

The Effective Theory of Inflation, Cosmological Observations Contrasted with Quantum Fields in (Quasi)-De Sitter Spacetime

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THE PHYSICS OF DE SITTER SPACETIME

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DM + Λ + Baryons + Radiation

The Standard Cosmological Model = Concordance Model

- Begins by the **inflationary** era. Slow-Roll inflation explains horizon and flatness.
- Gravity is described by Einstein's General Relativity.
- Particle Physics described by the Standard Model of Particle Physics: $SU(3) \otimes SU(2) \otimes U(1) =$ qcd+electroweak model.
- Dark matter is non-relativistic during the matter dominated era where structure formation happens. DM is outside the SM of particle physics. CDM \implies **WDM**.
- Dark energy described by the cosmological constant Λ .

Standard Model of the Universe: Concordance Model

$ds^2 = dt^2 - a^2(t) d\vec{x}^2$: spatially **flat** geometry.

The Universe starts by an **INFLATIONARY ERA**.

Inflation = Accelerated Expansion: $\frac{d^2 a}{dt^2} > 0$.

During inflation the universe expands by at least sixty or so e-folds: $e^{66} \simeq 10^{29}$. Inflation **lasts** $\simeq 10^{-36}$ sec and ends by $z \sim 10^{27}$ followed by a **radiation** dominated era.

Matter can be effectively described during inflation by a single Scalar Field $\phi(t, \mathbf{x})$: the **Inflaton**.

Lagrangian: $\mathcal{L} = a^3(t) \left[\frac{\dot{\phi}^2}{2} - \frac{(\nabla\phi)^2}{2 a^2(t)} - V(\phi) \right]$.

Friedmann eq.: $H^2(t) = \frac{1}{3 M_{Pl}^2} \left[\frac{\dot{\phi}^2}{2} + V(\phi) \right]$, $H(t) \equiv \dot{a}(t)/a(t)$.

Slow-roll \implies **quasi-de Sitter** space-time.

If $\dot{\phi}^2 \ll V(\phi) \simeq \bar{V} \implies a(t) = \bar{a} e^{\bar{H}t}$, $\bar{H} = \sqrt{\bar{V}} / [\sqrt{3} M_{Pl}]$

The Theory of Inflation

We describe inflation as an **effective** field in the Ginsburg-Landau sense.

D. Boyanovsky, C. Destri, H. J. de Vega, N. G. Sánchez, (**review article**),
arXiv:0901.0549, Int. J. Mod.Phys. A **24**, 3669-3864 (2009).
Relevant effective theories in physics:

- Ginsburg-Landau theory of superconductivity. It is an effective theory for Cooper pairs in the microscopic BCS theory of superconductivity.
- The $O(4)$ sigma model for pions, the sigma and photons at energies $\lesssim 1$ GeV. The microscopic theory is QCD: quarks and gluons. $\pi \simeq \bar{q}q$, $\sigma \simeq \bar{q}q$.
- The theory of second order phase transitions à la Landau-Kadanoff-Wilson... (ferromagnetic, antiferromagnetic, liquid-gas, Helium 3 and 4, ...)
-

Slow-roll evolution of the Inflaton

During slow-roll the inflaton derivatives are **small** and the evolution equations can be approximated by:

$$3 H(t) \dot{\phi} + V'(\phi) = 0 \quad , \quad H^2(t) = \frac{V(\phi)}{3M_{Pl}^2}$$

These first order equations can be solved in closed form as:

$$M_{Pl}^2 N[\phi] = - \int_{\phi}^{\phi_{end}} V(\varphi) \frac{d\varphi}{dV} d\varphi \quad , \quad e^{N[\phi]} = a(\phi_{end})/a(\phi) \quad ,$$

$N[\phi]$ = the number of e-folds since the field ϕ **exits** the horizon till the end of inflation. $N \sim 60$.

ϕ_{end} = absolute minimum of $V(\phi)$.

Therefore, $\phi^2 =$ **scales** as $N M_{Pl}^2$. We define:

$\chi \equiv \frac{\phi}{\sqrt{N} M_{Pl}}$ = is a **dimensionless** and **slow** field.

Universal form of the slow-roll inflaton potential:

$V(\phi) = N M^4 w(\chi)$, M = energy scale of inflation

SLOW and Dimensionless Variables

$$\chi = \frac{\phi}{\sqrt{N} M_{Pl}} \quad , \quad \tau = \frac{m t}{\sqrt{N}} \quad , \quad \mathcal{H}(\tau) = \frac{H(t)}{m \sqrt{N}} \quad ,$$

$$m \equiv \frac{M^2}{M_{Pl}} \quad , \quad |V''(0)| = m^2 = \text{inflaton mass}, \quad (|w''(0)| = 1)$$

slow inflaton, slow time, slow Hubble.

