

$$L_{CCS} = -\frac{1}{2}\partial_\mu\phi\partial_\nu\phi g^{\mu\nu}\sqrt{-g} + \frac{D-2}{8(D-1)}\phi^2 R\sqrt{-g} + \frac{1}{16\pi G}(R-(D-2)\Lambda)\sqrt{-g}$$

in different gauges:

de Sitter non-invariant

de Sitter invariant

2) Solving the effective field equations to get the mode functions using $M^2(x;x')$ at one-loop order.

- $ds^2 = -dt^2 + a^2(t)\, d\vec{x}\cdot d\vec{x}$ $\quad t \Rightarrow \textit{physical time}$
- $a(t) = e^{Ht} \quad H \Rightarrow \textit{constant}$ $\quad a(t)\|\vec{x}\| \Rightarrow \textit{physical distance}$

Technical detail: the calculations are done in position space.

•

•Quantum effects during Cosmology

3 messages of this talk

1) ***don't want further supression***

since we are after quantum effects which are already small.

for that need particles to be:

Massless

do NOT possess Conformal Invariance

2) ***calculations should be doable,***

de Sitter invariance love hurts

3) ***effects are small not necessarily zero.***

there are many non-zero examples with log enhancement

•*Persistence Time of Particles*

Flat Space

$\underline{\underline{Before}}$	$\underline{\underline{After}}$
$Vacuum$	$\leftarrow \bullet \quad \bullet \rightarrow$
$\vec{p} = 0$	$\vec{p} = \vec{k} - \vec{k}$
$E = 0$	$E = 2\sqrt{m^2 + k^2}$

- $\Delta E \;\; \Delta t \geq \hbar$
- $to\ NOT\ resolve\ \ \Delta E \;\Rightarrow\; \Delta t \leq \dfrac{1}{2\sqrt{m^2+k^2}}$
- $m = 0 \;\Rightarrow\; biggest\ effect$

$a(t) \neq 1$

- $x \rightarrow x + \Delta x \quad \Rightarrow \; \vec{k}\ \ a\ good\ \ Q.\#\ \ but\ ;$
    - $x_{phys} = a(t)\ x$
    - $k = \dfrac{2\pi}{\lambda}$

$$\therefore\ \vec{k}_{phys} = \frac{\vec{k}}{a(t)} \qquad \therefore\ E = \sqrt{m^2 + \frac{k^2}{a^2(t)}}$$

- $\int_t^{t+\Delta t} dt'\, E\left(t', k\right) \leq 1$

$$for\ \ m=0 \quad a(t) = e^{Ht} \quad \int_t^{t+\Delta t} dt'\, E\left(t', k\right) = \left[1 - \mathrm{e}^{-H\Delta t}\right] \frac{k}{H\,a\left(t\right)} \leq 1$$

- $k \leq H\ a(t) \quad can\ \ persist\ \ forever\ !$

•*Emergence Rate*

Conf. Inv.

$$g_{\mu\nu} = \Omega^2(x)\; g_{\mu\nu} \qquad A_\mu \to A_\mu \quad \phi \to \frac{\phi(x)}{\Omega(x)} \quad \Psi_i \to \frac{\Psi_i}{\Omega^{3/2}(x)}$$

$$e.g.\; E\,M \quad L = -\frac{1}{4} F_{\rho\sigma} F_{\mu\nu}\, g^{\rho\mu}\, g^{\sigma\nu} \sqrt{-g} \quad \to -\frac{1}{4} F_{\rho\sigma} F_{\mu\nu}\, \Omega^{-2}\, g^{\rho\mu}\, \Omega^{-2}\, g^{\sigma\nu}\, \Omega^4 \sqrt{-g}$$

Conf. Coords.

$$\left.\begin{array}{l} ds^2 = -dt^2 + a^2(t)\, d\vec{x}\cdot d\vec{x} \\ \bullet\, d\eta = \dfrac{dt}{a(t)} \end{array}\right\} \Rightarrow ds^2 = a^2\left[-d\eta^2 + d\vec{x}\cdot d\vec{x}\right] \quad \therefore g_{\mu\nu} = a^2\, \eta_{\mu\nu}$$

$$\bullet\, \Omega = \frac{1}{a} \quad \bullet\, L_{conf}(g_{\mu\nu}, A_\mu, \Psi, \phi) = L_{conf}(\eta_{\mu\nu}, A_\mu, a^{3/2}\Psi, a\,\phi)$$

- local field redef. don't change physics.
- conf. inv. theories are identical to flat space cousins.

Emergence Rate

$$\bullet\, \frac{dN}{d\eta} = \Gamma_{flat} \qquad \bullet\, \frac{dN}{dt} = \frac{dN}{d\eta}\,\frac{d\eta}{dt} = \frac{\Gamma_{flat}}{a(t)} \qquad \therefore\; few\; will\; emerge$$

MMCS and GRAVITONS

- massless
- do NOT possess Conformal Invariance

•*An Explicit Example*

• particle creation in Yukawa Theory:

$$L = -\frac{1}{2}\partial_\mu \phi \partial_\nu \phi\, g^{\mu\nu}\sqrt{-g} - \frac{1}{2}\xi \phi^2 R\sqrt{-g} + \bar{\psi} e^\mu_{\ b}\, \gamma^b (\partial_\mu - \frac{1}{2} A_{\mu c d}\, J^{cd})\psi \sqrt{-g} - f\, \phi \psi \bar{\psi} \sqrt{-g}$$

where $\{\gamma^a, \gamma^b\} = -2\eta^{ab} I$ $\qquad g_{\mu\nu} = e_{\mu b}\, e_{\nu c}\, \eta^{bc}$

and $A_{\mu b c} = e^\nu_{\ b}\, (e_{\nu c,\mu} - \Gamma^\rho_{\mu\nu}\, e_{\rho c})$ $\qquad J^{bc} = \frac{i}{4}[\gamma^b, \gamma^c]$

for homogeneous and isotropic metric

$$\Rightarrow\ e_{\mu b}(t,x)\Big|_{HI} = -\delta_{\mu 0}\delta_{b0} + a(t)\,\delta_{\mu i}\delta_{bi}$$

and in terms of conformally rescaled fermion fields

$$\Psi(t,x) = a^{3/2}(t)\,\psi(t,x) \quad \bar{\Psi}(t,x) = a^{3/2}(t)\,\bar{\psi}(t,x)$$

$$L\Big|_{HI} = \frac{a^3}{2}\left(\dot{\phi}^2 - \frac{1}{a^2}\vec{\nabla}\phi \cdot \vec{\nabla}\phi - \xi(12H^2 + 6\dot{H})\phi^2\right) + \bar{\Psi}\left(\gamma^0 \partial_0 + \frac{1}{a}\gamma^i \partial_i\right)\Psi - f\,\phi\bar{\Psi}\Psi$$

•R.P.Woodard astro-ph/0310757

• free field expansions:

$$\phi_I(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^3} \left\{ e^{i\vec{k}\cdot\vec{x}} A(t,k)\, \alpha(\vec{k}) + e^{-i\vec{k}\cdot\vec{x}} A^*(t,k)\, \alpha^\dagger(\vec{k}) \right\},$$

$$\Psi_I(t,\vec{x}) = \int \frac{d^3q}{(2\pi)^3} \left\{ e^{i\vec{q}\cdot\vec{x}} B(t,\vec{q},r)\, \beta(\vec{q},r) + e^{-i\vec{q}\cdot\vec{x}} C(t,\vec{q},r)\, \gamma^\dagger(\vec{q},r) \right\},$$

$$\bar{\Psi}_I(t,\vec{x}) = \int \frac{d^3p}{(2\pi)^3} \left\{ e^{i\vec{p}\cdot\vec{x}} \bar{C}(t,\vec{p},s)\, \gamma(\vec{p},s) + e^{-i\vec{p}\cdot\vec{x}} \bar{B}(t,\vec{p},s)\, \beta^\dagger(\vec{p}) \right\}$$

$$\left[\alpha(\vec{k}),\, \alpha^\dagger(\vec{k}\,') \right] = (2\pi)^3\, \delta^3(\vec{k} - \vec{k}\,') \;\;,\;\; \left\{ \beta(\vec{q},r), \beta^\dagger(\vec{p},s) \right\} = \left\{ \gamma(\vec{q},r), \gamma^\dagger(\vec{p},s) \right\} = \delta_{rs}\, (2\pi)^3\, \delta^3(\vec{q} - \vec{p})$$

• eqn. for scalar wave function:

$$\ddot{A}(t,k) + 3H\,\dot{A}(t,k) + \frac{k^2}{a^2} A(t,k) + \xi\, (12H^2 + 6\dot{H}) A(t,k) = 0$$

$$\bullet\; Flat\ Space\big(a(t)=1\big) \Rightarrow A(t,k) = \frac{1}{\sqrt{2k}} e^{-ik\Delta\eta(t)} = \frac{1}{\sqrt{2k}} e^{-ikt}$$

$$\bullet\; \left.\begin{matrix} Conformal \\ Coupling \end{matrix}\right\} \begin{pmatrix} any\; a(t) \\ \xi = 1/6 \end{pmatrix} \Rightarrow A(t,k) = \frac{1}{\sqrt{2k}\,a(t)} e^{-ik\Delta\eta(t)} = \frac{1}{\sqrt{2k}\,a(t)} e^{-ik\int_0^t \frac{dt'}{a(t')}} \quad since \quad \Delta\eta(t) = \int_0^t \frac{dt'}{a(t')}$$

$$\bullet\; \left.\begin{matrix} Minimally \\ Coupled \end{matrix}\right\} \begin{pmatrix} a(t) = e^{Ht} \\ \xi = 0 \end{pmatrix} \Rightarrow A(t,k) = \frac{1}{\sqrt{2k}\,a(t)} \left[1 - \frac{i\,Ha}{k} \right] e^{-ik\Delta\eta(t)} = \frac{1}{\sqrt{2k}\,a(t)} \left[1 - \frac{i\,Ha}{k} \right] e^{ik(1-e^{-Ht})/H}$$

$$B(t,\vec{q},r) = \frac{u(\vec{q},r)}{\sqrt{2q}} e^{-iq\Delta\eta(t)} \quad , \quad C(t,\vec{p},s) = \frac{v(\vec{p},s)}{\sqrt{2p}} e^{ip\Delta\eta(t)} .$$

• time evolution op. of the interaction picture : $U \equiv T\left\{\exp\left[-if\int_{t_{in}}^{t_{out}} dt\int d^3x\,\phi_I(t,\vec{x})\,\bar{\Psi}_I(t,\vec{x})\,\Psi_I(t,\vec{x})\right]\right\}$

where $\alpha^{out}(\vec{k}) = U^\dagger \alpha^{in}(\vec{k})U\ ,\ \beta^{out}(\vec{q},r) = U^\dagger \beta^{in}(\vec{q},r)U\ ,\ \gamma^{out}(\vec{p},s) = U^\dagger \gamma^{in}(\vec{p},s)U.$

