de Sitter Holography and Wavefunctionals

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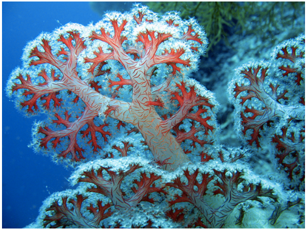
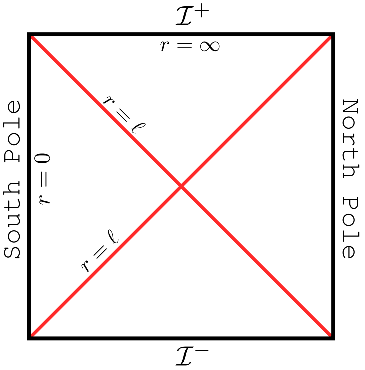


Outline

- Static Patch Curiosities
- Global de Sitter Space
- State Space at $\mathcal{I}^+$/Cosmic Corals
- dS/CFT
- Outlook

STATIC PATCH CURIOSITIES



The Static patch observer - Cosmic horizons

Observers surrounded by COSMOLOGICAL HORIZON. Geometry is given by:

$$ds^2 = -\left(1 - r^2/\ell^2\right) dt^2 + \left(1 - r^2/\ell^2\right)^{-1} dr^2 + r^2 d\Omega_2^2 \; .$$

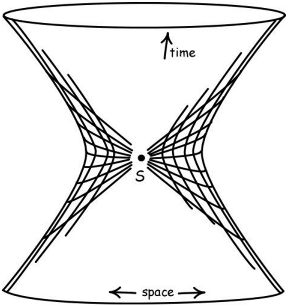
How to describe the de Sitter observer remains an open question. Matrix theory?

Gibbons-Bekenstein Hawking entropy of cosmological horizon? What is it counting? What are the microstates?

Maximal sized Nariai black holes have $1/3$ of full de Sitter entropy.

Basic Confusion: No asymptotic boundaries, no sharp observables.

GLOBAL DE SITTER SPACE: THE METAOBSERVER



The metaobserver - Structure of $\mathcal{I}^+$

More globally, at late times asymptotically de Sitter has FG expansion [Starobinsky]:

$$ds^2 = \frac{\ell^2}{\eta^2}\left(-d\eta^2 + g^{(0)}_{ij} dx_i dx_j + \eta^2 g^{(2)}_{ij} dx^i dx^j + \eta^3 g^{(3)}_{ij} dx_i dx_j + \dots\right)$$

$\mathcal{I}^+$ lives at $\eta \to 0^-$. Also, $\mathrm{Tr}\, g^{(3)}_{ij} = \nabla^i g^{(3)}_{ij} = 0$.

Though mathematically similar to boundary of AdS, $\mathcal{I}^+$ is also crucially different.

We have to consider deformations of (conformal) metric on $\mathcal{I}^+$, $g^{(0)}_{ij}$, in addition to deformations of subleading terms in η.

Classical Results

Interesting mathematical results (classical) relating the $\mathcal{I}^+$ data $(g_{ij}^{(0)}, g_{ij}^{(3)})$ to the presence of bulk singularities [Andersson,Galloway].

For instance if $g_{ij}^{(0)}$ is compact with negative curvature, the geometry is singular in the past.

Example, Taub-Nut de Sitter, where $g_{ij}^{(0)} =$ squashed S^3, is globally well defined for small squashing but develops singularities for large squashing.

Also there is the cosmic no hair theorem [Gibbons,Hawking]. Theories with $\Lambda > 0$ classically asymptote to dS at late times.

Quantum Mechanical Setup

Quantum Mechanically, we might wish to compute the transition amplitude from initial (quantum) state, i.e. Hartle-Hawking, to some late time configuration $\Phi(x)$.

In other words we may wish to consider the wavefunctional:

$$\Psi_{HH}[\Phi(x)] = \langle \mathrm{HH} | \Phi(x) \rangle \ ,$$

where $\Phi(x)$ is some late time configuration near $\mathcal{I}^+$ and $|\mathrm{HH}\rangle$ is the

Chernikov-Tagirov-Bunch-Davies-Schomblond-Spindel-Hartle-Hawking-Euclidean-Mottola-Allen-Sasaki

de Sitter invariant vacuum state. It is defined (perturbatively) by the modes which become ordinary positive frequency Minkowski modes well inside de Sitter horizon.