χ and $w(\chi)$ are of order **one**.

Evolution Equations:

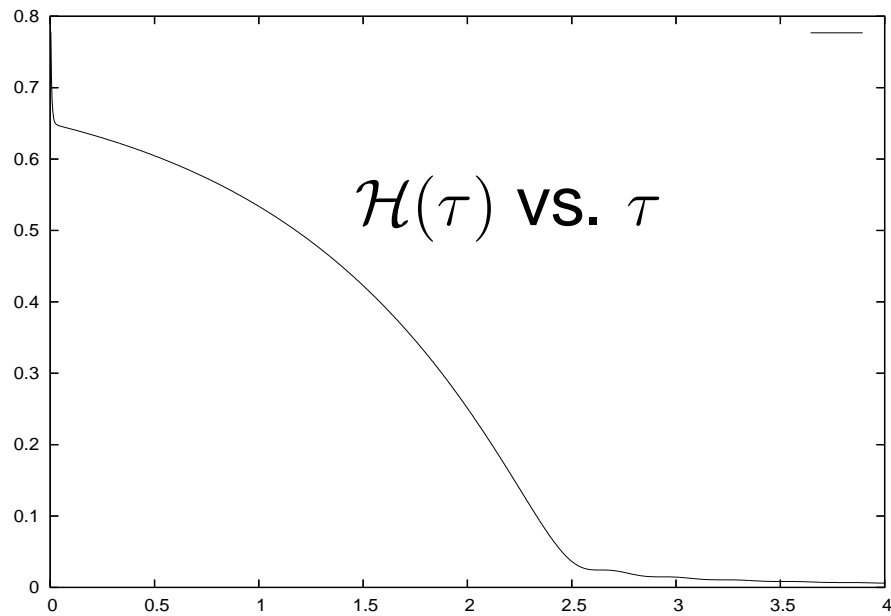
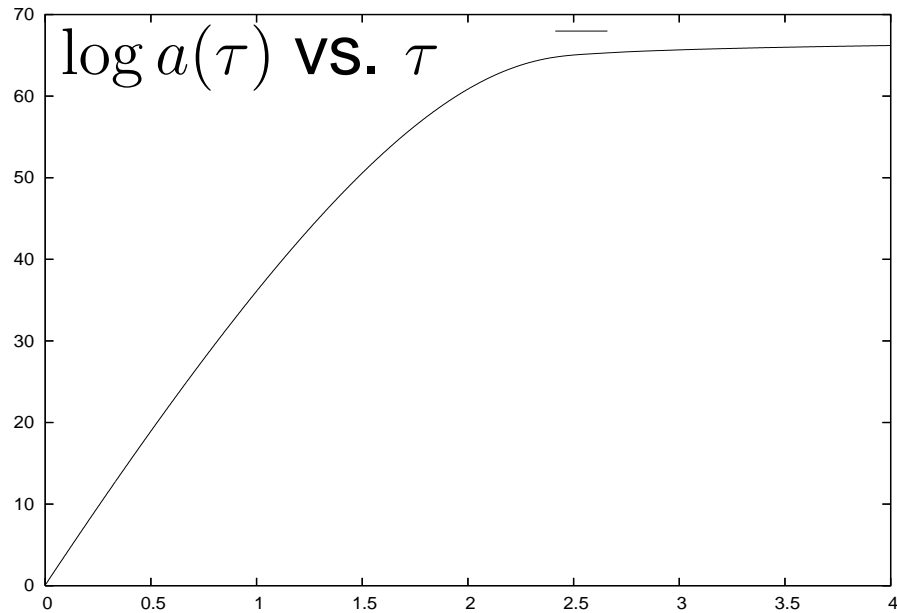
$$\mathcal{H}^2(\tau) = \frac{1}{3} \left[\frac{1}{2N} \left(\frac{d\chi}{d\tau} \right)^2 + w(\chi) \right] \quad ,$$

$$\frac{1}{N} \frac{d^2\chi}{d\tau^2} + 3 \mathcal{H} \frac{d\chi}{d\tau} + w'(\chi) = 0 \quad .$$

$1/N$ terms: **corrections** to slow-roll

Higher orders in slow-roll are obtained **systematically** by expanding the solutions in $1/N$.