$Amplitude$
$vac \to \phi\Psi\bar{\Psi}$

$$\left\langle \Omega \middle| \sqrt{2k}\ \alpha(\vec{k})\ \sqrt{2q}\ \beta(\vec{q},r)\sqrt{2p}\ \gamma(\vec{p},s)U \middle| \Omega \right\rangle$$

$$= -i\ f\ (2\pi)^3\,\delta^3(\vec{k}+\vec{p}+\vec{q})\sqrt{8kqp}\int_{t_{in}}^{t_{out}} dt\, A^*(t,k)\,\bar{B}(t,\vec{q},r)\,C(t,\vec{p},s) + O(f^3)$$

$$= -i\ f\ (2\pi)^3\,\delta^3(\vec{k}+\vec{p}+\vec{q})\ \bar{u}(\vec{q},r)\,v(\vec{p},s)\int_{t_{in}}^{t_{out}} dt\,F(t)\,e^{i(k+p+q)\Delta\eta(t)} + O(f^3)$$

$$Flat\ Space \Rightarrow \int_{-\infty}^{\infty} dt\, e^{i(k+p+q)\Delta\eta(t)} = \int_{-\infty}^{\infty} dt\, e^{i(k+p+q)t} = 2\pi\,\delta(k+q+p)$$

$$Conformal\ Coupling \Rightarrow \int_{t_{in}}^{t_{out}} \frac{dt}{a(t)}\, e^{i(k+p+q)\Delta\eta(t)} = \int_{t_{in}}^{t_{out}} \frac{dt}{a(t)}\, e^{i(k+p+q)\int_0^t \frac{dt'}{a'}}\ .$$

$$Minimal\ Coupling \Rightarrow \int_{t_{in}}^{t_{out}} dt\left(e^{-Ht} - \frac{iH}{k}\right)e^{i(k+p+q)\Delta\eta(t)} = \int_{t_{in}}^{t_{out}} dt\left(e^{-Ht} - \frac{iH}{k}\right)e^{i(k+p+q)\,(1-e^{-Ht})/H}$$

• *in short* ***: masless and conformal non-inv particles have bigger effect during Inflation***

•*PREVIOUS CALCULATIONS USING MMCS and/or GRAVITONS*

- Density Perturbations

  - Mukhanov Chibisov (1981) => Scalar WMAP sees!

  - Starobinsky (1979) => Gravitons, LISA or ET can see in future decades

- MMCS + λ φ^4 => 2 loop T_μν and -iM^2

  - Brunier, Onemli, Woodard (2005) and - Kahya, Onemli (2007)

    => violation of weak en. Condition

- SQED =>Woodard Kahya 1-loop -iM^2, Prokopec, Tsamis, Woodard 2-loop, stochastic formulation..

- Yukawa => Duffy Woodard 2004, S.P.Miao Woodard 2006 1-loop, stochastic formulation...

- Scalar driven inflation – Loop effects

  Back-reaction at one loop (mukhanov et al 97, Abramo et al 98, Ghosh et al 99, Unruh 98, Brandenberger et al 02...)

  Loop corrections to power spectrum (Maldacena 03, Weinberg 05, Boyanovsky et al 05, Sloth 05, Lyth 05, de Vega 06, Seery 07, Burgess Holman 07, Riotto 08, Adshead et al 08, Senatore Zaldarriaga 09, Giddings et al 10, Kahya et al 10, Hebecker et al 10, Urakawa 11 ,Pimentel, Senatore Zaldarriaga 12...)

  => Zeta-zeta correlator is time dependent or not ?

  Can we get a tilt on the power spectrum, is this effect big enough so that we can observe this in future experiments? Planck satellite not really! But future proposals of 21 cm of PS at high z (Furlanetto et al 06) tensor-scalar ratio r to one in 10^8 compared to now r < 0.2

Self-Mass Squared at one loop

What is it, mathematically and what do we do with it?
(MMCS as an example)

1. Compute $-iM^2(x;x')$
2. Quantum Corrected linearized eqns:

$$\partial_\mu(\sqrt{-g}\, g^{\mu\nu}\, \partial_\nu\varphi) \; - \; \int d^4x' M^2(x;x')\, \varphi(x') = 0$$

with Lagrangian

=> $$L = \frac{1}{16\pi G}(R - 2\Lambda) - \frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi\, g^{\mu\nu}\sqrt{-g}$$

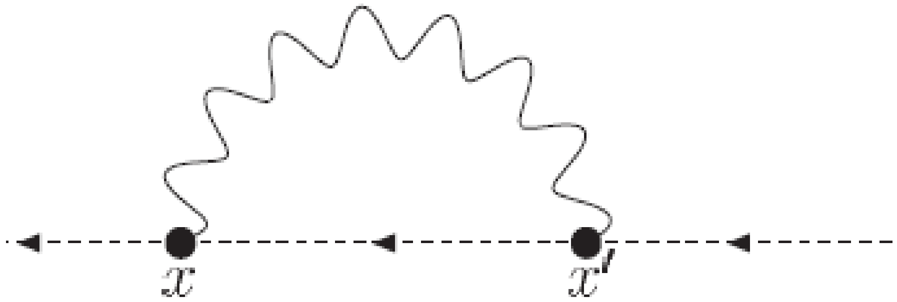
and in a locally de Sitter geometry.

explicitly self-mass-squared is:

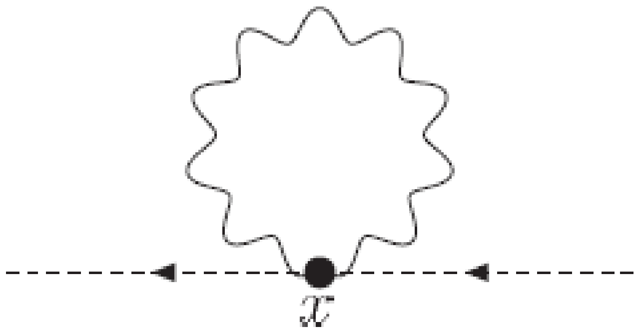
$$-i\,M^2(x;x') = \sum_{i=1}^{2} V_i^{\alpha\beta}(x) \sum_{j=1}^{2} V_j^{\rho\sigma}(x')\; i\left[{}_{\alpha\beta}\Delta_{\rho\sigma}\right](x;x')\, i\,\Delta(x;x')$$

$$+\sum_{i=1}^{4} U_i^{\alpha\beta\rho\sigma}(x)\, i\left[{}_{\alpha\beta}\Delta_{\rho\sigma}\right](x;x')\,\delta^D(x-x') + \sum_i C_i(x)\,\delta^D(x-x')$$

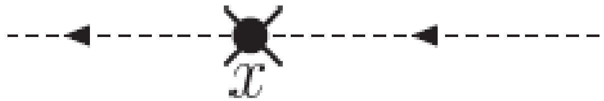
- Graphically 1-loop self-mass-squared is:



Contribution from two 3-point interactions.



Contribution from the 4-point interaction.



Contribution from counterterms.

- We need two things: Vertex operators and Propagators

•***Perturbative Formulation***

$$\bullet\ perturbation \Rightarrow g_{\mu\nu}(x) = a^2\left(\eta_{\mu\nu} + \kappa h_{\mu\nu}(x)\right) \quad \kappa^2 = 16\pi G$$

$$g^{\mu\nu} = \frac{1}{a^2}\left(\eta^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h^{\mu\rho} h_\rho^\nu + \ldots\right)$$

$$\sqrt{-g} = a^D\left(1 + \frac{1}{2}\kappa h + \frac{1}{8}\kappa^2 h^2 - \frac{1}{4}\kappa^2 h^{\rho\sigma} h_{\rho\sigma} + \ldots\right)$$

Lagrangian

$$L = -\frac{1}{2}\partial_\mu\phi\partial_\nu\phi\, g^{\mu\nu}\sqrt{-g}$$

$$\Rightarrow -\frac{1}{2}\partial_\mu\phi\partial_\nu\phi\, a^{D-2}\left(\eta^{\mu\nu} - \kappa h^{\mu\nu} + \frac{1}{2}\eta^{\mu\nu}\kappa h\right.$$

$$\left. + \frac{1}{8}\eta^{\mu\nu}\kappa^2 h^2 - \frac{1}{4}\eta^{\mu\nu}\kappa^2 h^{\rho\sigma} h_{\rho\sigma} - \frac{1}{2}\kappa^2 h\, h^{\mu\nu} + \kappa^2 h^{\mu\rho} h_\rho^\sigma + \ldots\right)$$

•*Working in Position Space*

- $4-momentum\, is\, not\, a\, good\, Quantum\ \#$
- $No\, integrations\, at\ 1-loop$

$\boxed{Vertices}$

•$3-\ pt\ Vertex:$

$$i\,\frac{\delta}{\delta\phi(x)}\frac{\delta}{\delta\phi(y)}\frac{\delta}{\delta h_{\mu\nu}(z)}S[\phi,h]\bigg|_{h=0} = i\,\kappa\, a^{D-2}(z)\partial_x^{\mu}\,\partial_y^{\nu}\,\delta^D\left(z-x\right)\delta^D\left(z-y\right)+\ldots$$

•$4-\ pt\ Vertex:$

$$i\,\frac{\delta}{\delta\phi(x)}\frac{\delta}{\delta\phi(y)}\frac{\delta}{\delta h_{\mu\nu}(z)}\frac{\delta}{\delta h_{\rho\sigma}(w)}S[\phi,h]\bigg|_{h=0} = i\frac{\kappa^2}{2}m^2\,a^D(w)\eta^{\mu(\rho}\,\eta^{\sigma)\nu}\ \delta^D\left(w-x\right)\delta^D\left(w-y\right)\delta^D\left(w-z\right)+\ldots$$

PROPAGATORS

•*Conformally Coupled Scalar*

$$i\Delta_{CF} = \frac{H^{D-2}}{(4\pi)^{D/2}}\,\Gamma\left(\frac{D}{2}-1\right)\left(\frac{4}{y}\right)^{\frac{D}{2}-1}$$

$$where\; y = H^2\, a a'\, \Delta x^2 \quad , \quad \Delta x^2 = \|\vec{x}-\vec{x}'\|^2 - \left(|\eta-\eta'| - i\delta\right)^2$$

•*Massless Minimally Coupled Scalar*

$$i\Delta_A(x;x') = i\Delta_{CF}(x;x')$$

$$+\ \frac{H^{D-2}}{(4\pi)^{D/2}}\ \frac{\Gamma(D-1)}{\Gamma\left(\frac{D}{2}\right)}\left\{\frac{D}{D-4}\,\frac{\Gamma^2\left(\frac{D}{2}\right)}{\Gamma(D-1)}\left(\frac{4}{y}\right)^{\frac{D}{2}-2} - \pi\cot\left(\frac{\pi}{2}D\right) + \ln(a a')\right\}$$

$$+\ \frac{H^{D-2}}{(4\pi)^{D/2}}\sum_{n=1}^{\infty}\left\{\frac{1}{n}\,\frac{\Gamma(n+D-1)}{\Gamma\left(n+\frac{D}{2}\right)}\left(\frac{y}{4}\right)^{n} - \frac{1}{n-\frac{D}{2}+2}\,\frac{\Gamma\left(n+\frac{D}{2}+1\right)}{\Gamma(n+2)}\left(\frac{y}{4}\right)^{n-\frac{D}{2}+2}\right\}$$

the MMCS propagator breaks the de Sitter invariance.