Computing Ψ_{HH}

For matter field φ born out of the Bunch-Davies vacuum in a fixed dS background one computes:

$$\Psi_{HH}[\phi(x)] \sim e^{iS_{cl}[\phi(x)]}$$

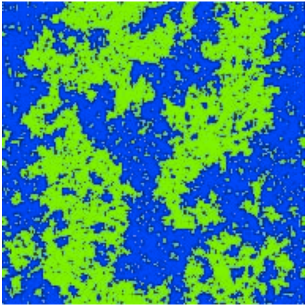
with boundary conditions $\varphi \sim e^{-ik\eta}/\sqrt{k}$ at early times and $\varphi \sim \phi(x)$ at late times.

Semiclassically, according to Hartle-Hawking, one computes (restricting to metric field below):

$$\Psi_{HH}[h_{ij}] \sim \int_{g(\partial\mathcal{M})=h} \mathcal{D}g \; e^{-S_E[g]} \ ,$$

for all compact Euclidean geometries with a single boundary with induced metric h_{ij}.

PERTURBATIVE Ψ_{HH} and STATE SPACE AT $\mathcal{I}^+$



Massless Fields in de Sitter Space

We consider the evolution of a free massless scalar field in a fixed dS_4 background:

$$ds^2 = \frac{\ell^2}{\eta^2}\left(-d\eta^2 + d\vec{x}^2\right) \ , \quad x^i \sim x^i + L \ , \quad \eta \in (-\infty, 0)$$

Solution of wave-equation:

$$\varphi^{cl}_{\vec{k}}(\eta) = \frac{1}{\ell(2kL)^{3/2}}\left(A^{+}_{\vec{k}}(1 - ik\eta)e^{ik\eta} + A^{-}_{\vec{k}}(1 + ik\eta)e^{-ik\eta}\right) \ .$$

Positive frequency mode for $|HH\rangle$ vacuum is: $v_k = \frac{1}{\ell(2kL)^{3/2}}(1 + ik\eta)e^{-ik\eta}$.

Late time profile distributions

Wavefunctional is given by evaluating on-shell action. For massless scalar φ emerging from $|\text{HH}\rangle$ with late time profile $\phi_{\vec{k}}$, i.e. $\varphi_{\vec{k}}(\eta = 0) = \phi_{\vec{k}}$, we compute:

$$\Psi_{HH}[\phi] \propto e^{iS_{cl}[\phi]} \implies \mathcal{P}[\phi] \propto |\Psi_{HH}[\phi]|^2 \; .$$

leading to probability distribution for late time configuration $\phi_{\vec{k}}$:

$$\mathcal{P}[\phi] \propto e^{-2\sum_{\vec{k}} \beta_k |\phi_{\vec{k}}|^2} \; , \quad \beta_k = \frac{1}{2}(Lk)^3 \ell^2 .$$

QUESTION: How (if at all) are late time spatial configurations organized?

Distances in Field Space

Inspired by SPIN GLASSES, we define a notion of overlap between late time profiles [D.A.,Denef].

We propose a regularized Euclidean distance:

$$d[\phi_1, \phi_2] = \frac{1}{L^3} \int d^3x \left[(\phi_1(\vec{x}) - \overline{\phi_1(\vec{x})}) - (\phi_2(\vec{x}) - \overline{\phi_2(\vec{x})}) \right]^2 .$$

Overline $=$ Spatial average $\implies$ subtraction of zero mode.

We further subtract the divergent late time average (w.r.t. $\mathcal{P}[\phi]$) from d_{12}.

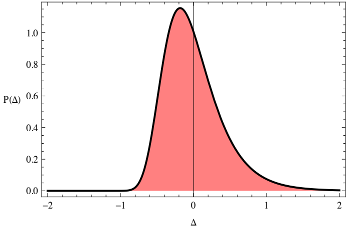
Our task is to compute the single and triple overlap distribution. e.g.

$$P(\Delta) = \int \mathcal{D}\phi_1 \mathcal{D}\phi_2 |\Psi_{HH}(\phi_1)|^2 |\Psi_{HH}(\phi_2)|^2 \delta\left(\Delta - d[\phi_1, \phi_2]\right) .$$

Single Overlap Distribution

For single overlap distribution, $P(\Delta) = \langle \delta(\Delta - d_{12}) \rangle$ we find the GUMBEL distribution (κ, γ are $\mathcal{O}(1)$ numbers):

$$P(\Delta) = \frac{\kappa^\kappa}{\Gamma(\kappa)} \exp\left[-\kappa\left(\Delta' + e^{-\Delta'}\right)\right] \;, \quad \Delta' \equiv \gamma + (2\pi\ell)^2 \Delta.$$



Also appears in extreme value physics. Asymmetry $\Longrightarrow$ There are more dissimilar configurations. (Also found by Benna using different 'distance'.)