Exact Inflaton Dynamics: $w(\chi) = \frac{y}{32} \left(\chi^2 - \frac{8}{y} \right)^2$



Slow-roll Inflaton Dynamics

Integrating the inflaton evolution eqs. to the order $1/N$:

$$\log \frac{a(t)}{a(0)} = \sqrt{\frac{2N}{3y}} m t - \frac{1}{8} \frac{\varphi^2(0)}{M_{Pl}^2} \left[e^{\sqrt{\frac{2y}{3N}} m t} - 1 \right]$$

$$H(t) = m \sqrt{\frac{2N}{3y}} \left[1 - \frac{y}{8N} \frac{\varphi^2(0)}{M_{Pl}^2} e^{\sqrt{\frac{2y}{3N}} m t} \right]$$

Hubble $H(t)$ **slowly** decreases during slow-roll.

De Sitter universe **plus** $1/N$ corrections:

During slow-roll the universe is **quasi-De Sitter**.

$$H(\text{end slow-roll}) \simeq H(\text{slow-roll beginning}) / \sqrt{N}$$
$$\sqrt{N} \simeq 8$$

$$m t_{\text{end slow-roll}} = \sqrt{\frac{3N}{2y}} \log \left[\frac{8N M_{Pl}^2}{y \varphi^2(0)} \right]$$

The Slow-roll regime is an **attractor**

(Belinsky-Grishchuk-Zeldovich-Khalatnikov, 1985)

Primordial Power Spectrum

Adiabatic Scalar Perturbations: $P(k) = |\Delta_{k ad}^{(S)}|^2 k^{n_s - 1}$.

To dominant order in slow-roll:

$$|\Delta_{k ad}^{(S)}|^2 = \frac{N^2}{12 \pi^2} \left(\frac{M}{M_{Pl}} \right)^4 \frac{w^3(\chi)}{w'^2(\chi)} .$$

Hence, for **all** slow-roll inflation models:

$$|\Delta_{k ad}^{(S)}| \sim \frac{N}{2 \pi \sqrt{3}} \left(\frac{M}{M_{Pl}} \right)^2$$

The WMAP result: $|\Delta_{k ad}^{(S)}| = (0.494 \pm 0.1) \times 10^{-4}$

determines the scale of inflation M (using $N \simeq 60$)

$$\left(\frac{M}{M_{Pl}} \right)^2 = 0.85 \times 10^{-5} \longrightarrow M = 0.70 \times 10^{16} \text{ GeV}$$

The inflation energy scale **turns to be** the grand unification energy scale !! We find the scale of inflation **without** knowing the tensor/scalar ratio r !!

The scale M is independent of the shape of $w(\chi)$.

spectral index n_s , the ratio r and the running of n_s

$r \equiv$ ratio of tensor to scalar fluctuations.
tensor fluctuations = primordial **gravitons**.

$$n_s - 1 = -\frac{3}{N} \left[\frac{w'(\chi)}{w(\chi)} \right]^2 + \frac{2}{N} \frac{w''(\chi)}{w(\chi)}, \quad r = \frac{8}{N} \left[\frac{w'(\chi)}{w(\chi)} \right]^2$$

$$\frac{dn_s}{d \ln k} = -\frac{2}{N^2} \frac{w'(\chi) w'''(\chi)}{w^2(\chi)} - \frac{6}{N^2} \frac{[w'(\chi)]^4}{w^4(\chi)} + \frac{8}{N^2} \frac{[w'(\chi)]^2 w''(\chi)}{w^3(\chi)},$$

χ is the inflaton field at horizon exit.

$n_s - 1$ and r are **always** of order $1/N \sim 0.02$ (model indep.)

Running of n_s of order $1/N^2 \sim 0.0003$ (model independent).

Primordial Non-gaussianity $f_{NL} =$ order $1/N$

D. Boyanovsky, H. J. de Vega, N. G. Sanchez,
Phys. Rev. D 73, 023008 (2006), astro-ph/0507595.

Ginsburg-Landau Approach

Ginsburg-Landau potentials:

polynomials in the field starting by a constant term.

Linear terms can always be eliminated by a **constant** shift of the inflaton field.

The quadratic term can have a positive or a negative sign:

$$\begin{cases} w''(0) > 0 \rightarrow \text{single well potential} \rightarrow \text{large field (chaotic) inflation} \\ w''(0) < 0 \rightarrow \text{double well potential} \rightarrow \text{small field (new) inflation} \end{cases}$$

The inflaton potential must be **bounded** from below \implies **highest** order term must be **even** with a **positive** coefficient.

Renormalizability \implies degree of the inflaton potential ≤ 4 .