•Photon Propagator

• in de Sitter non-invariant gauge:

$$i\left[{}_{\mu}\Delta_{\nu}\right](x;x') = a a' \bar{\eta}_{\mu\nu}\, i\Delta_B(x;x') - a a' \delta_{\mu}^{\ 0}\,\delta_{\nu}^{\ 0}\, i\Delta_C(x;x')$$

$$where\ \bar{\eta}_{\mu\nu} = \eta_{\mu\nu} + \delta_{\mu}^{\ 0}\,\delta_{\nu}^{\ 0}$$

$$i\Delta_B(x;x') = \frac{H^{D-2}}{(4\pi)^{D/2}} \; \frac{\Gamma(D-2)\,\Gamma(1)}{\Gamma\left(\frac{D}{2}\right)} \; {}_2F_1\left(D-2,1;\frac{D}{2};1-\frac{y}{4}\right)$$

$$i\Delta_C(x;x') = \frac{H^{D-2}}{(4\pi)^{D/2}} \; \frac{\Gamma(D-3)\,\Gamma(2)}{\Gamma\left(\frac{D}{2}\right)} \; {}_2F_1\left(D-3,2;\frac{D}{2};1-\frac{y}{4}\right)$$

if we expand them in the same form as MMCS propagator;

$$i\Delta_B(x;x') = i\Delta_{CF}(x;x') - \frac{H^{D-2}}{(4\pi)^{D/2}} \sum_{n=0}^{\infty} \left\{ \frac{\Gamma(n+D-2)}{\Gamma\left(n+\frac{D}{2}\right)} \left(\frac{y}{4}\right)^{n} - \frac{\Gamma\left(n+\frac{D}{2}\right)}{\Gamma(n+2)} \left(\frac{y}{4}\right)^{n-\frac{D}{2}+2} \right\}$$

$$i\Delta_C(x;x') = i\Delta_{CF}(x;x') - \frac{H^{D-2}}{(4\pi)^{D/2}} \sum_{n=0}^{\infty} \left\{ (n+1)\frac{\Gamma(n+D-3)}{\Gamma\left(n+\frac{D}{2}\right)} \left(\frac{y}{4}\right)^{n} - \left(n-\frac{D}{2}+3\right) \frac{\Gamma\left(n+\frac{D}{2}-1\right)}{\Gamma(n+2)} \left(\frac{y}{4}\right)^{n-\frac{D}{2}+2} \right\}$$

• in de Sitter invariant gauge of Allen and Jacobson:

$$i\left[{}_{\mu}\Delta_{\nu}^{AJ}\right](x;x') = B(y)\,\frac{\partial^2 y}{\partial x^{\mu}\,\partial x'^{\nu}} + C(y)\,\frac{\partial y}{\partial x^{\mu}}\frac{\partial y}{\partial x'^{\nu}}\ ,$$

$$B(y) = \frac{H^{D-4}}{(4\pi)^{D/2}}\frac{\Gamma(D-3)}{\Gamma(\frac{D}{2})}\Big\{-\Big(\frac{D-3}{2}\Big)\,{}_2F_1\Big(D-2,1;\frac{D}{2};1-\frac{y}{4}\Big)$$
$$+\Big[\frac{1}{2}-\Big(\frac{D-2}{D}\Big)(\psi(D)-\psi(1))\Big]\,{}_2F_1\Big(D,1;\frac{D}{2}+1;\frac{y}{4}\Big)$$
$$+\frac{(D-3)}{(4y-y^2)^{D/2}}\int_0^y dy'(4y'-y'^2)^{(D/2)-1}$$
$$\times\int_{y'}^4 dy''\,{}_2F_1\Big(D-2,1;\frac{D}{2};1-\frac{y''}{4}\Big)\Big\},$$

de Sitter inv, function of y only

$$C(y) = \frac{H^{D-4}}{(4\pi)^{D/2}}\frac{\Gamma(D-3)}{\Gamma(\frac{D}{2})}\frac{\partial}{\partial y}\Big\{\Big[\frac{1}{2}-\Big(\frac{D-2}{D}\Big)(\psi(D)$$
$$-\psi(1))\Big]\,{}_2F_1\Big(D,1;\frac{D}{2}+1;\frac{y}{4}\Big)+\frac{(D-3)}{(4y-y^2)^{D}}\times\int_0^y dy'(4y'-y'^2)^{(D/2)-1}$$
$$\times\int_{y'}^4 dy''\,{}_2F_1\Big(D-2,1;\frac{D}{2};1-\frac{y''}{4}\Big)\Big\},$$

where $\psi(z) = \Gamma'(z)/\Gamma(z)$ *and*

• corrected photon propagator in de Sitter invariant gauge of Allen and Jacobson: (Tsamis Woodard 2006)

$$i\Big[{}_{\mu}\Delta_{\nu}\Big](x;x') = B(y)\frac{\partial^2 y}{\partial x^{\mu}\partial x'^{\nu}} + C(y)\frac{\partial y}{\partial x^{\mu}}\frac{\partial y}{\partial x'^{\nu}}$$

$$B(y) \;=\; \frac{1}{4(D-1)H^2}\Big[-(4y-y^2)\gamma'(y) - (D-1)(2-y)\gamma(y)\Big]\,,$$

$$C(y) \;=\; \frac{1}{4(D-1)H^2}\Big[+(2-y)\gamma'(y) - (D-1)\gamma(y)\Big]\,.$$

$$\gamma(y) \equiv \frac{1}{2}(D-1)\frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}}\Bigg\{\frac{\Gamma(\frac{D}{2})}{\frac{D}{2}-1}\Big(\frac{4}{y}\Big)^{\frac{D}{2}-1} + \frac{1}{D-3}\sum_{n=0}^{\infty}\Bigg[\frac{\Gamma(n+D-1)}{\Gamma(n+\frac{D}{2}+1)}\Big(\frac{y}{4}\Big)^{n}$$

$$\times(n+1)\Big[\psi\Big(2-\frac{D}{2}\Big)-\psi\Big(\frac{D}{2}-1\Big)+\psi\Big(n+D-1\Big)-\psi\Big(n+2\Big)\Big]$$

$$-\frac{\Gamma(n+\frac{D}{2}+1)}{\Gamma(n+3)}\Big(\frac{y}{4}\Big)^{n+3-\frac{D}{2}}\Big(n+3-\frac{D}{2}\Big)$$

$$\times\Big[\psi\Big(2-\frac{D}{2}\Big)-\psi\Big(\frac{D}{2}-1\Big)+\psi\Big(n+\frac{D}{2}+1\Big)-\psi\Big(n+4-\frac{D}{2}\Big)\Big]\Bigg]\Bigg\}.$$

Graviton Propagator

- To get the graviton propagator add gauge fixing term(de Sitter non inv.):

$$\mathcal{L}_{\rm GF} = -\tfrac{1}{2} a^{D-2} \eta^{\mu\nu} F_\mu F_\nu, \qquad F_\mu \equiv \eta^{\rho\sigma}(h_{\mu\rho,\sigma} - \tfrac{1}{2} h_{\rho\sigma,\mu} + (D-2) H a h_{\mu\rho} \delta^0_\sigma).$$

- Graviton propagator has the following form:

$$i[{}_{\mu\nu}\Delta_{\rho\sigma}](x;x') = \sum_{I=A,B,C} [{}_{\mu\nu}T^I_{\rho\sigma}] i\Delta_I(x;x').$$

where, $$[{}_{\mu\nu}T^A_{\rho\sigma}] = 2\bar{\eta}_{\mu(\rho}\bar{\eta}_{\sigma)\nu} - \frac{2}{D-3}\bar{\eta}_{\mu\nu}\bar{\eta}_{\rho\sigma},$$

$$[{}_{\mu\nu}T^B_{\rho\sigma}] = -4\delta^0_{(\mu}\bar{\eta}_{\nu)(\rho}\delta^0_{\sigma)},$$

$$[{}_{\mu\nu}T^C_{\rho\sigma}] = \frac{2}{(D-2)(D-3)}[(D-3)\delta^0_\mu\delta^0_\nu + \bar{\eta}_{\mu\nu}] \times [(D-3)\delta^0_\rho\delta^0_\sigma + \bar{\eta}_{\rho\sigma}].$$

$$\bar{\eta}_{\mu\nu} \equiv \eta_{\mu\nu} + \delta^0_\mu\delta^0_\nu \quad \text{and} \quad \bar{\delta}^\mu_\nu \equiv \delta^\mu_\nu - \delta^\mu_0\delta^0_\nu.$$

$$i\Delta_A(x;x') = A(y) + k\ln(aa') \qquad \text{where } k \equiv \frac{H^{D-2}}{(4\pi)^{D/2}}\frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})}.$$

$$A(y) \equiv \frac{H^{D-2}}{(4\pi)^{D/2}}\Bigg\{\frac{\Gamma(\frac{D}{2}-1)}{\frac{D}{2}-1}\Big(\frac{4}{y}\Big)^{(D/2)-1} + \frac{\Gamma(\frac{D}{2}+1)}{\frac{D}{2}-2}\Big(\frac{4}{y}\Big)^{(D/2)-2}$$

$$-\pi\cot\Big(\frac{\pi D}{2}\Big)\frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} + \sum_{n=1}^{\infty}\Bigg[\frac{1}{n}\frac{\Gamma(n+D-1)}{\Gamma(n+\frac{D}{2})}\Big(\frac{y}{4}\Big)^n$$

$$-\frac{1}{n-\frac{D}{2}+2}\frac{\Gamma(n+\frac{D}{2}+1)}{\Gamma(n+2)}\Big(\frac{y}{4}\Big)^{n-(D/2)+2}\Bigg]\Bigg\}$$

which is the MMCS propagator

$$i\Delta_B(x;x') = i\Delta_{\rm cf}(x;x') - \frac{H^{D-2}}{(4\pi)^{D/2}}\sum_{n=0}^{\infty}\Bigg\{\frac{\Gamma(n+D-2)}{\Gamma(n+\frac{D}{2})}\Big(\frac{y}{4}\Big)^n - \frac{\Gamma(n+\frac{D}{2})}{\Gamma(n+2)}\Big(\frac{y}{4}\Big)^{n-(D/2)+2}\Bigg\}$$

$$i\Delta_C(x;x') = i\Delta_{\rm cf}(x;x') + \frac{H^{D-2}}{(4\pi)^{D/2}}\sum_{n=0}^{\infty}\Bigg\{(n+1)\frac{\Gamma(n+D-3)}{\Gamma(n+\frac{D}{2})}\Big(\frac{y}{4}\Big)^n$$

$$-\Big(n-\frac{D}{2}+3\Big)\frac{\Gamma(n+\frac{D}{2}-1)}{\Gamma(n+2)}\Big(\frac{y}{4}\Big)^{n-(D/2)+2}\Bigg\}$$

same two propagators that appeared for photon propagator

$$i\left[{}_{\mu\nu}\Delta^{\rm dS}_{\rho\sigma}\right](x;z) = \sum_{k=1}^{5}\left[\mathcal{C}_0^k(y) + \mathcal{C}_2^k(y)\right] \times \left[{}_{\mu\nu}\mathcal{T}^k_{\rho\sigma}\right](x;z) \; .$$