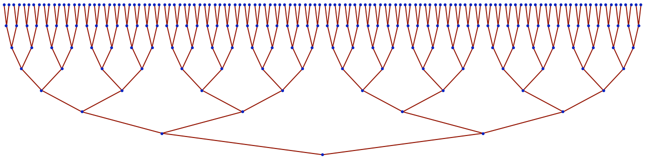
Triple Overlap Distribution

Similarly, we can compute the triple overlap distribution:

$$P(\Delta_1, \Delta_2, \Delta_3) \equiv \langle \delta(\Delta_1 - d_{23})\delta(\Delta_2 - d_{13})\delta(\Delta_3 - d_{12}) \rangle$$

$P(\vec{\Delta})$ peaks at $\Delta_3 = \max\{\Delta_1, \Delta_2\}$.

This is a clear signal of ultrametricity directly analogous to that found for spin glasses.



dS/CFT CORRESPONDENCE

and

NON-PERTURBATIVE Ψ_{HH}?

CONJECTURE [Strominger,Witten,Maldacena]

Boundary-to-boundary correlators at $\mathcal{I}^+$ with Dirichlet (future) boundary conditions are those of a Euclidean (non-unitary) 3d CFT. Late time (non-normalizable) profiles of fields are sources in the CFT.

Partition function of CFT computes Hartle-Hawking wavefunctional $\Psi_{HH}[\Phi_I(x)]$:

$$Z_{CFT}[\Phi_I(x)] \propto \Psi_{HH}[\Phi_I(x)] \approx e^{iS_{cl}[\Phi_I(x)]} .$$

CFT correlators are variational derivatives of $\Psi_{HH}[\Phi_I(x)]$ w.r.t $\Phi_I(x)$.

Challenge: Computing CFT partition function for finite, space dependent sources (dissordered systems)?

For 4d Vassiliev gravity with infinite tower of even spin modes we have a proposal [D.A.,Hartman,Strominger]. It is the 3d $Sp(N)$ model [LeClair]:

$$\mathcal{S}_{CFT} = \int d^3x \; \delta^{ij} \; \partial_i \chi^A \partial_j \chi^B \Omega_{AB}$$

The χ^A are anti-commuting scalars which transform as $Sp(N)$ vectors. $N \equiv \ell_{dS}^2 / \ell_{Pl}^2$.

The above theory is a CFT whose correlators conjecturally reproduce those in higher spin gravity. Three-point functions checked [Giombi,Yin].

We must impose a $Sp(N)$ SINGLET constraint on the operator content.

Bulk graviton is dual to stress tensor, T_{ij}, bulk conformally coupled scalar is dual to $J^{(0)} \sim \Omega_{AB}\chi^A\chi^B$ and higher spin fields are dual to higher spin currents.

Perturbatively this is related to $O(N)$ model by $N \to -N$.

Since the dual theory is a free theory we can compute Z_{CFT}.

For simplicity, we only consider turning on a source, $\sigma(x)$ for $J^{(0)} \sim \Omega_{AB}\chi^A\chi^B$ and couple the theory to a background metric g_{ij}, i.e. source for T_{ij}.

$$\mathcal{S}_{CFT} = \int d^3x\sqrt{g}\left(g^{ij}\partial_i\chi^A\partial_j\chi^B\Omega_{AB} + \frac{1}{8}R[g]\Omega_{AB}\chi^A\chi^B + \sigma(x)\Omega_{AB}\chi^A\chi^B\right)$$