The theory of inflation is an **effective** theory \implies higher degree potentials are **acceptable**

Stability under the addition of terms of higher order.

Otherwise, the description obtained could not be trusted.

Fourth order Ginsburg-Landau inflationary models

$$w(\chi) = w_o \pm \frac{\chi^2}{2} + G_3 \chi^3 + G_4 \chi^4 \quad , \quad G_3 = \mathcal{O}(1) = G_4$$

$$V(\phi) = N M^4 w \left(\frac{\phi}{\sqrt{N} M_{Pl}} \right) = V_o \pm \frac{m^2}{2} \phi^2 + g \phi^3 + \lambda \phi^4 .$$

$$m = \frac{M^2}{M_{Pl}} \quad , \quad g = \frac{m}{\sqrt{N}} \left(\frac{M}{M_{Pl}} \right)^2 G_3 \quad , \quad \lambda = \frac{G_4}{N} \left(\frac{M}{M_{Pl}} \right)^4$$

Notice that

$$\left(\frac{M}{M_{Pl}} \right)^2 \simeq 10^{-5} \quad , \quad \left(\frac{M}{M_{Pl}} \right)^4 \simeq 10^{-10} \quad , \quad N \simeq 60 .$$

- Small couplings arise **naturally** as ratio of two energy scales: inflation and Planck.
- The inflaton is a **light** particle:

$$m = \frac{M^2}{M_{Pl}} \simeq 0.003 M \quad , \quad m = 2.5 \times 10^{13} \text{ GeV}$$

$$H \sim \sqrt{N} m \simeq 2 \times 10^{14} \text{ GeV}.$$

MCMC Results for double-well inflaton potential

Bounds: $r > 0.023$ (95% CL) , $r > 0.046$ (68% CL)

Most probable values: $n_s \simeq 0.964$, $r \simeq 0.051$ \leftarrow measurable!!

The most probable double-well inflaton potential has a moderate nonlinearity with the quartic coupling $y \simeq 1.26 \dots$

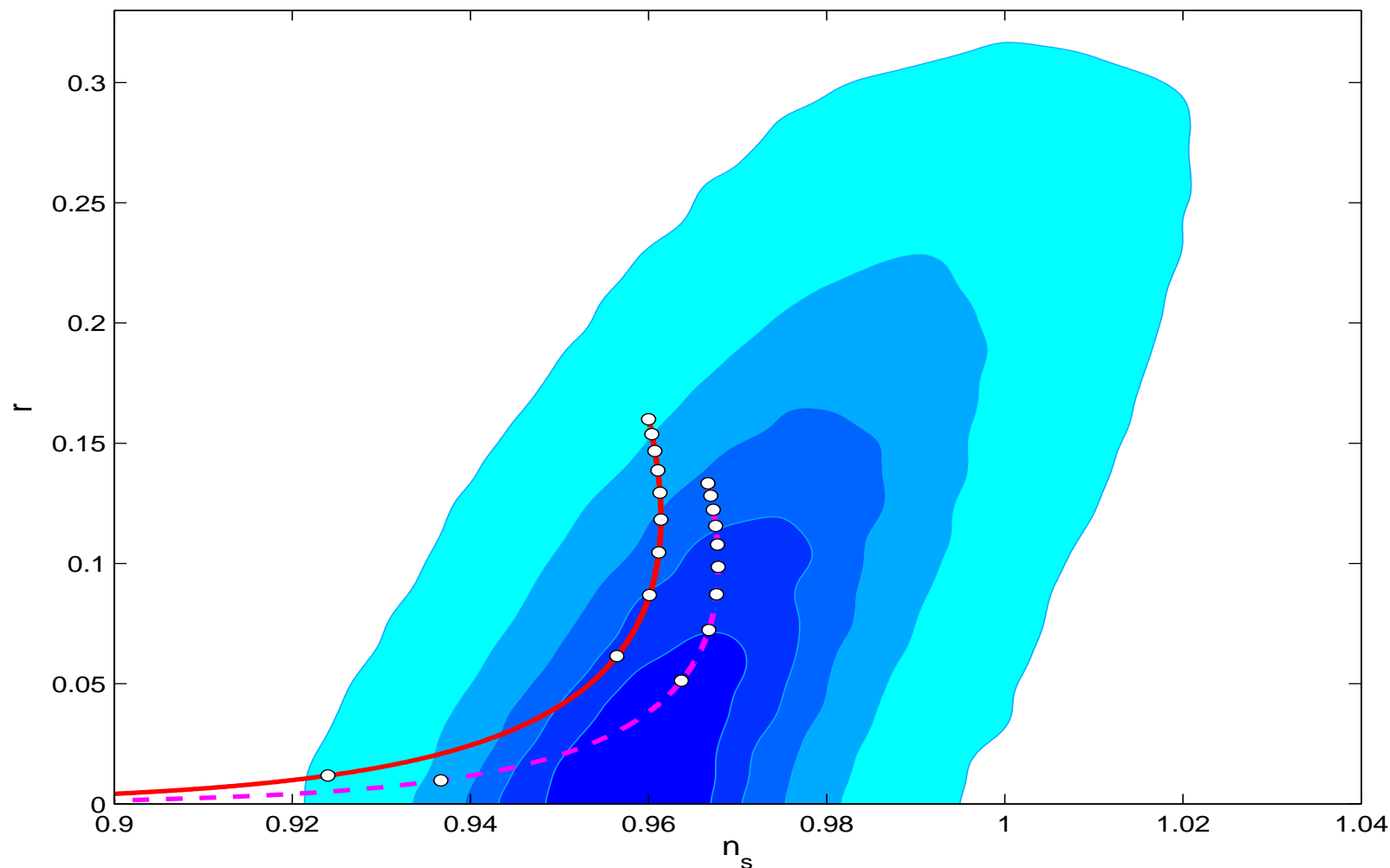
The $\chi \rightarrow -\chi$ symmetry is here spontaneously broken since the absolute minimum of the potential is at $\chi \neq 0$