$$D^\mu h_{\mu\nu} - \frac{1}{2} D_\nu h^\mu{}_\mu = 0$$

k	$\left[{}_{\mu\nu}\mathcal{T}^k_{\rho\sigma}\right](x;z)$
1	$\frac{\partial^2 y}{\partial x^\mu \partial z^{(\rho}} \frac{\partial^2 y}{\partial z^{\sigma)} \partial x^\nu}$
2	$\frac{\partial y}{\partial x^{(\mu}} \frac{\partial^2 y}{\partial x^{\nu)} \partial z^{(\rho}} \frac{\partial y}{\partial z^{\sigma)}}$
3	$\frac{\partial y}{\partial x^\mu} \frac{\partial y}{\partial x^\nu} \frac{\partial y}{\partial z^\rho} \frac{\partial y}{\partial z^\sigma}$
4	$H^2 \left[g_{\mu\nu}(x) \frac{\partial y}{\partial z^\rho} \frac{\partial y}{\partial z^\sigma} + \frac{\partial y}{\partial x^\mu} \frac{\partial y}{\partial x^\nu} g_{\rho\sigma}(z)\right]$
5	$H^4 g_{\mu\nu}(x) g_{\rho\sigma}(z)$

$$\begin{aligned}
\mathcal{C}_0^1(y) &= 2S_0''(y) \; , \\
\mathcal{C}_0^2(y) &= 4S_0'''(y) \; , \\
\mathcal{C}_0^3(y) &= S_0''''(y) \; , \\
\mathcal{C}_0^4(y) &= -2S_0''(y) + (2-y)S_0'''(y) \\
&\qquad - \frac{2}{D-1} \frac{MW''(y)}{H^2} \; , \\
\mathcal{C}_0^5(y) &= -(2-y)S_0'(y) + (2-y)^2 S_0''(y) \\
&\qquad - \frac{4}{D-1} \frac{(2-y)MW'(y)}{H^2} - \frac{2}{(D-2)(D-1)} \frac{W(y)}{H^4} \; .
\end{aligned}$$

$$\begin{aligned}
K\mathcal{C}_2^1(y) &= -\frac{1}{4}(4y-y^2)^2 G'''' - \frac{1}{2}(D+1)(2-y)(4y-y^2)G''' \\
&\qquad + \frac{1}{4} D(D+1)(4y-y^2)G'' - \frac{(D-2)D(D+1)}{D-1} G'' \; , \\
K\mathcal{C}_2^2(y) &= \frac{1}{2}(2-y)(4y-y^2)G'''' - (D+1)(4y-y^2)G''' \\
&\qquad + \frac{2D(D+1)}{D-1} G''' - \frac{1}{2} D(D+1)(2-y)G'' \; , \\
K\mathcal{C}_2^3(y) &= -\Bigl(\frac{D-2}{D-1}\Bigr)G'''' + \frac{1}{4}(4y-y^2)G'''' + \frac{1}{2}(D+1)(2-y)G''' \\
&\qquad - \frac{1}{4} D(D+1)G'' \; , \\
K\mathcal{C}_2^4(y) &= -\frac{1}{D-1}(4y-y^2)G'''' - 2\Bigl(\frac{D+1}{D-1}\Bigr)(2-y)G''' + \frac{D(D+1)}{D-1} G'' \; , \\
K\mathcal{C}_2^5(y) &= \frac{1}{D-1}(4y-y^2)^2 G'''' + 2\Bigl(\frac{D+1}{D-1}\Bigr)(2-y)(4y-y^2)G''' \\
&\qquad - \frac{D(D+1)}{D-1}(4y-y^2)G'' + \frac{4(D-2)(D+1)}{D-1} G'' \; ,
\end{aligned}$$

where the constant prefactor is $K \equiv 4(D-2)^2/(D-3)^2$.

$$\begin{aligned}
G(y) &\equiv -\frac{1}{4} \frac{\square}{H^2}\Bigl[\frac{\square}{H^2} - (D-2)\Bigr] S_2(y) \; , \\
&= -\frac{1}{4}(4y-y^2)^2 S_2'''' - \Bigl(\frac{D+2}{2}\Bigr)(2-y)(4y-y^2)S_2''' \\
&\qquad + \frac{3D}{4}(4y-y^2)S_2'' - \frac{D(D+2)}{4}(2-y)^2 S_2'' \\
&\qquad + \frac{D(D-1)}{2}(2-y)S_2' \; .
\end{aligned}$$

S P Miao, N.Tsamis, RP Woodard 11

EO Kahya, SP Miao, RP Woodard 11

$$S_0(y) = \Big(\frac{D-2}{D-1}\Big)\frac{H^{D-6}}{(4\pi)^{\frac{D}{2}}}\Big\{-\frac{\Gamma(\frac{D}{2}-1)}{(\frac{D}{2}-3)(\frac{D}{2}-2)}\Big(\frac{4}{y}\Big)^{\frac{D}{2}-3} + \text{constant} - (S_0)^b_1\Big(\frac{y}{4}\Big) \quad + \sum_{n=2}^{\infty}\Big[(S_0)^a_n\Big(\frac{y}{4}\Big)^{n-\frac{D}{2}+2} - (S_0)^b_n\Big(\frac{y}{4}\Big)^n\Big]\Big\}$$

$$\begin{aligned}(S_0)^a_n &= -\frac{2(WM)^a_n}{D-2} + \frac{\Gamma(\frac{D}{2}-1)\Gamma(1-\frac{D}{2})}{2\Gamma(3-\frac{D}{2}+n)}\frac{\Gamma(\frac{3}{2}+b_M+n)\Gamma(\frac{3}{2}-b_M+n)}{b_M\Gamma(\frac{1}{2}+b_M)\Gamma(\frac{1}{2}-b_M)(n+1)!} \qquad b_B \equiv \Big(\frac{D-3}{2}\Big)\ ,\ \ b_A \equiv \Big(\frac{D-1}{2}\Big) \\ &\qquad \times\Big\{\psi\Big(\frac{3}{2}+b_M+n\Big) - \psi\Big(\frac{3}{2}-b_M+n\Big) - \psi\Big(\frac{1}{2}+b_M\Big) + \psi\Big(\frac{1}{2}-b_M\Big)\Big\},\end{aligned}$$

$$\begin{aligned}(S_0)^b_n &= -\frac{2(WM)^b_n}{D-2} + \frac{\Gamma(\frac{D}{2}-1)\Gamma(1-\frac{D}{2})}{2\Gamma(\frac{D}{2}+n)}\frac{\Gamma(b_A+b_M+n)\Gamma(b_A-b_M+n)}{b_M\Gamma(\frac{1}{2}+b_M)\Gamma(\frac{1}{2}-b_M)n!} \qquad b_W \equiv \Big(\frac{D+1}{2}\Big)\ ,\ \ b_M \equiv \sqrt{\frac{(D-1)(D+7)}{4}}. \\ &\qquad \times\Big\{\psi\big(b_A+b_M+n\big) - \psi\big(b_A-b_M+n\big) - \psi\Big(\frac{1}{2}+b_M\Big) + \psi\Big(\frac{1}{2}-b_M\Big)\Big\}.\end{aligned}$$

$$(MW)^a_n = \frac{\Gamma(n+\frac{D}{2}+2)}{(D-2)(n-\frac{D}{2}+2)(n-\frac{D}{2}+1)(n+1)!} + \frac{\Gamma(\frac{D}{2}-1)\Gamma(1-\frac{D}{2})}{2\Gamma(3-\frac{D}{2}+n)}\frac{\Gamma(\frac{3}{2}+b_M+n)\Gamma(\frac{3}{2}-b_M+n)}{\Gamma(\frac{1}{2}+b_M)\Gamma(\frac{1}{2}-b_M)(n+1)!},$$

$$(MW)^b_n = \frac{\Gamma(n+D)}{(D-2)\Gamma(n+\frac{D}{2})n(n-1)} + \frac{\Gamma(\frac{D}{2}-1)\Gamma(1-\frac{D}{2})}{2\Gamma(\frac{D}{2}+n)}\frac{\Gamma(b_A+b_M+n)\Gamma(b_A-b_M+n)}{\Gamma(\frac{1}{2}+b_M)\Gamma(\frac{1}{2}-b_M)n!},$$

$$MW(y) = \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}}\Big\{-\frac{\Gamma(\frac{D}{2}-1)}{\frac{D}{2}-2}\Big(\frac{4}{y}\Big)^{\frac{D}{2}-2} + \text{constant} \qquad + \sum_{n=1}^{\infty}\Big[(MW)^a_n\Big(\frac{y}{4}\Big)^{n-\frac{D}{2}+2} - (MW)^b_n\Big(\frac{y}{4}\Big)^n\Big]\Big\}. \quad (53)$$

$$W_1 = \frac{\Gamma(D+1)}{\Gamma(\frac{D}{2}+1)}\Big\{\frac{D+1}{2D}\Big\}, \qquad W_2 = \frac{\Gamma(D+1)}{\Gamma(\frac{D}{2}+1)}\Big\{\psi\Big(-\frac{D}{2}\Big) - \psi\Big(\frac{D+1}{2}\Big) - \psi(D+1) - \psi(1)\Big\}.$$

$$S_2(y) = 32\Big(\frac{D-2}{D-3}\Big)^2 \frac{H^{D-6}}{(4\pi)^{\frac{D}{2}}}\Bigg\{\frac{\Gamma(\frac{D}{2}-1)(\frac{4}{y})^{\frac{D}{2}-5}}{4!(\frac{D}{2}-5)(\frac{D}{2}-4)(\frac{D}{2}-3)(\frac{D}{2}-2)} + \text{constant} \;\; + \sum_{n=4}^{\infty}(S_2)^a_n \times \Big(\frac{y}{4}\Big)^{n-\frac{D}{2}+2} - \sum_{n=1}^{\infty}(S_2)^b_n \times \Big(\frac{y}{4}\Big)^n\Bigg\}.$$

$$(S_2)^a_n = \frac{-\Gamma(\frac{D}{2}+1+n)}{(D-2)^2(2-\frac{D}{2}+n)(n+1)!}\Bigg\{\frac{\Psi^{a\prime}_n(b_A)+[\Psi^a_n(b_A)]^2}{2(D-1)^2} - \frac{\Psi^a_n(b_A)}{(D-1)^3}$$
$$-\frac{2\Psi^a_n(b_A)}{(D-2)(D-1)} + \frac{6}{(D-2)^2} - \frac{[3(D-3)+(\frac{D}{2}-2-n)\Psi^a_n(b_B)]}{(\frac{D}{2}+n)(D-3)(D-2)}\Bigg\},$$
$$(S_2)^b_n = -\frac{\Gamma(D-1+n)}{(D-2)^2 n\Gamma(\frac{D}{2}+n)}\Bigg\{\frac{\Psi^{b\prime}_n(b_A)+[\Psi^b_n(b_A)]^2}{2(D-1)^2} - \frac{\Psi^b_n(b_A)}{(D-1)^3}$$
$$-\frac{2\Psi^b_n(b_A)}{(D-2)(D-1)} + \frac{6}{(D-2)^2} - \frac{[3(D-3)-n\Psi^b_n(b_B)]}{(D-2+n)(D-3)(D-2)}\Bigg\}.$$