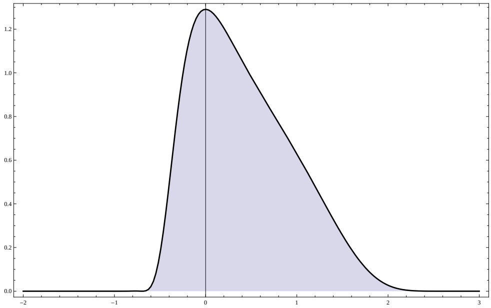
This corresponds to fixing late time profiles, g_{ij} and σ, for the bulk metric and scalar field, with all other fields switched off. Then:

$$Z_{CFT}[g,\sigma] = e^{-S_{ct}} \det\left(\frac{-\nabla^2 + \sigma(x) + \frac{1}{8}R[g]}{\Lambda_{UV}^2}\right)^{N/2} .$$

We can explore $Z_{CFT} = \Psi_{HH}$ for some special slices [D.A.,Denef,Harlow].

Squashed S^3

On a squashed S^3 with $ds^2 = (d\theta^2 + \cos^2\theta d\phi^2) + e^{-\rho}(d\psi + \sin\theta d\phi)^2$:

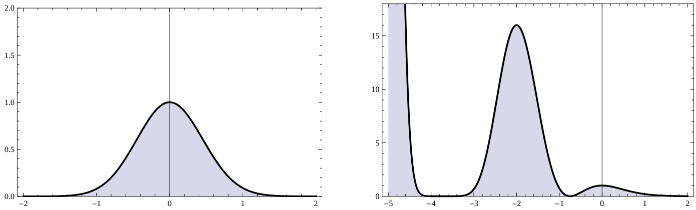


Horizontal axis is the squashing parameter ρ. Vertical axis is $|\Psi_{HH}[\rho]|^2$. We have $N = 2$.

Notice wavefunction is NORMALIZABLE in ρ direction.

Finite constant σ on a round S^3

For a round metric on S^3 with varying constant σ (source of $\chi \cdot \chi$):



Horizontal axis is σ. Vertical axis is $|\Psi_{HH}[\sigma]|^2$. We have $N = 1000, 2$.

Notice there is a local maximum near $\sigma = 0$. Perturbatively near this region we have the Gaussian wavefunction of a free scalar field in a fixed dS background. As we proceed out in σ space the wavefunction acquires local maxima larger than $\sigma = 0$.

Wavefunction is NON-NORMALIZABLE in σ direction = INSTABILITY?

$S^2 \times S^1$

We can also study on CFT on a topology different from S^3. For example we can compute Z_{CFT} on an $S^2 \times S^1$ as a function of the relative size $\beta = R_{S^1}/R_{S^2}$.

Singlet constraint becomes non-trivial due to a non-contractible cycle. Thus, we couple the theory to a Chern-Simons sector and take the Chern-Simons coupling to infinity. This leads to integrating over flat connections of the $Sp(N)$ gauge field.

One finds that for small β, $Z_{CFT}[\beta] \sim e^{4\zeta(3)N\beta^{-2}}$.

Such a divergence for small β can also for Ψ_{HH} in Einstein gravity if we include certain complex instanton solutions and the Hartle-Hawking (no boundary) prescription for Ψ_{HH}.

Some Lessons

Some lessons from higher spin dS/CFT:

dS/CFT can actually work (at least in this setup).

$N \sim \ell^2/\ell_{Pl}^2$ is an INTEGER and hence the bulk cosmological constant is quantized. Brane constructions?

In special cases, we can compute wavefunction (i.e. Z_{CFT}), find interesting structure, possibly (non-perturbative) instabilities even in this simple setup.

dS/CFT is NOT an analytic continuation of AdS/CFT beyond perturbation theory.

OUTLOOK

How is bulk unitarity encoded in CFT?

Can we compare Ψ_{HH} for different topologies, i.e. Lens Space?

To what extent is cosmology an RG flow in the CFT (cosmological solutions in Vasiliev)?

Can we accomodate bubble nucleation in the language of dS/CFT (e.g. boundary CFTs)?

Resolution of big bang singularities (simple example: de Sitter foliated by compact quotients of $\mathcal{H}_3$)?

Further Concluding Remarks

de Sitter space is a tale of two observers. In our own Universe we are interpolating between two de Sitter eras.

Whether and how these two pictures are connected remains an open question, i.e. is the static patch observer encoded in the CFT?

How much can we learn from the higher spin toy model? e.g. de Sitter entropy, RG flow/cosmology...

How is the late time configuration space of geometries organized more generally?