$$w(\chi) = \frac{y}{32} \left(\chi^2 - \frac{8}{y} \right)^2$$

MCMC analysis calls for $w''(\chi) < 0$ at horizon exit
 \implies double well potential **favoured**.

C. Destri, H. J. de Vega, N. Sanchez, MCMC analysis of WMAP data points to broken symmetry inflaton potentials and provides a lower bound on the tensor to scalar ratio, Phys. Rev. D77, 043509 (2008), astro-ph/0703417.

MCMC Results for the double-well inflaton potential

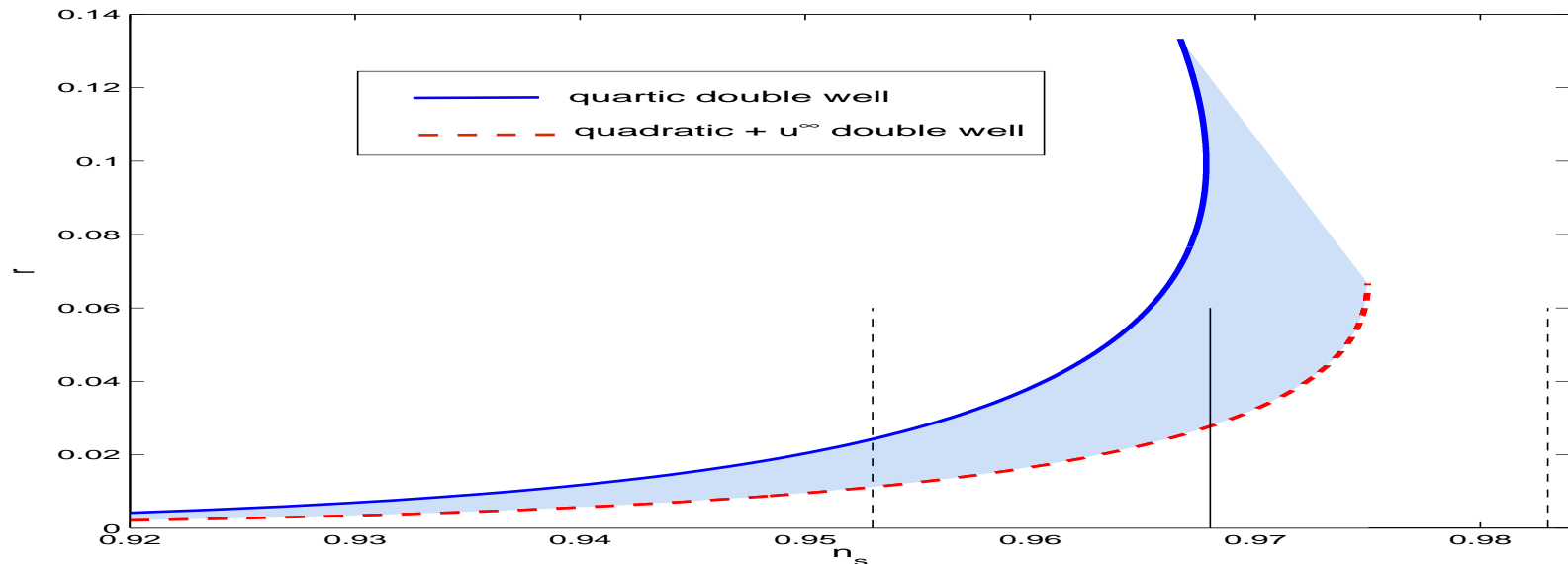


Solid line for $N = 50$ and dashed line for $N = 60$

White dots: $z = 0.01 + 0.11 * n$, $n = 0, 1, \dots, 9$,

y increases from the uppermost dot $y = 0$, $z = 1$.