$$\Psi^a_n(\nu) \equiv \psi\Big(\frac{3}{2}+\nu+n\Big) - \psi\Big(\frac{1}{2}+\nu\Big) - \psi\Big(\frac{3}{2}-\nu+n\Big) + \psi\Big(\frac{1}{2}-\nu\Big),$$
$$\Psi^b_n(\nu) \equiv \psi\Big(b_A+\nu+n\Big) - \psi\Big(\frac{1}{2}+\nu\Big) - \psi\Big(b_A-\nu+n\Big) + \psi\Big(\frac{1}{2}-\nu\Big).$$

$$\Psi^a_n(\nu) = -\sum_{m=0}^{n}\frac{2\nu}{(m+\nu+\frac{1}{2})(m-\nu+\frac{1}{2})}.$$

$$(S_2)^a_n = \frac{-\Gamma(\frac{D}{2}+1+n)}{(D-2)^2(2-\frac{D}{2}+n)(n+1)!}\Bigg\{\sum_{m=0}^{n-1}\frac{1}{(\frac{D}{2}+m)(\frac{D}{2}-1-m)}$$
$$\times\sum_{\ell=m+1}^{n}\frac{1}{(\frac{D}{2}+\ell)(\frac{D}{2}-1-\ell)} - \frac{2}{D-2}\sum_{m=0}^{n}\frac{1}{(\frac{D}{2}+m)(\frac{D}{2}-1-m)} + \frac{6}{(D-2)^2}$$
$$-\frac{3}{(D-2)(\frac{D}{2}+n)} - \frac{(\frac{D}{2}-2-n)}{(D-2)(\frac{D}{2}+n)}\sum_{m=0}^{n}\frac{1}{(\frac{D}{2}+m-1)(\frac{D}{2}-2-m)}\Bigg\}.$$

$$i\left[{}_{\mu\nu}\Delta^{\rm br}_{\rho\sigma}\right](x;z) = \sum_{k=1}^{14}\left[\delta\mathcal{C}_0^k(u,v,y) + \delta\mathcal{C}_2^k(u,v,y)\right] \times \left[{}_{\mu\nu}\mathcal{T}^k_{\rho\sigma}\right](x;z)\,.$$

k	$\left[{}_{\mu\nu}\mathcal{T}^k_{\rho\sigma}\right](x;z)$
6	$\frac{\partial y}{\partial x^{(\mu}}\frac{\partial^2 y}{\partial x^{\nu)}\partial z^{(\rho}}\frac{\partial u}{\partial z^{\sigma)}} + \frac{\partial u}{\partial x^{(\mu}}\frac{\partial^2 y}{\partial x^{\nu)}\partial z^{(\rho}}\frac{\partial y}{\partial z^{\sigma)}}$
7	$\frac{\partial u}{\partial x^{(\mu}}\frac{\partial^2 y}{\partial x^{\nu)}\partial z^{(\rho}}\frac{\partial u}{\partial z^{\sigma)}}$
8	$\frac{\partial y}{\partial x^{\mu}}\frac{\partial y}{\partial x^{\nu}}\frac{\partial u}{\partial z^{\rho}}\frac{\partial u}{\partial z^{\sigma}} + \frac{\partial u}{\partial x^{\mu}}\frac{\partial u}{\partial x^{\nu}}\frac{\partial y}{\partial z^{\rho}}\frac{\partial y}{\partial z^{\sigma}}$
9	$\frac{\partial y}{\partial x^{(\mu}}\frac{\partial u}{\partial x^{\nu)}}\frac{\partial y}{\partial z^{(\rho}}\frac{\partial u}{\partial z^{\sigma)}}$
10	$\frac{\partial y}{\partial x^{(\mu}}\frac{\partial u}{\partial x^{\nu)}}\frac{\partial u}{\partial z^{\rho}}\frac{\partial u}{\partial z^{\sigma}} + \frac{\partial u}{\partial x^{\mu}}\frac{\partial u}{\partial x^{\nu}}\frac{\partial y}{\partial z^{(\rho}}\frac{\partial u}{\partial z^{\sigma)}}$
11	$\frac{\partial u}{\partial x^{\mu}}\frac{\partial u}{\partial x^{\nu}}\frac{\partial u}{\partial z^{\rho}}\frac{\partial u}{\partial z^{\sigma}}$
12	$H^2[\frac{\partial y}{\partial x^{(\mu}}\frac{\partial u}{\partial x^{\nu)}}g_{\rho\sigma}(z) + g_{\mu\nu}(x)\frac{\partial y}{\partial z^{(\rho}}\frac{\partial u}{\partial z^{\sigma)}}]$
13	$H^2[\frac{\partial u}{\partial x^{\mu}}\frac{\partial u}{\partial x^{\nu}}g_{\rho\sigma}(z) + g_{\mu\nu}(x)\frac{\partial u}{\partial z^{\rho}}\frac{\partial u}{\partial z^{\sigma}}]$
14	$H^2[\frac{\partial u}{\partial x^{\mu}}\frac{\partial u}{\partial x^{\nu}}g_{\rho\sigma}(z) - g_{\mu\nu}(x)\frac{\partial u}{\partial z^{\rho}}\frac{\partial u}{\partial z^{\sigma}}]$

$$u \equiv \ln(a_x a_z) \qquad , \qquad v \equiv \ln\Big(\frac{a_x}{a_z}\Big)\,.$$

$k=5$, $k=11$ and $k=13$ the coefficients take the form,

$$\delta\mathcal{C}_0^k(u,v,y) = \frac{k_M}{H^4}\Big\{\Big[(u+C_M)A_1^k + B_1^k\Big] \times e^{(b_M-b_A)u} + \Big[(u+C_M)A_2^k + B_2^k\Big]$$

$$\times e^{(b_M-b_W)u} \times (y-2) + \Big[(u+C_M)A_3^k + B_3^k\Big] \times e^{(b_M-b_W)u} \times \cosh(v)\Big\}. \qquad k_M \equiv \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}}\frac{\Gamma(b_M)\Gamma(2b_M)}{\Gamma(b_A)\Gamma(b_M+\frac{1}{2})}$$

For $k=7$, $k=10$ and $k=12$ only the f_2 terms contribute,

$$\delta\mathcal{C}_0^k(u,v,y) = \frac{k_M}{H^4}\Big[(u+C_M)A_2^k + B_2^k\Big] \times e^{(b_M-b_W)u}\,, \quad \delta\mathcal{C}_0^{14}(u,v,y) = \frac{k_M}{H^4}\Big[(u+C_M)A_3^k + B_3^k\Big] \times e^{(b_M-b_W)u} \times \sinh(v)\,.$$

Constants	Values
A_1^5	$\frac{b_A-b_M}{2b_A b_M}$
B_1^5	$\frac{1+6b_A+2b_A^2+(-4-2b_A)b_M}{2(D-2)b_A b_M}$
A_2^5	$\frac{4b_A+2b_A^2-2b_A b_M}{8b_A^2 b_M(b_M-b_W)}$
B_2^5	$\frac{-22b_A-74b_A^2-48b_A^3-8b_A^4+(2+26b_A+32b_A^2+8b_A^3)b_M}{8(D-2)b_A^2 b_M(b_M-b_W)^2}$
A_3^5	$\frac{6b_A+2b_A^2+(-2-2b_A)b_M}{4b_A^2 b_M(b_M-1)}$
B_3^5	$\frac{-2-48b_A-84b_A^2-36b_A^3-4b_A^4+(6+44b_A+28b_A^2+4b_A^3)b_M}{4(D-2)b_A^2 b_M(b_M-1)^2}$
A_2^7	$\frac{-1-b_A+b_M}{2b_A^2 b_M}$
B_2^7	$\frac{-1-6b_A-2b_A^2+(2+2b_A)b_M}{2(D-2)b_A^2 b_M}$
A_2^{10}	$\frac{2+7b_A+2b_A^2+(-3-2b_A)b_M}{4b_A^2 b_M}$
B_2^{10}	$\frac{3+20b_A+18b_A^2+4b_A^3+(-6-10b_A-4b_A^2)b_M}{4(D-2)b_A^2 b_M}$
A_1^{11}	$\frac{9b_A+16b_A^2+4b_A^3+(-1-8b_A-4b_A^2)b_M}{2b_A b_M}$
B_1^{11}	$\frac{1+22b_A+40b_A^2+36b_A^3+8b_A^4+(-4-16b_A-20b_A^2-8b_A^3)b_M}{2(D-2)b_A b_M}$
A_2^{11}	$\frac{-4-28b_A-22b_A^2-4b_A^3+(8+14b_A+4b_A^2)b_M}{8b_A^2 b_M}$
B_2^{11}	$\frac{-8-70b_A-90b_A^2-48b_A^3-8b_A^4+(18+42b_A+32b_A^2+8b_A^3)b_M}{8(D-2)b_A^2 b_M}$
A_3^{11}	$\frac{50b_A+110b_A^2+56b_A^3+8b_A^4+(-6-46b_A-40b_A^2-8b_A^3)b_M}{4b_A^2 b_M(b_M-1)}$
B_3^{11}	$\frac{-6-168b_A-720b_A^2-1060b_A^3-668b_A^4-176b_A^5-16b_A^6}{4(D-2)b_A^2 b_M(b_M-1)^2} + \frac{(22+204b_A+460b_A^2+412b_A^3+144b_A^4+16b_A^5)b_M}{4(D-2)b_A^2 b_M(b_M-1)^2}$
A_2^{12}	$\frac{b_A-b_M}{4b_A^2 b_M}$
B_2^{12}	$\frac{1+6b_A+2b_A^2+(-2-2b_A)b_M}{4(D-2)b_A^2 b_M}$

Constants	Values
A_1^{13}	$\frac{5b_A+2b_A^2+(-1-2b_A)b_M}{2b_A b_M}$
B_1^{13}	$\frac{1+16b_A+16b_A^2+4b_A^3+(-4-8b_A-4b_A^2)b_M}{2(D-2)b_A b_M}$
A_2^{13}	$\frac{-6b_A-2b_A^2+(2+2b_A)b_M}{8b_A^2 b_M}$
B_2^{13}	$\frac{-1-11b_A-9b_A^2-2b_A^3+(3+5b_A+2b_A^2)b_M}{4(D-2)b_A^2 b_M}$
A_3^{13}	$\frac{20b_A+20b_A^2+4b_A^3+(-4-12b_A-4b_A^2)b_M}{4b_A^2 b_M(b_M-1)}$
B_3^{13}	$\frac{-4-104b_A-302b_A^2-256b_A^3-80b_A^4-8b_A^5+(14+108b_A+144b_A^2+64b_A^3+8b_A^4)b_M}{4(D-2)b_A^2 b_M(b_M-1)^2}$
A_3^{14}	$\frac{-6b_A-2b_A^2+(2+2b_A)b_M}{4b_A^2 b_M(b_M-1)}$
B_3^{14}	$\frac{2+56b_A+86b_A^2+36b_A^3+4b_A^4+(-8-44b_A-28b_A^2-4b_A^3)b_M}{4(D-2)b_A^2 b_M(b_M-1)^2}$

$$i\Big[{}_{\mu\nu}\Delta^{\rm br,2}_{\rho\sigma}\Big](x;z) = k\Big[\ln(4a_x a_z)+2\psi\Big(\frac{D-1}{2}\Big)-4+\frac{1}{D-1}\Big] \times\Big\{\mathcal{R}^{\perp}_{\mu\rho}\mathcal{R}^{\perp}_{\nu\sigma}+\mathcal{R}^{\perp}_{\mu\sigma}\mathcal{R}^{\perp}_{\nu\sigma}-\frac{2}{D-1}\,g^{\perp}_{\mu\nu}g^{\perp}_{\rho\sigma}\Big\}$$

$$k\equiv\frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}}\frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \qquad g^{\perp}_{\mu\nu}(x)\equiv g_{\mu\nu}(x)+\frac{1}{H^2}\frac{\partial u}{\partial x^\mu}\frac{\partial u}{\partial x^\nu}=a_x^2\Big[\eta_{\mu\nu}+\delta^0_\mu\delta^0_\nu\Big]\,.$$

$$\begin{aligned}\mathcal{R}^{\perp}_{\mu\rho}(x;z) \;&\equiv\; -\frac{1}{2H^2}\Big\{\frac{\partial^2 y}{\partial x^\mu\partial z^\rho}-\frac{\partial y}{\partial x^\mu}\frac{\partial u}{\partial z^\rho}-\frac{\partial u}{\partial x^\mu}\frac{\partial y}{\partial z^\rho}-(2-y)\frac{\partial u}{\partial x^\mu}\frac{\partial u}{\partial z^\rho}\Big\}\\ &=\; a_x a_z\Big[\eta_{\mu\rho}+\delta^0_\mu\delta^0_\rho\Big]\,.\end{aligned}$$