The universal banana region



We find that all $r = r(n_s)$ curves for double-well inflaton potentials in the Ginsburg-Landau spirit fall **inside** the **universal** banana region \mathcal{B} .

The lower border of \mathcal{B} corresponds to the limiting potential:

$$w(\chi) = \frac{4}{y} - \frac{1}{2} \chi^2 \quad \text{for } \chi < \sqrt{\frac{8}{y}} \quad , \quad w(\chi) = +\infty \quad \text{for } \chi > \sqrt{\frac{8}{y}}$$

This gives a **lower bound** for r for **all** potentials in the Ginsburg-Landau class: $r > 0.021$ for the current best value of the spectral index $n_s = 0.964$.

Loop Quantum Corrections to Slow-Roll Inflation

$$\phi(\vec{x}, t) = \Phi_0(t) + \varphi(\vec{x}, t), \quad \Phi_0(t) \equiv \langle \phi(\vec{x}, t) \rangle, \quad \langle \varphi(\vec{x}, t) \rangle = 0$$

$$\varphi(\vec{x}, t) = \frac{1}{a(\eta)} \int \frac{d^3 k}{(2\pi)^3} \left[a_{\vec{k}} \chi_k(\eta) e^{i\vec{k}\cdot\vec{x}} + \text{h.c.} \right],$$

$a_{\vec{k}}^\dagger, a_{\vec{k}}$ are creation/annihilation operators,

$\chi_k(\eta)$ are mode functions. $\eta =$ conformal time.

To one loop order the equation of motion for the inflaton is

$$\ddot{\Phi}_0(t) + 3H \dot{\Phi}_0(t) + V'(\Phi_0) + g(\Phi_0) \langle [\varphi(\mathbf{x}, t)]^2 \rangle = 0$$

where $g(\Phi_0) = \frac{1}{2} V'''(\Phi_0)$.

The mode functions obey:

$$\chi_k''(\eta) + \left[k^2 + M^2(\Phi_0) a^2(\eta) - \frac{a''(\eta)}{a(\eta)} \right] \chi_k(\eta) = 0$$

where $M^2(\Phi_0) = V''(\Phi_0) = 3H_0^2 \eta_V + \mathcal{O}(1/N^2)$

Quantum Corrections to the Friedmann Equation

The mode functions equations for slow-roll become,

$$\chi_k''(\eta) + \left[k^2 - \frac{\nu^2 - \frac{1}{4}}{\eta^2} \right] \chi_k(\eta) = 0 \quad , \quad \nu = \frac{3}{2} + \epsilon_V - \eta_V + \mathcal{O}(1/N^2).$$

The scale factor during slow roll is $a(\eta) = -\frac{1}{H_0 \eta (1 - \epsilon_V)}$.

Slow-roll parameters of order $1/N$:

$$\epsilon_V = \frac{1}{2} M_{Pl}^2 \left(\frac{V'(\Phi_0)}{V(\Phi_0)} \right)^2 \quad , \quad \eta_V = M_{Pl}^2 \frac{V''(\Phi_0)}{V(\Phi_0)}$$

D. Boyanovsky, H. J. de Vega, N. G. Sanchez,

Quantum corrections to slow roll inflation and new scaling of superhorizon fluctuations. Nucl. Phys. B 747, 25 (2006), astro-ph/0503669.

Quantum corrections to the inflaton potential and the power spectra from superhorizon modes and trace anomalies, Phys. Rev. D 72, 103006 (2005), astro-ph/0507596.

Quantum Corrections in slow-roll

Scale invariant case: $\nu = \frac{3}{2}$. $\Delta \equiv \frac{3}{2} - \nu = \eta_V - \epsilon_V$ controls the departure from scale invariance.