- **pushing for de Sitter invariant gauge makes things very complicated**

Self-Mass SQED

•*for Massless Minimally Coupled Scalar*

• in de Sitter non-invariant gauge:

$$M^2_{ren}(x;x') = \frac{e^2}{8\pi^2} a a' \ln(aa') \partial^2 \delta^4(x-x') - \frac{e^2 H^2}{4\pi^2} a^4 \ln(a) \delta^4(x-x') - \frac{i e^2}{2^8 \pi^4} a a' \partial^6 \left\{ \ln^2\left(\mu^2 \Delta x^2\right) - 2\ln\left(\mu^2 \Delta x^2\right) \right\}$$

$$-\frac{i e^2 H^4}{2^6 \pi^4} (a a')^3 \left\{ \partial_0^2 \left[\ln^2\left(2^{-2} H^2 \Delta x^2\right) - \ln\left(2^{-2} H^2 \Delta x^2\right) \right] + \frac{\partial^2}{2} \left[3\ln^2\left(2^{-2} H^2 \Delta x^2\right) - 5\ln\left(2^{-2} H^2 \Delta x^2\right) \right] \right\}.$$

• in de Sitter invariant gauge of Allen and Jacobson:

$$M^2_{AJ\,ren}(x;x') = \frac{e^2}{8\pi^2} a a' \ln(aa') \partial^2 \delta^4(x-x') - \frac{e^2 H^2}{4\pi^2} a^4 \ln(a) \delta^4(x-x') - \frac{i e^2}{2^8 \pi^4} a a' \partial^6 \left\{ \ln^2\left(\mu^2 \Delta x^2\right) - 2\ln\left(\mu^2 \Delta x^2\right) \right\}$$

$$+\frac{i e^2 H^6}{16 \pi^4} (a a')^4 \left\{ \frac{1}{2y} - \frac{8}{y(4-y)} \ln\left(\frac{y}{4}\right) + a a' H^2 \Delta\eta^2 \left[\frac{2}{y^2} + \frac{1}{y} \right] - \frac{3}{4} \ln\left(\frac{\sqrt{e}}{4} H^2 \Delta x^2 \right) \right\}.$$

$$-iM^2_{\substack{++\\ \text{ren}}}(x;x') = ie^2 \delta Z_{\text{fin}} \sqrt{-g}\,\Box \delta^4(x-x') - i12e^2 H^2 \delta\xi_{\text{fin}} \sqrt{-g}\,\delta^4(x-x')$$
$$-\frac{3e^2 H^2}{(4\pi)^4} \sqrt{-g}\sqrt{-g'}\,\Box^2 \left\{ \frac{4}{y} \ln\left(\frac{y}{4}\right) \right\} + \frac{3e^2 H^4}{(4\pi)^4} \sqrt{-g}\sqrt{-g'}\,\Box \left\{ 7\left(\frac{4}{y}\right) \right.$$
$$\left. +8\sum_{n=1}^{\infty} \frac{1}{n^2} \left(\frac{y}{4}\right)^n + \left[4\left(\frac{4}{y}\right) - 4\ln\left(\frac{y}{4}\right) + 8\ln\left(1-\frac{y}{4}\right) - \frac{6}{1-\frac{y}{4}} \right] \ln\left(\frac{y}{4}\right) \right\} .$$

MMCS + GR Self-mass

$$-iM^2_{\rm ren}(x;x') = i\kappa^2 a^4\left(\Delta\alpha_1\Box^2 + \Delta\alpha_2\Box + \Delta\alpha_3\frac{\nabla^2}{a^2}\right)\delta^4(x-x') + \text{Table 10.}$$

TABLE X. All finite nonlocal contributions with $x = \frac{y}{4}$, where $y(x;x')$ is defined in Eq. (11).

External operator	Coefficient of $\frac{\kappa^2 H^4}{(4\pi)^4}$
$(aa')^4\Box^3/H^2$	$-\frac{\ln x}{3x}$
$(aa')^4\Box^2$	$\frac{26\ln x}{3x}+\frac{38}{3x}-6\ln^2 x-18\ln x$
$(aa')^4 H^2\Box$	$-\frac{6\ln x}{x}+\frac{4}{x}-41\ln x$
$(aa')^4 H^4$	$\frac{4\ln x}{x}+\frac{28}{x}-120-108\ln x$
$(aa')^3(a^2+a'^2)\Box^3/H^2$	$\frac{\ln x}{6x}$
$(aa')^3(a^2+a'^2)\Box^2$	$-\frac{\ln x}{3x}+\frac{1}{6x}$
$(aa')^3(a^2+a'^2)H^2\Box$	$-\frac{2\ln x}{3x}+\frac{5}{x}-18\ln x$
$(aa')^3(a^2+a'^2)H^4$	$\frac{4\ln x}{3x}-\frac{32}{3x}-54$
$(aa')^3 H^2\nabla^2$	$-\frac{2\ln x}{3x}-\frac{16}{x}+84\ln x-48x\ln x+96x$
$(aa')^3\nabla^2\Box$	$\frac{\ln x}{3x}$
$(aa')^2(a^2+a'^2)H^2\nabla^2$	$-\frac{7\ln x}{3x}+\frac{11}{2x}-12\ln x+48x\ln x+12x$
$(aa')^2(a^2+a'^2)\nabla^2\Box$	$-\frac{17\ln x}{6x}-\frac{49}{6x}+4\ln^2 x+10\ln x+12x\ln x$
$(aa')^2(a^2+a'^2)\nabla^2\Box$	$-\frac{17\ln x}{6x}-\frac{49}{6x}+4\ln^2 x+10\ln x+12x\ln x$
$(aa')^2\nabla^4$	$\frac{10}{3}\ln x-24x\ln x+24x^2\ln x-36x^2$

where, $\alpha_i = -\beta_i + \Delta\alpha_i$.

$$\beta_1 = 0,$$

$$\beta_2 = \frac{H^{D-4}}{(4\pi)^{D/2}}\left\{\frac{(-D^3+D-4)\Gamma(\frac{D}{2}-1)}{4(D-1)(D-3)} - \frac{(D+1)(D-4)\Gamma(D)\pi\cot(\frac{\pi}{2}D)}{4(D-3)\Gamma(\frac{D}{2})}\right\}$$

$$= \frac{H^{D-4}}{(4\pi)^{D/2}}\left\{-\frac{61}{3}+O(D-4)\right\},$$

$$\beta_3 = \frac{H^{D-4}}{(4\pi)^{D/2}}\left\{\frac{(D^3-16D^2+28D-16)\Gamma(\frac{D}{2})}{2(D-1)(D-2)(D-3)(D-4)} + \frac{(D^2-4D+1)\Gamma(D-1)\pi\cot(\frac{\pi}{2}D)}{(D-3)\Gamma(\frac{D}{2})} - 3\right\}$$

$$= \frac{H^{D-4}}{(4\pi)^{D/2}}\left\{-\frac{4}{D-4}+\frac{58}{3}+2\gamma+O(D-4)\right\}.$$

• Schwinger-Kelydsh Formalism (In-In Formalism)

In - Out

- free vacuum ➔ ends up the same way
- valid for scattering in flat space
- not valid for cosmological settings [future will contribute even more, not casual]
- gives complex results for the matrix elements of Hermitian operators.

- Answers the question:

  - What must the field be in order to make the universe to evolve from in vacuum to out vacuum?

In - In

- start from an initial state (free at finite time, e.g. Bunch-Davies)

  ➔ unknown state in the asymptotic future
- casual
- gives real results
- Answers the question:

  - What happens when the universe is released from a prepared state at some finite time and allowed to evolve as it will?

$$\Phi_{io}(x;\vec{k}) = \langle \Omega_{out} | [\varphi(x), \alpha_{in}^{\dagger}(\vec{k})] | \Omega_{in} \rangle. \qquad \Phi(x;\vec{k}) = \langle \Omega | [\varphi(x), \alpha^{\dagger}(\vec{k})] | \Omega \rangle.$$

$$i\Delta_{++}(x;x'): y \to y_{++}(x;x') \equiv a(\eta)a(\eta')[\|\vec{x} - \vec{x}'\|^2 - (|\eta - \eta'| - i\delta)^2],$$

- it turns out to result in + / - polarity

$$i\Delta_{+-}(x;x'): y \to y_{+-}(x;x') \equiv a(\eta)a(\eta')[\|\vec{x} - \vec{x}'\|^2 - (\eta - \eta' + i\delta)^2],$$

$$-iM_{--}^2(x;x') = (-iM_{++}^2(x;x'))^*,$$
$$-iM_{-+}^2(x;x') = (-iM_{+-}^2(x;x'))^*.$$

$$i\Delta_{-+}(x;x'): y \to y_{-+}(x;x') \equiv a(\eta)a(\eta')[\|\vec{x} - \vec{x}'\|^2 - (\eta - \eta' - i\delta)^2],$$

$$i\Delta_{--}(x;x'): y \to y_{--}(x;x') \equiv a(\eta)a(\eta')[\|\vec{x} - \vec{x}'\|^2 - (|\eta - \eta'| + i\delta)^2].$$

$$\Gamma[\varphi_+, \varphi_-] = S[\varphi_+] - S[\varphi_-]$$
$$-\frac{1}{2}\int d^4x \int d^4x' \left\{ \begin{array}{l} \varphi_+(x)M_{++}^2(x;x')\varphi_+(x') + \varphi_+(x)M_{+-}^2(x;x')\varphi_-(x') \\ +\varphi_-(x)M_{-+}^2(x;x')\varphi_+(x') + \varphi_-(x)M_{--}^2(x;x')\varphi_-(x') \end{array} \right\} + O(\varphi_\pm^3),$$