Explicit solutions in slow-roll:

$$\chi_k(\eta) = \frac{1}{2} \sqrt{-\pi\eta} i^{\nu+\frac{1}{2}} H_\nu^{(1)}(-k\eta), \quad H_\nu^{(1)}(z) = \text{Hankel function}$$

Quantum fluctuations: $\langle [\varphi(\mathbf{x}, t)]^2 \rangle = \frac{1}{a^2(\eta)} \int \frac{d^3k}{(2\pi)^3} |\chi_k(\eta)|^2$

$$\frac{1}{2} \langle [\varphi(\mathbf{x}, t)]^2 \rangle = \left(\frac{H_0}{4\pi}\right)^2 \left[\Lambda_p^2 + \ln \Lambda_p^2 + \frac{1}{\Delta} + 2\gamma - 4 + \mathcal{O}(\Delta) \right]$$

UV cutoff $\Lambda_p = \text{physical cutoff}/H$, $\frac{1}{\Delta} = \text{infrared pole}$.

$\langle \dot{\varphi}^2 \rangle$, $\langle (\nabla\varphi)^2 \rangle$ are **infrared finite**

Quantum Corrections to the Inflaton Potential

Upon UV renormalization the Friedmann equation results

$$H^2 = \frac{1}{3M_{Pl}^2} \left[\frac{1}{2} \dot{\Phi}_0^2 + V_R(\Phi_0) + \left(\frac{H_0}{4\pi}\right)^2 \frac{V_R''(\Phi_0)}{\Delta} + \mathcal{O}\left(\frac{1}{N}\right) \right]$$

Quantum corrections are **proportional** to $\left(\frac{H}{M_{Pl}}\right)^2 \sim 10^{-9} !!$

The Friedmann equation gives for the effective potential:

$$V_{eff}(\Phi_0) = V_R(\Phi_0) + \left(\frac{H_0}{4\pi}\right)^2 \frac{V_R''(\Phi_0)}{\Delta}$$

$$V_{eff}(\Phi_0) = V_R(\Phi_0) \left[1 + \left(\frac{H_0}{4\pi M_{Pl}}\right)^2 \frac{\eta_V}{\eta_V - \epsilon_V} \right]$$

in terms of slow-roll parameters

Very **DIFFERENT** from the one-loop effective potential in **Minkowski** space-time:

$$V_{eff}(\Phi_0) = V_R(\Phi_0) + \frac{[V_R''(\Phi_0)]^2}{64\pi^2} \ln \frac{V_R''(\Phi_0)}{M^2}$$

Quantum Fluctuations:

Scalar Curvature, Tensor, Fermion, Light Scalar.

All these quantum fluctuations **contribute** to the inflaton potential **and** to the primordial power spectra.

In de Sitter space-time: $\langle T_{\mu\nu} \rangle = \frac{1}{4} g_{\mu\nu} \langle T^\alpha_\alpha \rangle$

Hence, $V_{eff} = V_R + \langle T^0_0 \rangle = V_R + \frac{1}{4} \langle T^\alpha_\alpha \rangle$

Sub-horizon (Ultraviolet) contributions appear through the **trace anomaly** and only depend on the spin of the particle. Superhorizon (Infrared) contributions are of the order N^0 and can be expressed in terms of the **slow-roll parameters**.

$$V_{eff}(\Phi_0) = V(\Phi_0) \left[1 + \frac{H_0^2}{3(4\pi)^2 M_{Pl}^2} \left(\frac{\eta_\nu - 4\epsilon_\nu}{\eta_\nu - 3\epsilon_\nu} + \frac{3\eta_\sigma}{\eta_\sigma - \epsilon_\nu} + \mathcal{T} \right) \right]$$

$\mathcal{T} = \mathcal{T}_\Phi + \mathcal{T}_s + \mathcal{T}_t + \mathcal{T}_F = -\frac{2903}{20}$ is the total trace anomaly.

$$\mathcal{T}_\Phi = \mathcal{T}_s = -\frac{29}{30}, \quad \mathcal{T}_t = -\frac{717}{5}, \quad \mathcal{T}_F = \frac{11}{60}$$

→ the **graviton** (t) dominates.

Corrections to the Primordial Scalar and Tensor Power

$$\begin{aligned} |\Delta_{k,eff}^{(S)}|^2 &= |\Delta_k^{(S)}|^2 \left\{ 1 + \right. \\ &\quad \left. + \frac{2}{3} \left(\frac{H_0}{4\pi M_{Pl}} \right)^2 \left[1 + \frac{\frac{3}{8} r (n_s - 1) + 2 \frac{dn_s}{d \ln k}}{(n_s - 1)^2} + \frac{2903}{40} \right] \right\} \\ |\Delta_{k,eff}^{(T)}|^2 &= |\Delta_k^{(T)}|^2 \left\{ 1 - \frac{1}{3} \left(\frac{H_0}{4\pi M_{Pl}} \right)^2 \left[-1 + \frac{1}{8} \frac{r}{n_s - 1} + \frac{2903}{20} \right] \right\}. \end{aligned}$$

The anomaly contribution $-\frac{2903}{20} = -145.15$ **DOMINATES** as long as the number of fermions less than 783.

The scalar curvature fluctuations $|\Delta_k^{(S)}|^2$ are **ENHANCED** and the tensor fluctuations $|\Delta_k^{(T)}|^2$ **REDUCED**.

However, $\left(\frac{H}{M_{Pl}} \right)^2 \sim 10^{-9}$.

D. Boyanovsky, H. J. de Vega, N. G. Sanchez, Phys. Rev. D 72, 103006 (2005), astro-ph/0507596.

The Energy Scale of Inflation

Grand Unification Idea (GUT)

- Renormalization group running of electromagnetic, weak and strong couplings shows that they **all meet** at $E_{GUT} \simeq 2 \times 10^{16}$ GeV
- Neutrino masses are explained by the **see-saw** mechanism: $m_\nu \sim \frac{M_{\text{Fermi}}^2}{M_R}$ with $M_R \sim 10^{16}$ GeV.
- Inflation energy scale: $M \simeq 10^{16}$ GeV.

Conclusion: the GUT energy scale appears in at least **three** independent ways.

Moreover, moduli potentials: $V_{\text{moduli}} = M_{\text{SUSY}}^4 v \left(\frac{\phi}{M_{\text{Pl}}} \right)$
resemble inflation potentials provided $M_{\text{SUSY}} \sim 10^{16}$ GeV.
First observation of SUSY in nature??

De Sitter Geometry and Scale Invariance

The De Sitter metric **is scale invariant**:

$$ds^2 = \frac{1}{(H\eta)^2} \left[(d\eta)^2 - (d\vec{x})^2 \right] , \quad \eta = \text{conformal time.}$$

But inflation **only lasts** for N e-folds !

Corrections to scale invariance:

$|n_s - 1|$ as well as the ratio r are of order $\sim 1/N$,

The Harrison-Zeldovich point $n_s = 1$ and $r = 0$ corresponds to a critical point.

It is a gaussian fixed point around which the inflation model **hovers** in the renormalization group (RG) sense with an almost scale invariant spectrum during the slow roll stage.

The quartic coupling:

$$\lambda = \frac{G_4}{N} \left(\frac{M}{M_{Pl}} \right)^4 , \quad N = \log \frac{a(\text{inflation end})}{a(\text{horizon exit})}$$

runs like in four dimensional RG in flat euclidean space.

Summary and Conclusions

- We formulate inflation as an **effective** field theory in the Ginsburg-Landau spirit and obtain $M \sim M_{GUT} \sim 10^{16}$ GeV as inflation energy scale.
- This effective theory **is consistent** because:
 $H \ll M \ll M_{Pl}$. Inflaton mass turns to be **small**:
 $m \sim H/\sqrt{N}$. Infrared regime !!
- The slow-roll approximation is a $1/N$ expansion,
 $N \sim 60$. For all slow-roll models $n_s - 1$ and r are $\sim 1/N$.
Running: $dn_s/d \ln k \sim 1/N^2$.
- MCMC analysis of WMAP+LSS data **plus** this theory input indicates a spontaneously symmetry breaking inflaton potential: $w(\chi) = \frac{y}{32} \left(\chi^2 - \frac{8}{y} \right)^2$, $y \simeq 1.26$.
- Lower Bound: $r > 0.023$ (95% CL). Most probable values: $r \simeq 0.051$ (\Leftarrow measurable !!) $n_s \simeq 0.964$.

Summary and Conclusions 2

- Primordial Non-gaussianity $f_{NL} = \text{order } 1/N$. Too small to be detected.
- Quantum (loop) corrections in the effective theory are of the order $(H/M_{Pl})^2 \sim 10^{-8}$. Same order of magnitude as loop graviton corrections.

Preinflationary and inflationary fast-roll eras and their signatures in the low CMB multipoles,

C. Destri, H. J. de Vega, N. G. Sanchez, arXiv:0912.2994, Phys. Rev. D **81**, 063520 (2010).

Higher order terms in the inflaton potential and the lower bound on the tensor to scalar ratio r .

C. Destri, H. J. de Vega, N. G. Sanchez, arXiv:0906.4102, Annals of Physics, **326**, 578 (2011).

The Universe is our ultimate physics laboratory

THANK YOU VERY MUCH
FOR YOUR ATTENTION!!