- **Meaning of To Solve:**

- • ***Equation of motion:***

$$a^4\Box\Phi(x) - \int_{\eta_i}^{0} d\eta' \int d^3x' \{M^2_{++}(x;x') + M^2_{+-}(x;x')\}\Phi(x') = 0.$$

where, $M^2_{++}(x;x') + M^2_{+-}(x;x') = \sum_{\ell=1}^{\infty} \kappa^{2\ell} \mathcal{M}^2_{\ell}(x;x')$ and $\Phi(x;\vec{k}) = \sum_{\ell=0}^{\infty} \kappa^{2\ell} \Phi_{\ell}(\eta, k) \times e^{i\vec{k}\cdot\vec{x}}$

- • ***Zeroth order(classical) solution*:**

$$\Phi_0(\eta, k) = u(\eta, k) \equiv \frac{H}{\sqrt{2k^3}}\Big(1 - \frac{ik}{aH}\Big)\exp\Big[\frac{ik}{aH}\Big].$$

$$u(\eta, k) = \frac{H}{\sqrt{2k^3}}\Big\{1 + \frac{k^2}{2H^2a^2} + \frac{ik^3}{3H^3a^3} + O\Big(\frac{k^4}{H^4a^4}\Big)\Big\}.$$

- • ***First order solution:***

$$a^2\Big[\partial_0^2 + 2Ha\partial_0 + k^2\Big]\Phi_1(\eta, k) = -\int_{\eta_i}^{0} d\eta' \int d^3x' \, \mathcal{M}^2_1(x;x') u(\eta', k) e^{i\vec{k}\cdot(\vec{x}-\vec{x}')}$$

• • Late time limit

u(η,k) $\Rightarrow$ u(0,k)

$$\frac{1}{a^4}\int d^4x' a^4\Big(\Delta\alpha_1\Box^2 + \Delta\alpha_2\Box + \Delta\alpha_3\frac{\nabla^2}{a^2}\Big) \times \delta^4(x - x')u(\eta', \vec{k})e^{i\vec{k}\cdot(\vec{x}'-\vec{x})} = -\Delta\alpha_3\frac{k^2}{a^2}u(\eta, k).$$

=> we can take the u term out of the integrals

$$-\frac{1}{a^4}\int_{\eta_i}^{0} d\eta' \int d^3x' \mathcal{M}_1^2(x;x')u(\eta', k)e^{i\vec{k}\cdot(\vec{x}'-\vec{x})} \longrightarrow -\frac{u(0,k)}{a^4}\int_{\eta_i}^{0} d\eta' \int d^3x' \mathcal{M}_1^2(x;x');$$

=> we have the following type of integrals to calculate:

$$-u(0,k)\frac{iH^8a^{4-K}}{(4\pi)^4}\Big(\frac{\Box}{H^2}\Big)^N \int_{\eta_i}^{0} d\eta' a'^K \times \int d^3x' \Big\{f\Big(\frac{y_{++}}{4}\Big) - f\Big(\frac{y_{+-}}{4}\Big)\Big\},$$

$f(x)$	$-\frac{iH^4}{16\pi^2} \times \int d^4x' a'^3 \{ f(\frac{y_{++}}{4}) - f(\frac{y_{+-}}{4}) \}$
$\frac{1}{x}$	$-\frac{\ln(a)}{a} + \frac{1}{a} + O(\frac{1}{a^2})$
$\frac{\ln(x)}{x}$	$-\frac{\ln^2(a)}{2a} + \frac{2\ln(a)}{a} - \frac{3}{a} + \frac{\pi^2}{3a} + O(\frac{\ln(a)}{a^2})$
$\ln(x)$	$\frac{1}{6} - \frac{\ln(a)}{2a} + \frac{1}{4a} + O(\frac{1}{a^2})$
$\ln^2(x)$	$\frac{1}{3}\ln(a) - \frac{11}{9} - \frac{\ln^2(a)}{2a} + \frac{2\ln(a)}{a} - \frac{9}{4a} + \frac{\pi^2}{3a} + O(\frac{\ln(a)}{a^2})$

Table 5-1. Integrals with a'^3.

$f(x)$	$-\frac{iH^4}{16\pi^2} \times \int d^4x' a'^4 \{ f(\frac{y_{++}}{4}) - f(\frac{y_{+-}}{4}) \}$
$\frac{1}{x}$	$-\frac{1}{2} + O(\frac{1}{a})$
$\frac{\ln(x)}{x}$	$\frac{3}{4} + O(\frac{1}{a})$
$\ln(x)$	$\frac{1}{6}\ln(a) - \frac{11}{36} + O(\frac{1}{a})$
$\ln^2(x)$	$\frac{1}{6}\ln^2(a) - \frac{8}{9}\ln(a) + \frac{7}{4} - \frac{\pi^2}{9} + O(\frac{\ln(a)}{a})$

Table 5-2. Integrals with a'^4.

$f(x)$	$-\frac{iH^4}{16\pi^2} \times \int d^4x' a'^5 \{ f(\frac{y_{++}}{4}) - f(\frac{y_{+-}}{4}) \}$
$\frac{1}{x}$	$-\frac{1}{8}a + O(\frac{1}{a})$
$\frac{\ln(x)}{x}$	$\frac{17}{36}a + O(\frac{1}{a})$
$\ln(x)$	$\frac{1}{24}a - \frac{1}{6} + O(\frac{1}{a})$
$\ln^2(x)$	$-\frac{13}{144}a - \frac{1}{3}\ln(a) + \frac{5}{9} + O(\frac{\ln(a)}{a})$

Table 5-3. Integrals with a'^5.

External Operator $\times f(x)$	Coefficient of $u(0,k) \times \frac{H^4}{16\pi^2}$
$(aa')^4\Box^3/H^2 \times -\frac{\ln x}{3x}$	0
$(aa')^4\Box^2 \times \frac{26\ln x}{3x}$	0
$(aa')^4\Box^2 \times \frac{38}{3x}$	0
$(aa')^4\Box^2 \times -6\ln^2 x$	-18
$(aa')^4\Box^2 \times -18\ln x$	0
$(aa')^4 H^2\Box \times -\frac{6\ln x}{x}$	0
$(aa')^4 H^2\Box \times \frac{4}{x}$	0
$(aa')^4 H^2\Box \times -4\ln x$	2
$(aa')^4 H^4 \times \frac{4\ln x}{x}$	3
$(aa')^4 H^4 \times \frac{18}{x}$	-9
$(aa')^4 H^4 \times -108\ln x$	$33\ -18\ \ln(a)$
$(aa')^3(a^2+a'^2)\Box^3/H^2 \times \frac{\ln x}{6x}$	$(\frac{-325+12\pi^2}{27}) + \frac{14}{3}\ln(a) - \frac{2}{3}\ln^2(a)$
$(aa')^3(a^2+a'^2)\Box^2 \times -\frac{\ln x}{3x}$	$(\frac{85-12\pi^2}{27}) - 4\ln(a) + \frac{2}{3}\ln^2(a)$
$(aa')^3(a^2+a'^2)\Box^2 \times \frac{1}{6x}$	$\frac{8}{9}\ -\frac{2}{3}\ln(a)$
$(aa')^3(a^2+a'^2)H^2\Box \times -\frac{2\ln x}{3x}$	$(\frac{160-12\pi^2}{27}) - \frac{10}{3}\ln(a) + \frac{2}{3}\ln^2(a)$
$(aa')^3(a^2+a'^2)H^2\Box \times \frac{5}{x}$	$\frac{55}{3}\ -10\ln(a)$
$(aa')^3(a^2+a'^2)H^2\Box \times -18\ln x$	$-15\ +18\ln(a)$
$(aa')^3(a^2+a'^2)H^4 \times \frac{4\ln x}{3x}$	$(\frac{-91+12\pi^2}{27}) + \frac{8}{3}\ln(a) - \frac{2}{3}\ln^2(a)$
$(aa')^3(a^2+a'^2)H^4 \times -\frac{32}{3x}$	$-\frac{80}{9} + \frac{32}{3}\ln(a)$
Total	0

Table 5-4. The $-\frac{u(0,k)}{a^4}\frac{iH^8}{(4\pi)^4}\int d^4x'$ (Ext. Operator) $\times \{f(\frac{y_{++}}{4}) - f(\frac{y_{+-}}{4})\}$ terms.

EFFECTIVE FIELD EQUATIONS

- ***Results for SQED:***

  - It is possible to renormalize the theory in such a way that no large late time modification to the scalar wave functions occur at order $\mathbf{e^2}$.

  - This confirms the stochastic prediction that there should be no large, late time corrections until order $\mathbf{e^6}$. ()

  - Using de-Sitter non-inv and de Sitter inv gauge for photon prop. didn't change the result. No need to sweat more than you need to...

- ***Results for MMCS+GR:***

  - The leading order terms add up to zero

  - This confirms the stochastic expectations, and the fact that they only interact through derivatives which redshift like momentum.

•***Zeta - Zeta Correlator*** *(EO Kahya, VK Onemli, RP Woodard, 2010)*

$$\mathcal{L} = \left[\frac{\mathbf{R}}{16\pi G} - \frac{1}{2}\varphi_{,\mu}\varphi_{,\nu}\mathbf{g}^{\mu\nu} - V(\varphi) \; - \frac{1}{2}\sigma_{,\mu}\sigma_{,\nu}\mathbf{g}^{\mu\nu} - U(\sigma)\right]\sqrt{-\mathbf{g}},$$

φ is inflaton, σ is a spectator field

ADM decompose 3 + 1

$$\mathbf{g}_{\mu\nu}\, dx^{\mu}\, dx^{\nu} = -N^2\, dt^2 + g_{ij}\big(dx^i - N^i\, dt\big)\big(dx^j - N^j\, dt\big).$$

ADM showed that L becomes $\mathcal{L} = (\text{Surface Terms}) - \frac{\sqrt{g}}{16\pi G}\left[N \cdot A + \frac{B}{N}\right].$

one can vary this wrt N and get a simpler form => $\mathcal{L}_{\text{const}} = (\text{Surface Terms}) - \frac{\sqrt{g}}{8\pi G}\sqrt{AB}.$

$$A = -R + 16\pi G\left[V(\varphi) + U(\sigma) + \frac{1}{2}g^{ij}(\varphi_{,i}\varphi_{,j} + \sigma_{,i}\sigma_{,j})\right],$$

$$B = \big(E^i_i\big)^2 - E^{ij}E_{ij} - 8\pi G\big[\big(\dot{\varphi} - \varphi_{,i}N^i\big)^2 + \big(\dot{\sigma} - \sigma_{,i}N^i\big)^2\big],$$

$E_{ij} \equiv \frac{1}{2}[N_{i;j} + N_{j;i} - \dot{g}_{ij}]$, R is the $(D-1)$-dimensional Ricci scalar formed from g_{ij}.

nonzero background fields are $g_{ij} = a^2(t)\delta_{ij}$ and $\varphi = \varphi_0(t)$.

two nontrivial Einstein equations can be used to eliminate the background scalar,

$$\dot{\varphi}_0^2 = -\frac{(D-2)}{8\pi G}\dot{H}, \qquad V(\varphi_0) = \frac{(D-2)}{16\pi G}\left[\dot{H} + (D-1)H^2\right].$$

use perturbation theory where A and B have the following form

$$A = A_0 - R + 16\pi G\left[U(\sigma) + \frac{e^{-2\zeta}}{2a^2}\tilde{g}^{ij}\sigma_{,i}\sigma_{,j}\right]$$

$$\equiv A_0(1+\alpha).$$

$$B = A_0 + 2(D-2)H\left[(D-1)\left(\dot{\zeta} - \zeta_{,k}\tilde{N}^k\right) - \tilde{N}^k_{,k}\right]$$

$$+ (D-2)\left(\dot{\zeta} - \zeta_{,k}\tilde{N}^k\right)\left[(D-1)\left(\dot{\zeta} - \zeta_{,k}\tilde{N}^k\right) - 2\tilde{N}^k_{,k}\right]$$

$$+ \left(\tilde{N}^k_{,k}\right)^2 - \tilde{E}^{k\ell}\tilde{E}_{k\ell} - 8\pi G\left(\dot{\sigma} - \sigma_{,k}\tilde{N}^k\right)^2$$

$$\equiv A_0(1+\beta).$$

where $A_0 = B_0 = (D-2)[\dot{H} + (D-1)H^2]$.

gauge conditions: $$G_0(t,\vec{x}) \equiv \varphi(t,\vec{x}) - \varphi_0(t) = 0.$$

unimodular part of the metric: $$g_{ij} = a^2(t)e^{2\zeta(t,\vec{x})}\tilde{g}_{ij}(t,\vec{x}) \quad \Rightarrow \quad \sqrt{g} = a^{D-1}e^{(D-1)\zeta}.$$

$\tilde{g}_{ij} \equiv \delta_{ij} + h_{ij}$ to be transverse, $$G_i(t,\vec{x}) \equiv \partial_j \tilde{g}_{ij}(t,\vec{x}) = \partial_j h_{ij}(t,\vec{x}) = 0.$$

$$\begin{aligned}
-\frac{\sqrt{g}}{8\pi G}\sqrt{AB} &= -\frac{a^{D-1}e^{(D-1)\zeta}}{8\pi G}A_0\sqrt{(1+\alpha)(1+\beta)} \\
&= -\frac{a^{D-1}e^{(D-1)\zeta}}{8\pi G}A_0 \\
&\quad \times \left\{1 + \frac{(\alpha+\beta)}{2} - \frac{(\alpha-\beta)^2}{8} + \cdots\right\}.
\end{aligned}$$

Time dependence from self-interactions of ζ

The lowest order effect of zeta interaction by quadratic Lagrangians

$$\mathcal{L}^{(2)}_{\zeta} = \frac{(D-2)\epsilon a^{D-1}}{16\pi G}\left\{\dot{\zeta}^2 - \frac{1}{a^2}\partial_k\zeta\,\partial_k\zeta\right\}, \quad \mathcal{L}^{(2)}_{\sigma} = \frac{a^{D-1}}{2}\left\{\dot{\sigma}^2 - \frac{1}{a^2}\partial_k\sigma\,\partial_k\sigma\right\}. \quad \epsilon \equiv -\frac{\dot{H}}{H^2}.$$

$$\mathcal{L}^{(2)}_{h} = \frac{a^{D-1}}{64\pi G}\left\{\dot{h}_{ij}\dot{h}_{ij} - \frac{1}{a^2}\partial_k h_{ij}\partial_k h_{ij}\right\},$$

mmcs

solving for $\ddot{u} + (D-1)H\dot{u} + \frac{k^2}{a^2}u = 0 \qquad u\dot{u}^* - \dot{u}u^* = \frac{i}{a^{D-1}}.$

$$u(t,k) = \frac{\sqrt{\frac{\pi}{4(1-\epsilon)H}}}{a^{\frac{D-1}{2}}} H^{(1)}_{\nu}\left(\frac{k}{(1-\epsilon)Ha}\right), \qquad \nu \equiv \frac{D-1-\epsilon}{2(1-\epsilon)}. \quad \text{for constant } \epsilon$$

curvature power spectrum $\Delta^2_{\mathcal{R}}(k,t) \equiv \frac{k^3}{2\pi^2}\int d^3x\, e^{-i\vec{k}\cdot\vec{x}}\langle\Omega|\mathcal{R}(t,\vec{x})\mathcal{R}(t,\vec{0})|\Omega\rangle.$

measured value from CMB and large scale structure surveys E Komatsu et al, 1001.4538

$$\Delta^2_{\mathcal{R}}(k) = (2.441^{+0.088}_{-0.092}) \times 10^{-9}\left(\frac{k}{0.002\ \mathrm{Mpc}^{-1}}\right)^{-0.037\pm0.012}.$$

remember $\mathcal{R}(t,\vec{x}) \equiv -\frac{a^2(t)}{4\nabla^2}R = \left(\frac{D-2}{2}\right)\zeta(t,\vec{x}) + O(\zeta^2, \zeta h, h^2)$

The tensor power spectrum is,

$$\Delta^2_h(k,t) \equiv \frac{k^3}{2\pi^2}\int d^3x\, e^{-i\vec{k}\cdot\vec{x}}\langle\Omega|h_{ij}(t,\vec{x})h_{ij}(t,\vec{0})|\Omega\rangle.$$ not yet observed

$$\ddot{u}_\zeta + \Big[(D-1)H + \frac{\dot{\epsilon}}{\epsilon}\Big]\dot{u}_\zeta + \frac{k^2}{a^2}u_\zeta = 0 \qquad \text{with} \qquad u_\zeta \dot{u}_\zeta^* - \dot{u}_\zeta u_\zeta^* = \frac{i}{\epsilon a^{D-1}} \; .$$

$$\Delta^2_{\mathcal{R}}(k) \;\; = \;\; \frac{k^3}{2\pi^2} \lim_{t \gg t_k} \Big\{ 8\pi G \times |u_\zeta(t,k)|^2 + O(G^2) \Big\} \; ,$$

$$D - 4 = 0 = \dot{\epsilon} \quad \Rightarrow \quad \lim_{t\to\infty} u(t,k) = C(\epsilon) \times \frac{H(t_k)}{\sqrt{2k^3}}, \qquad C(\epsilon) \equiv \frac{\Gamma(\frac{2}{1-\epsilon})}{\Gamma(\frac{1}{1-\epsilon})}\Big[\frac{1-\epsilon}{2^\epsilon}\Big]^{\frac{1}{1-\epsilon}} \; .$$

well-known tree order results

$$\dot{\epsilon}(t) = 0 \implies \Delta^2_{\mathcal{R}} = C(\epsilon) \times \frac{GH^2(t_k)}{\pi\epsilon(t_k)} + O(G^2) \; ,$$

$$\dot{\epsilon}(t) = 0 \implies \Delta^2_{h} = C(\epsilon) \times \frac{16GH^2(t_k)}{\pi} + O(G^2) \; .$$

The latest data from the South Pole Telescope implies $r < 0.17$

Tensor to scalar ratio $\quad r \equiv \dfrac{\Delta^2_h(k_0)}{\Delta^2_{\mathcal{R}}(k_0)} < 0.17 \qquad \epsilon < 0.011$

R. Keisler et al., Astrophys. J. **743** (2011) 88, arXiv:1105.3182.

minimum generalization of quadratic

$$\mathcal{L}_\zeta = \frac{(D-2)\epsilon}{16\pi G} a^{D-1} e^{(D-1)\zeta} \left\{ \dot{\zeta}^2 - \frac{e^{-2\zeta}}{a^2} \partial_k \zeta \partial_k \zeta \right\}.$$

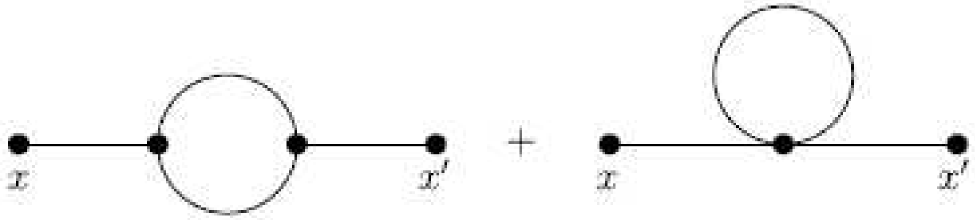
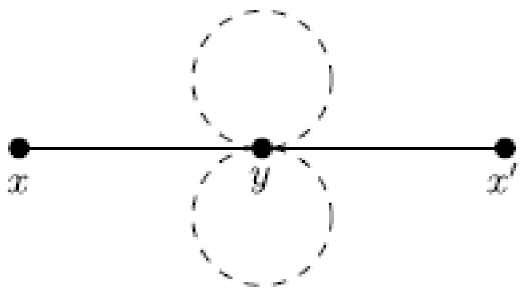


Fig. 1. One loop correction from cubic and quartic self-interactions of ζ.

$$[\Delta^2_{\mathcal{R}}(k,t)]_{\zeta \text{ loops}} \approx \frac{GH^2}{\pi\epsilon} \left\{ \frac{27GH^2}{4\pi\epsilon} \ln(a) + O(G^2H^4) \right\}.$$



$$\mathcal{L}_U = \frac{\epsilon}{D-1} a^{D-1} e^{(D-1)\zeta} U(\sigma). \qquad U(\sigma) = \lambda\sigma^4/4!.$$

$$[\Delta^2_{\mathcal{R}}(k,t)]_{\sigma \text{ loops}} \approx \frac{GH^2}{\pi\epsilon} \left\{ \frac{\lambda GH^2}{48\pi^3} \ln^3(a) + O(\lambda^2) \right\}.$$

One can see that it is possible to get time dependence, which was Weinberg's theorem Senatore and Zaldarriaga claim was no time dependence can arise from zeta-zeta and we gave two examples that one can get that for particular cases. Effect is small but not hopelessly small.

SUMMARY

• To get big effects we need two things: masslessness and conformal non-invariance.

• Showed that one can get a time dependent correction to power spectrum from self interactions of ζ and interaction between ζ and spectator σ field.

• Computed one loop quantum gravitational corrections to the scalar self-mass-squared on a locally de Sitter background. Used that to solve the one loop-corrected, linearized effective field equation for a MMCS+GR

• The sea of infrared gravitons produced by inflation has little effect on the scalar.

• ***de Sitter Love hurts***: Pushing for de Sitter invariance makes the propagator so difficult that makes it extremely difficult to work with and the physics doesn't change.

• ***there are non-zero cases***: Loop effects can be time dependent and grow with time logarithmically and possibly even observable if in future experiments. We gave two possible theories where one can see this.

CURRENT and FUTURE WORK

- Is it possible to say anything about the cosmological observables by doing loop calculations?
- Calculating 1-loop self mass of CCS+GR since this time non-derv interaction exist. (with Sohyun Park, Penn State)
- Use Graviton propagator for de Sitter Inv gauge(de Donder) =>
  Looks very complicated possibly some aspects are advantageous?
- See if the results depend on gauge, calculate the iM^2 for same theories and compare it with existing results. Also investigate the effect of correcting the vacuum state (Bunch-Davies)
- Non-perturbative treatment for MMCS, summation of leading logs(scalar only Starobinsky, Yokoyama, 94, Yukawa Woodard Miao 07, SQED Prokopec, Tsamis, Woodard 07)

Rising of Cosmology in HEP-TH

http://www.physics.utoronto.ca/~poppitz/Jobs94-08.pdf

DISTRIBUTION BETWEEN HEP-THEORY SUBFIELDS, AS PERCENTAGE OF TOTAL FACULTY HIRED IN A GIVEN YEAR: