F-theory Model Building

Jonathan J. Heckman

Review: arXiv:1001.0577 [hep-th]

arXiv:1005.3033 [hep-th] w/ H. Verlinde

arXiv:1006.5459 [hep-th] w/ C. Vafa

Based On...

This talk is based on the work of several groups

For brevity I will focus on only a few themes

and some potentially interesting future directions

Plan of the Talk

- Bottom up Stringy Standard Models
- Unification in F-theory
- Extra Sectors
- Fuzzy Local Models
- Conclusions

SM / MSSM

Gauge Group: $SU(3)_C \times SU(2)_L \times U(1)_Y$

Matter:

Quarks: $3 \times (Q \oplus U \oplus D)$

Leptons: $3 \times (L \oplus E)$

Higgs: $H_u \oplus H_d$

Interactions: $W = H_u Q U + H_d Q D + H_d L E + \cdots$

Some Questions

Q1: Standard Model (SM) from Strings?

Q2: Stringy ingredients of the SM?

Q3: Given a stringy SM, what does it predict?

Top Down Perspective

Compactify: $\mathcal{M}_{10} \simeq \mathcal{M}_4^{us} \times \mathcal{M}_6^{internal}$

$$+ \text{ Branes} + \text{Fluxes} + \text{Moduli} + \cdots$$

$\Rightarrow$ Many moving parts interacting non-trivially

Where to Look First?

Bottom Up Perspective

SM is well-tested. Planck scale less clear

Focus on gauge sector, defer gravity

$$M_{pl} \sim 10^{19}\ \text{GeV} \gg M_{weak} \sim 10^{2}\ \text{GeV}$$

$$M_{pl} \sim 10^{19}\ \text{GeV} \gg M_{GUT} \sim 10^{16}\ \text{GeV}$$

Local Models

Antoniadis, Kiritsis, Tomaras ’00
Aldazabel, Ibanez, Quevedo, Uranga ’00
Verlinde Wijnholt ’05,…

Build up in steps:

1) Decouple Gravity

2) Realize SM in IR

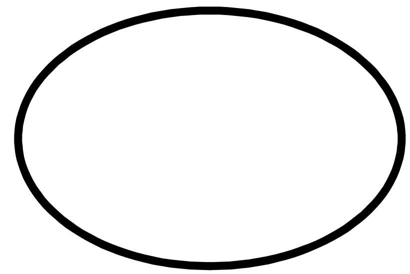
3) Extend SM (e.g. SUSY, unification, ...)

4) Recouple to gravity

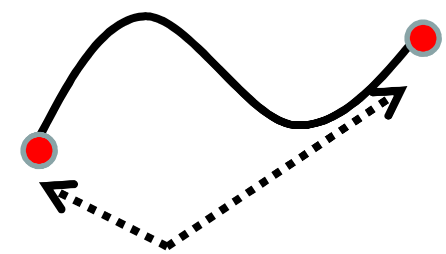
Won’t discuss much about this

Strategy:

Closed strings for gravity



Open strings for particle physics



$\Rightarrow$ Put SM on a spacetime filling brane

Perturbative Building Blocks

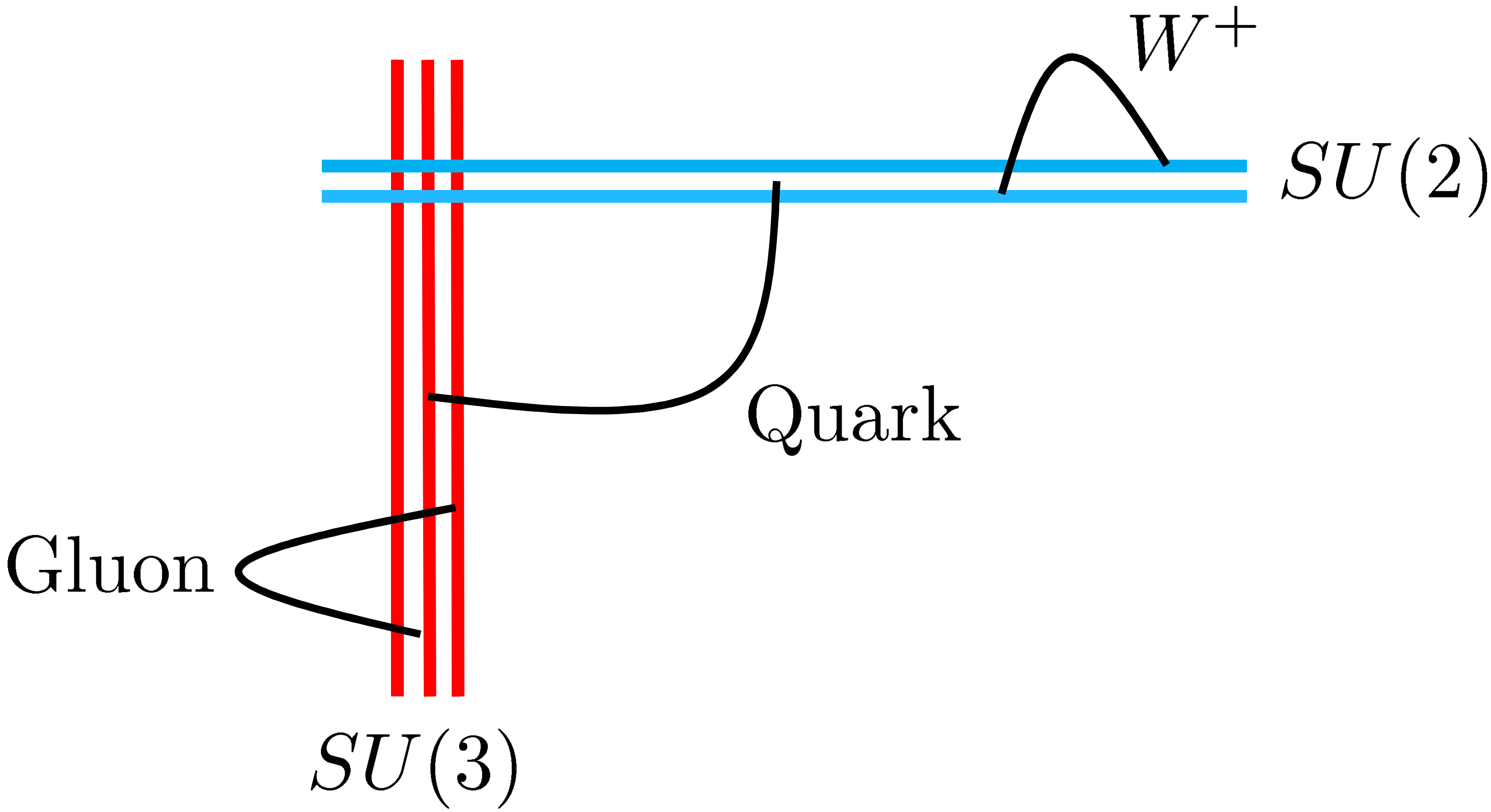
Gauge groups: U, SO, USp but no $E_{6,7,8}$

Matter: $\square \otimes \overline{\square} \qquad \begin{array}{|c|}\hline \\ \hline \\ \hline \end{array} \qquad \begin{array}{|c|c|}\hline & \\ \hline \end{array}$

Yukawas: $\#\square = \#\overline{\square}$ *e.g.* $\overline{\square} \times \overline{\square} \times \begin{array}{|c|}\hline \\ \hline \\ \hline \end{array}$

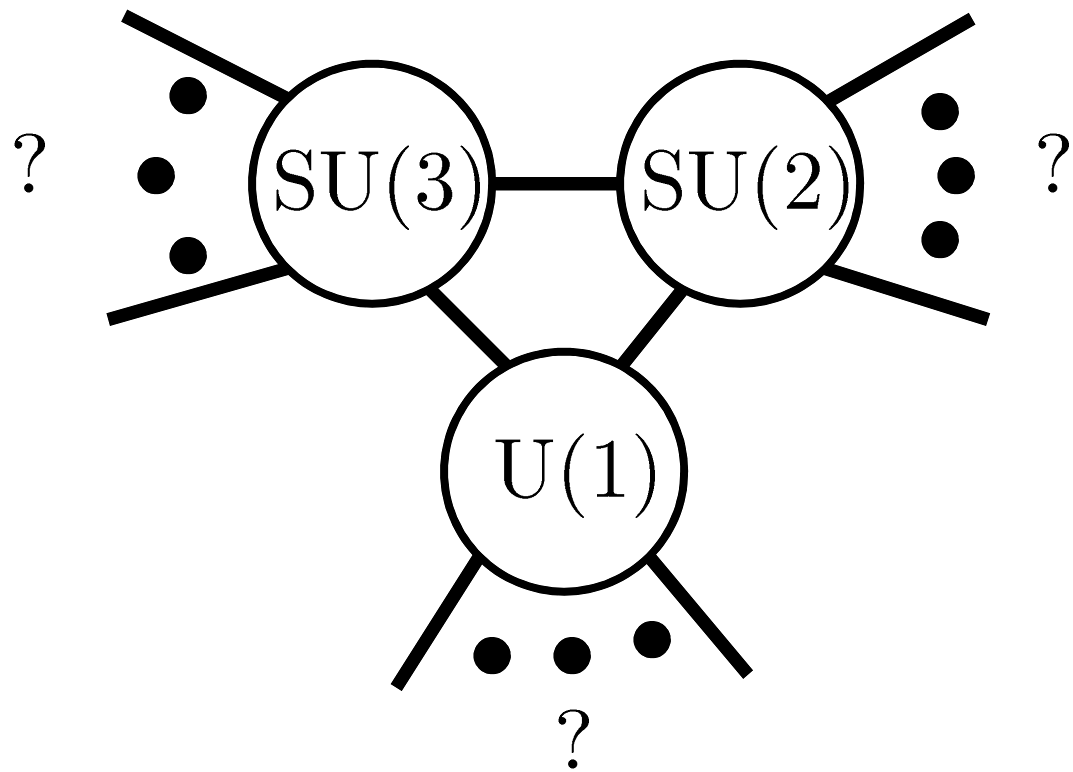
but not $\square \times \begin{array}{|c|}\hline \\ \hline \\ \hline \end{array} \times \begin{array}{|c|}\hline \\ \hline \\ \hline \end{array}$

SM is Like This!



Too Flexible?

Encourages building big quivers:



Is SM completely arbitrary?

Unification

But quivers also discourage unification

Circumstantial evidence for unification:

1) MSSM gauge coup.'s unify @ $\sim 10^{16}$ GeV

2) Matter in irreps of $SU(5)$, $SO(10)$, E_6,...

3) $m_{neutrino} \sim m_{weak}^2 / M_{GUT}$

• • •

$SU(5)$ GUT

$$SU(5) \supset SU(3)_C \times SU(2)_L \times U(1)_Y$$

Matter:

$$10_M \to Q \oplus U \oplus E$$

$$\overline{5}_M \to D \oplus L$$

$$5_H \to \cancel{T}_u \oplus H_u$$

$$\overline{5}_H \to \cancel{T}_d \oplus H_d$$

Interactions: $W = \underbrace{5_H 10_M 10_M}_{\Rightarrow \text{ t mass}} + \underbrace{\overline{5}_H \overline{5}_M 10_M}_{\Rightarrow \text{ b and } \tau \text{ masses}} + \cdots$

More GUT Structures

$$SU(5): \begin{array}{l} \text{Matter: } 5_H,\, \overline{5}_H,\, \overline{5}_M,\, 10_M \\ \\ W_{SU(5)} \supset 5_H 10_M 10_M + \overline{5}_H \overline{5}_M 10_M \end{array}$$

$$SO(10): \begin{array}{l} \text{Matter: } 10_H,\, 16_M \\ \\ W_{SO(10)} \supset 16_M 16_M 10_H \end{array}$$

In 4D, stop after E_6 (need chirality)

Open Strings & GUTs

GUTs problematic with open strings at $g_s \ll 1$

No 16_M for $SO(10)$ GUTs

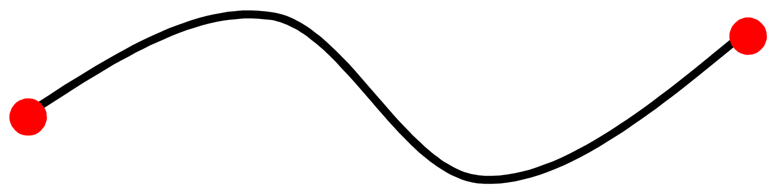
No $5_H 10_M 10_M$ for $SU(5)$ GUTs

$\yng(1) \times \yng(1,1) \times \yng(1,1)$ violates $\#\yng(1) = \#\overline{\yng(1)}$

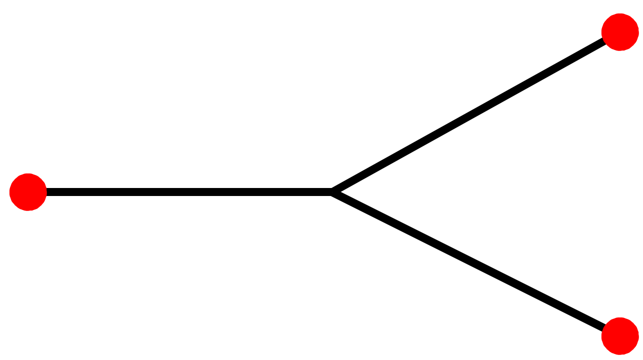
Main Issue: No E-type structures

Main Point:

Perturbative open strings obstruct unification



New ingredients as $g_s \to O(1)$



Roadmap

- Bottom up Stringy Standard Models
- Unification in F-theory

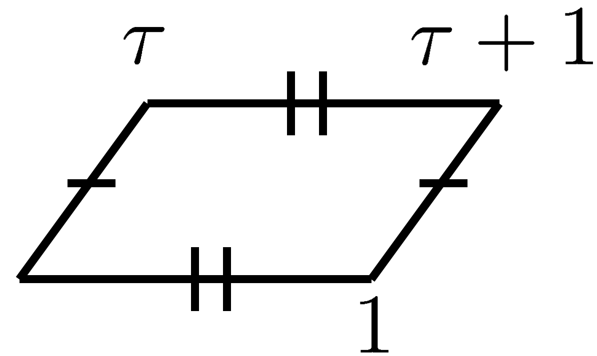
F-theory Review

Vafa '96

Strongly coupled IIB string theory = F-theory

$$\tau_{IIB} = C_0 + \frac{i}{g_s} \sim O(1)$$

Interpret τ_{IIB} as τ of a T^2

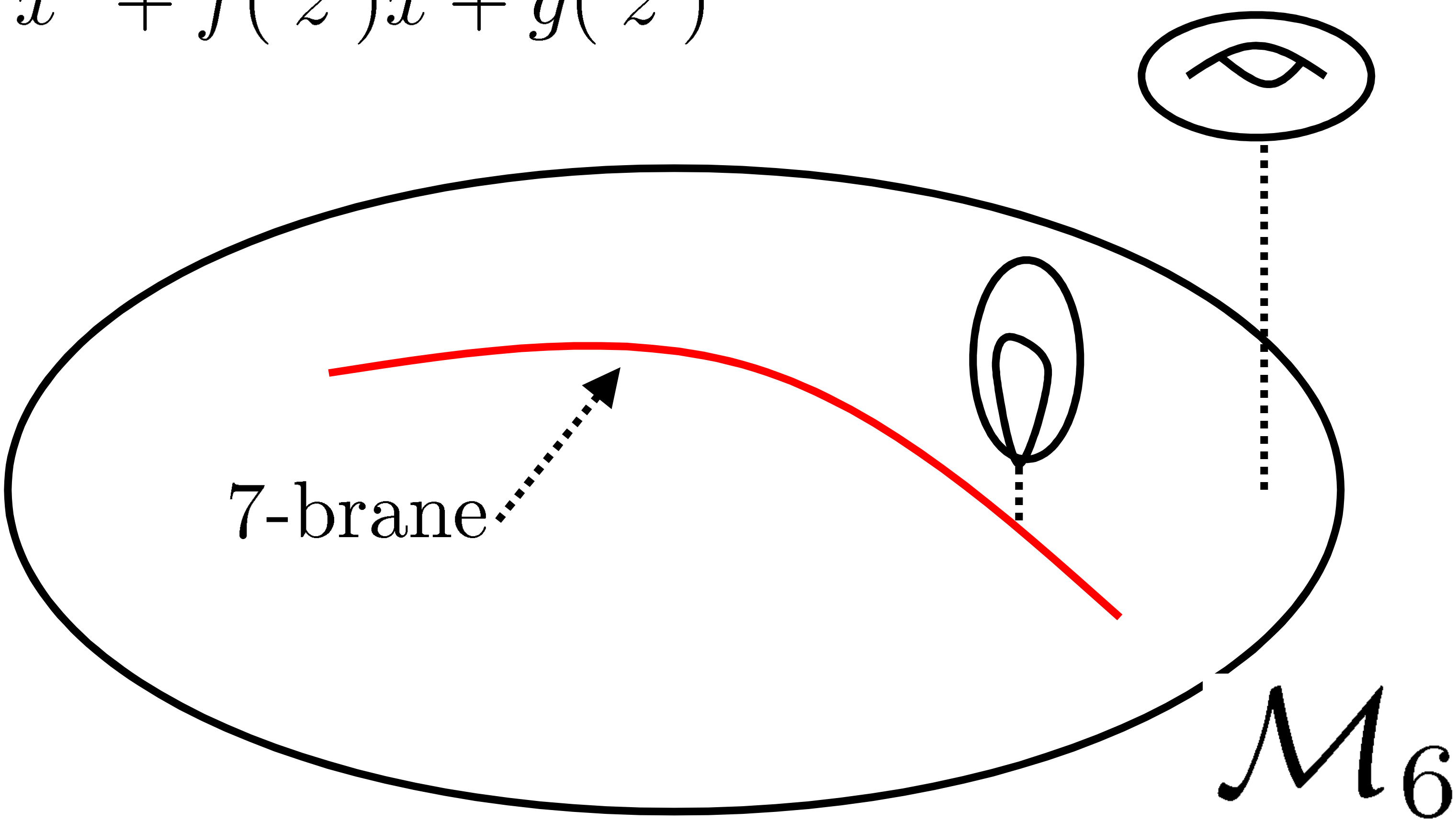


Allow $\tau_{IIB}(y)$ non-trivial position dependence

Geometric Formulation

F / $CY_4 \Rightarrow$ 4D $\mathcal{N} = 1$ SUSY

$$y^2 = x^3 + f(\overrightarrow{z})x + g(\overrightarrow{z})$$



Geometry $\Rightarrow$ Gauge Theory

C^2/Γ_{ADE}

Locally, $CY_4 \simeq ADE \to \mathcal{M}_4$

$\Rightarrow$ 7-brane on $R^{3,1} \times \mathcal{M}_4$ with ADE gauge gp.

$$\frac{4\pi}{g^2_{GUT}} = \frac{1}{g_s}\frac{\mathrm{Vol}(\mathcal{M}_4)_{\mathrm{open}}}{l_*^4} \Rightarrow$$

Weakly coupled when

$\mathrm{Vol}(\mathcal{M}_4)_{\mathrm{open}} \gg \mathrm{l}_*^4$

$(\alpha_{GUT} \sim 1/25)$

Expansion Parameters

$g_s \sim \mathcal{O}(1) \Rightarrow$ no expansion in g_s

Instead, perform expansion in: $\frac{1}{\mathrm{Vol}(\mathcal{M}_4)_{\mathrm{open}}}$

$\frac{1}{\mathrm{Vol}(\mathcal{M}_4)_{\mathrm{open}}} \sim M_{GUT}^4 \Rightarrow$ Expand in $\frac{M_{GUT}}{M_{pl}}$

Intersecting 7-Branes I

Each 7-brane fills out 4 directions in $\mathcal{M}_6$

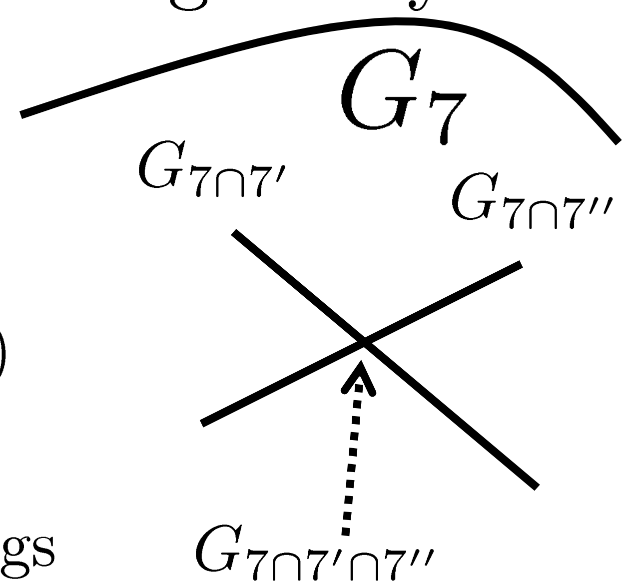
$\Rightarrow$ Generically expect intersections

$\Rightarrow$ Further jumps in ADE type of singularity

$\dim = 4 : 7_{\mathrm{GUT}} \Rightarrow$ Gauge group

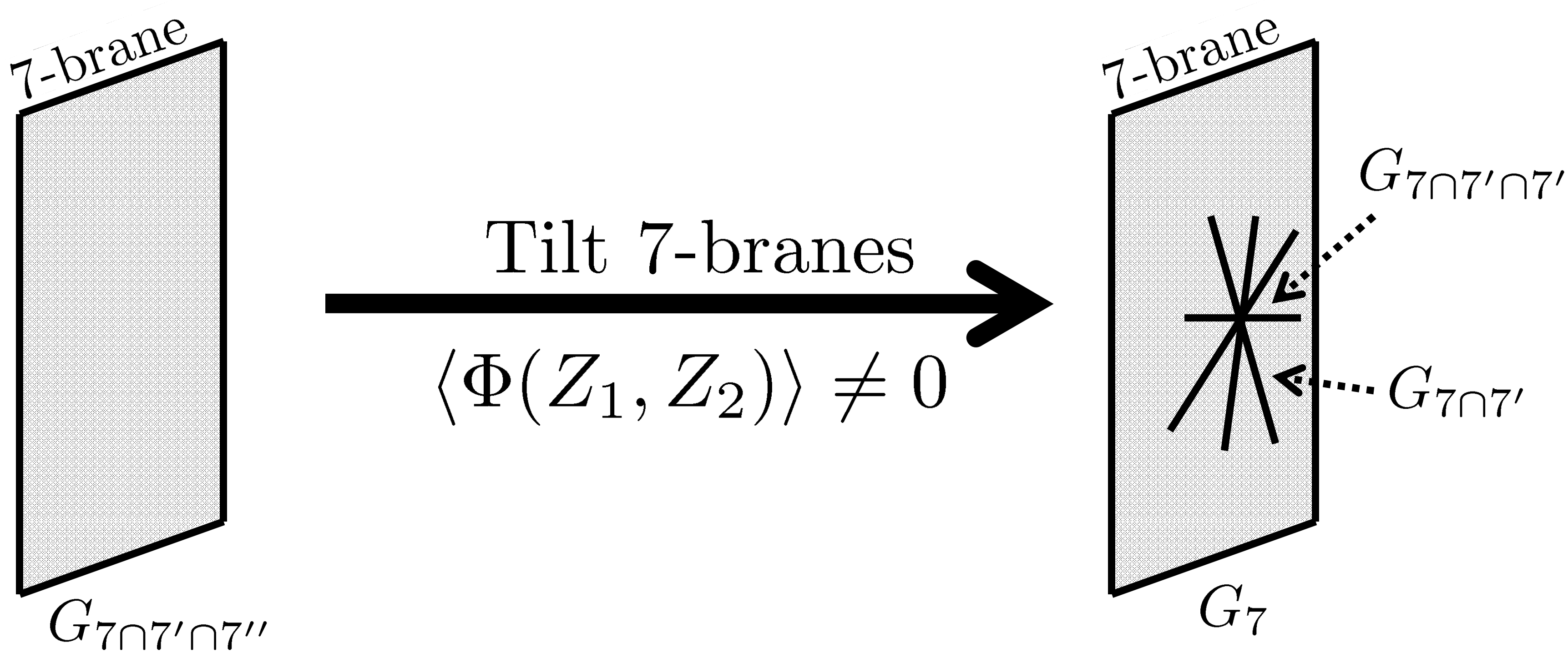
$\dim = 2 : 7 \cap 7' \Rightarrow$ Matter (non-chiral)

$\dim = 0 : 7 \cap 7' \cap 7'' \Rightarrow$ Cubic Couplings



Intersecting 7-Branes II

Locally model as $G_{7\cap 7'\cap 7''}$ 7-Brane which is tilted:



Gauge Theory Perspective

Model as 8D $G_{7\cap 7'\cap 7''}$ gauge theory

Katz Vafa '96
Beasley JJH Vafa '08
Donagi Wijnholt '08

8D EOMs:

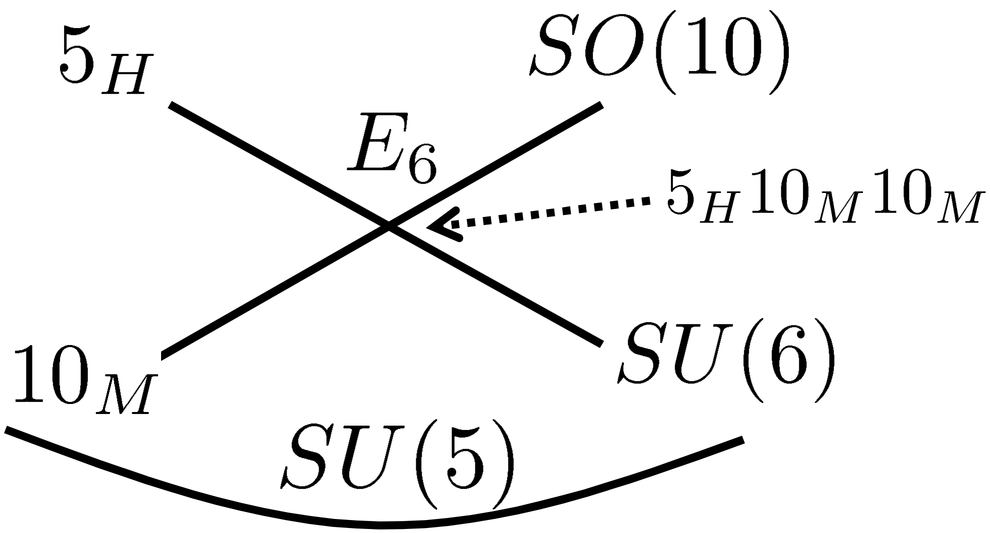
$$F^{(0,2)} = F^{(2,0)} = \overline{\partial}_A \Phi = 0$$

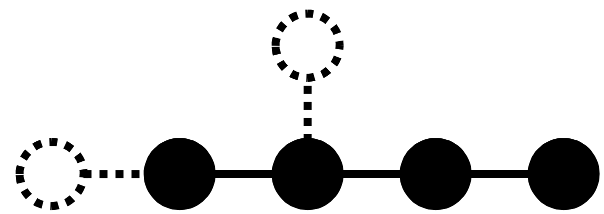
$$\omega \wedge F_{(1,1)} + \frac{i}{2}\left[\Phi, \Phi^\dagger\right] = 0$$

6D Matter: Fluctuations δA and $\delta\Phi$

4D Yukawas: $\int \delta A \wedge \delta A \wedge \delta\Phi$

Example: $5_H 10_M 10_M$ Yukawa





Locally describe as Higgsing E_6 gauge theory

$$E_6 \rightarrow SU(5) \times \underbrace{SU(2) \times U(1)}_{\langle \Phi \rangle \neq 0} \rightarrow SU(5)$$

Locally Unfolding E_8

$$\text{Eigenvalues}(\Phi(Z_1, Z_2)) = \text{7-Brane "Positions"}$$

$$E_8 \to SU(5)_{GUT} \times SU(5)_{\perp} \qquad \Phi \in SU(5)_{\perp}$$

$$b_0 \Phi^5 + b_2(Z_1, Z_2)\Phi^3 + \cdots + b_5(Z_1, Z_2) = 0$$

Hayashi et al. '09
Donagi Wijnholt '09

$$y^2 = x^3 + b_0 z^5 + b_2 x z^3 + b_3 y z^2 + b_4 x^2 z + b_5 x y$$

valid in a local patch

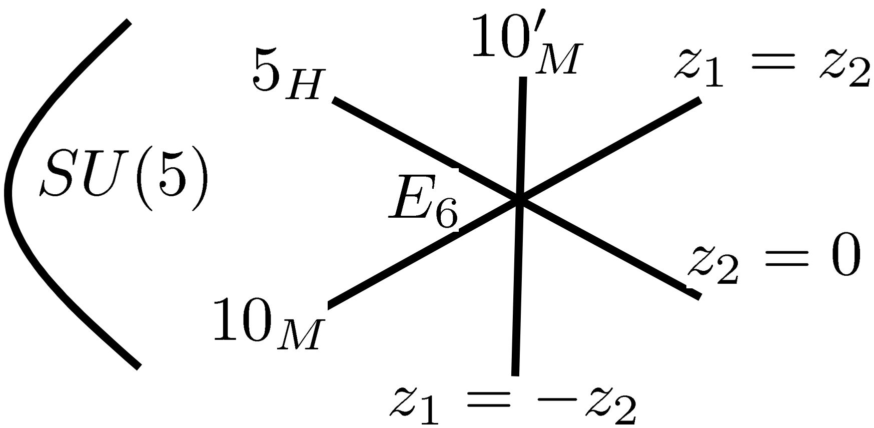
Monodromy I

Hayashi et al. '09
Cecotti Cordova JJH Vafa
(to appear)

Generically: $\mathrm{Eigen}(\Phi(Z_1, Z_2))$ has branch cuts

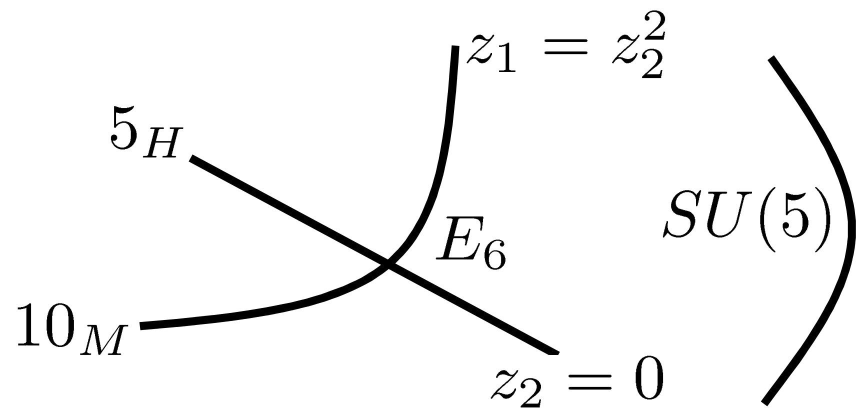
Example: Unfolding $E_6 \to SU(5) \times SU(2) \times U(1)$

$$\Phi = \# \begin{bmatrix} z_1 & \\ & -z_1 \end{bmatrix} \oplus z_2$$



$\Rightarrow$ 2 or more heavy gens.

$$\Phi = \# \begin{bmatrix} & 1 \\ z_1 & \end{bmatrix} \oplus z_2$$



$\Rightarrow$ 1 heavy gen.

Monodromy II

$$P(\Phi) = \Phi^5 + b_2(Z_1, Z_2)\Phi^3 + \cdots + b_5(Z_1, Z_2) = 0$$

Mono. group for $P(\Phi)$ affects pheno of model

$\equiv$ Galois group of $P(\Phi) = \prod(\Phi - \lambda_i)$; acts on λ_i's

S_5 Mono: 1 5-curve and 1 10-curve (not viable)

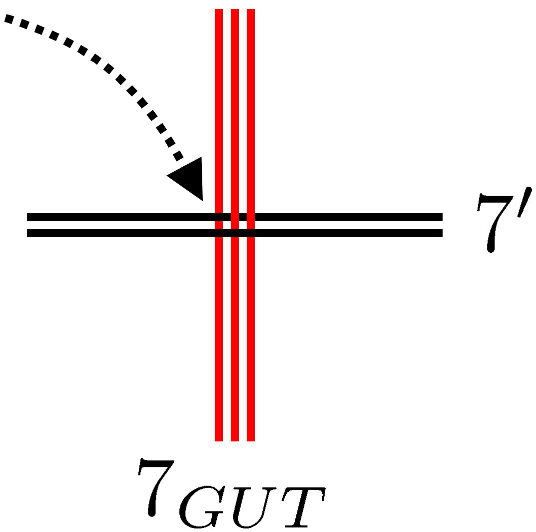
No Mono: Trouble with flavor (not viable)

Viable Cases: G_{mono} non-triv. proper s/gp of S_5

+ Flux

Flux through 7-brane ⇒ 4D chiral matter

On a curve Σ



$$(\not{D}_{4D} + \overline{\mathcal{D}}_{\Sigma})\Psi_{6D} = 0$$

Zero Modes: $\overline{\mathcal{D}}_{\Sigma}\Psi_{6D}^{(0)} = 0$

$$\boxed{\frac{1}{2\pi}\int\limits_{\Sigma} F_{7'} = N_{\text{generations}}}$$

$+$ Hyperflux

Can also turn on "hyperflux" F_Y in $U(1)_Y$

Breaks $SU(5) \to SU(3) \times SU(2) \times U(1)_Y$

Beasley JJH Vafa '08, Donagi Wijnholt '08

Distinguishes Higgs and Matter Fields:

$$\underset{\text{Higgs}}{\int F_Y \neq 0} \qquad\qquad \underset{\text{Matter}}{\int F_Y = 0}$$

Flux and Unification

F_Y splits gauge couplings at GUT scale

Donagi Wijnholt '08
Blumenhagen '08
Conlon Palti '09

$$\alpha_i^{-1} \to \alpha_{GUT}^{-1} + k_i + \text{other corrections}$$

$k_i \propto \int F_Y \wedge F_Y \sim \mathcal{O}(1)$

Same as $\mathcal{O}$(KK thresholds) and $\mathcal{O}$(2-loop MSSM) corrections

$\Rightarrow$ Can retain unification

Donagi Wijnholt '08

F-theory GUT Ingredients

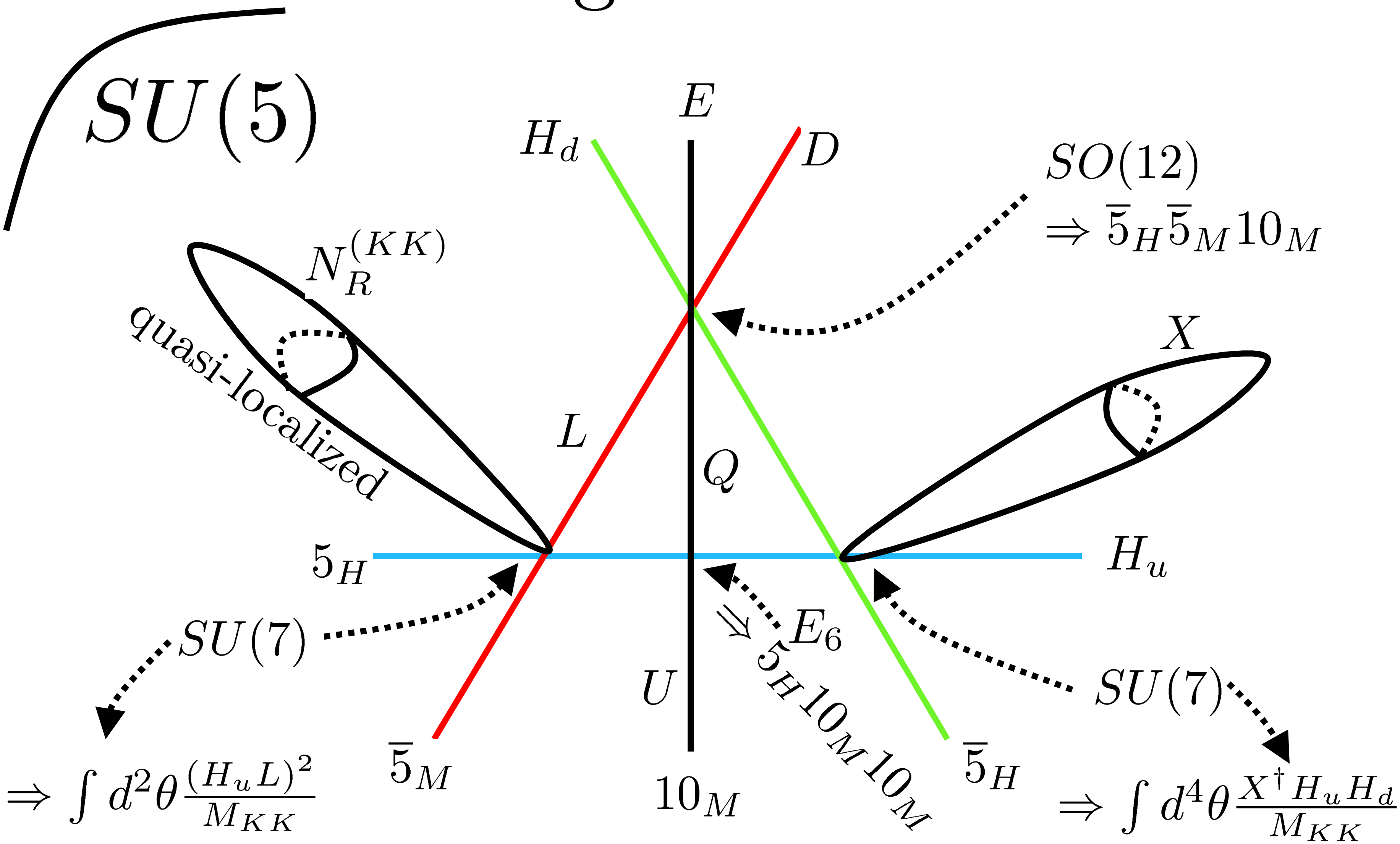
Beasley JJH Vafa ’08
Donagi Wijnholt ’08
Hayashi et al. ’08

Matter Curves for 5_H, $\overline{5}_H$, $\overline{5}_M$, 10_M

Yukawa Points for $5_H 10_M 10_M$, $\overline{5}_H \overline{5}_M 10_M$

$+\ 7'$-Brane Flux + Hyperflux + Monodromy

Rough Picture



Geometric Unification?

Aesthetic: This does not look very unified

Practical: Also has problematic phenomenology:

CKM Matrix measures wave function alignment

If no alignment, predict $\mathcal{O}(1)$ mixing

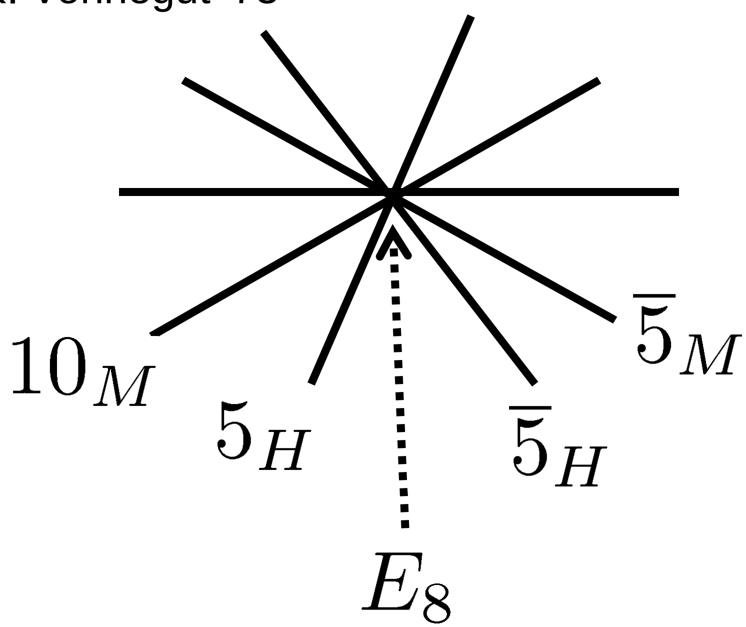
The Point of E_8

Flavor hierarchy $\Rightarrow$ keep Ψ's overlaps aligned

To keep Yukawas aligned, unify Yukawa points

JJH Vafa '08
Bouchard, JJH, Seo, Vafa '09
JJH, Tavanfar Vafa '09

K. Vonnegut '73



Nowhere to go beyond E_8

$\Rightarrow$ far less flexible than generic quiver

Fitting More In E_8

$E_8 \supset SU(5)_{GUT} \times SU(5)_{\perp}$ can accommodate:

$$248 \rightarrow (24,1) + (1,24) + (5,10) + (10,\overline{5}) + c.c.$$

Monodromy $G_{mono} \subset S_5$ removes some irreps

Can also include:

- ~~SUSY~~ sector
- Minimal gauge mediation
- Dynamically generated μ-term
- $\bullet \bullet \bullet$

Not Flexible Enough?

When E_8 gauge theory description extends

over *compact* $\mathcal{M}_4$, find tight restrictions

If no special factorization, generically "exotics"

Marsano, Saulina, Schafer-Nameki '09

But, can lift these from low energy spectrum

Important to stress that no global E_8 may exist

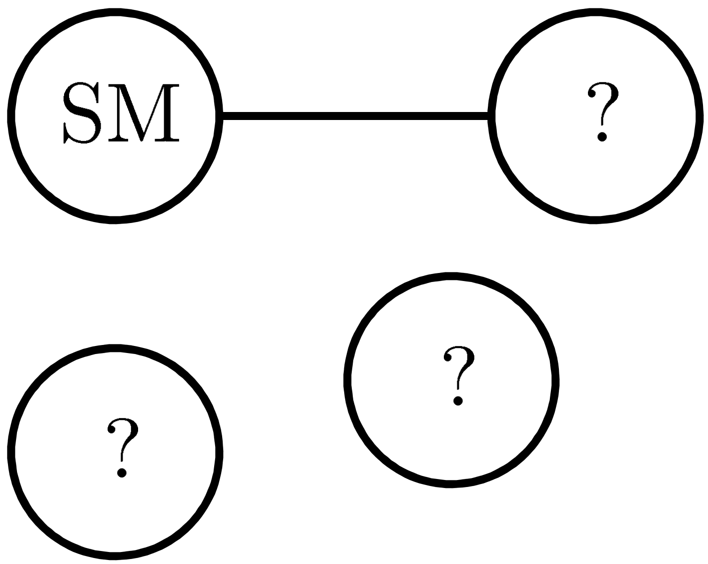
No known obstruction in general

Roadmap

- Unification in F-theory
- Extra Sectors

Extra Stuff?

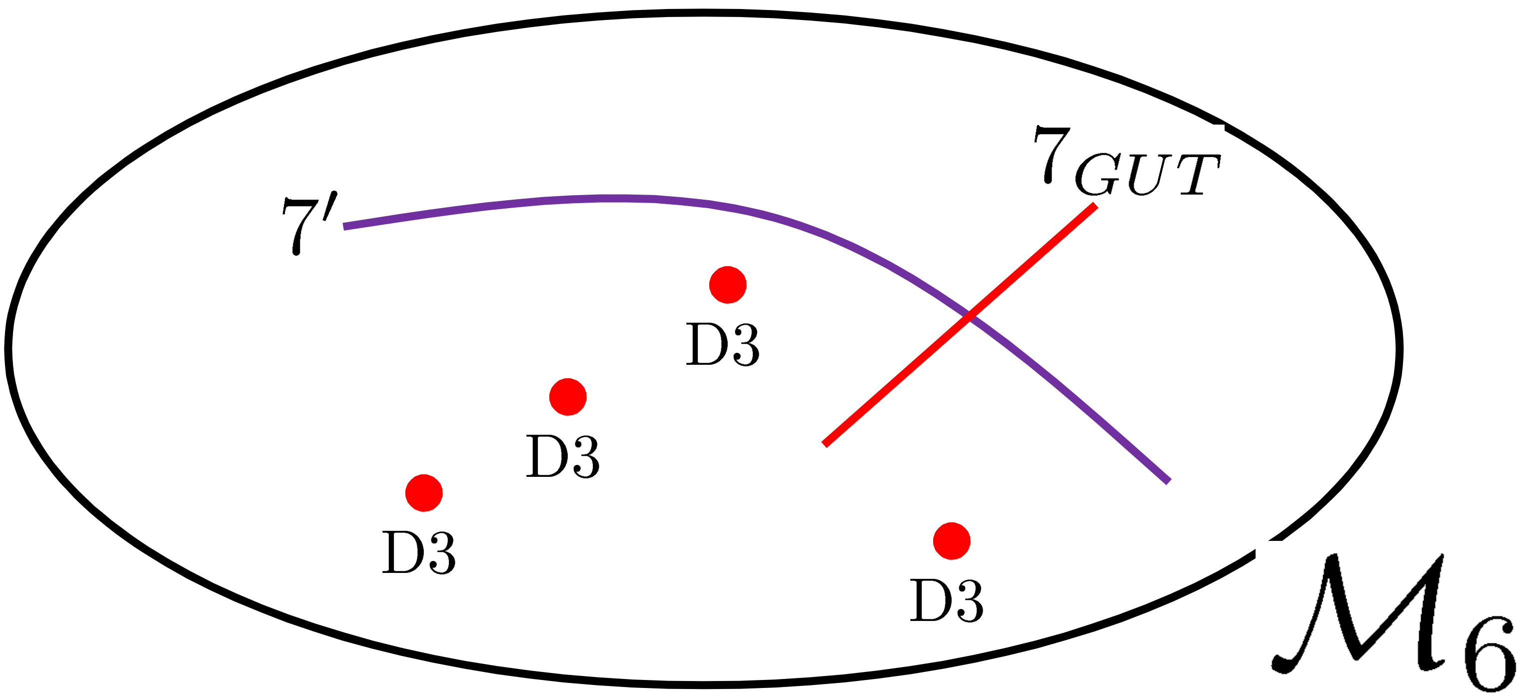
Intuition from quivers: Keep adding nodes



Where are these extra sectors?

A Hint From Global Models

No D3 Tadpole: $\frac{\chi(CY_4)}{24} = \int_{\mathcal{M}_6} H_{RR} \wedge H_{NS} + N_{D3}$



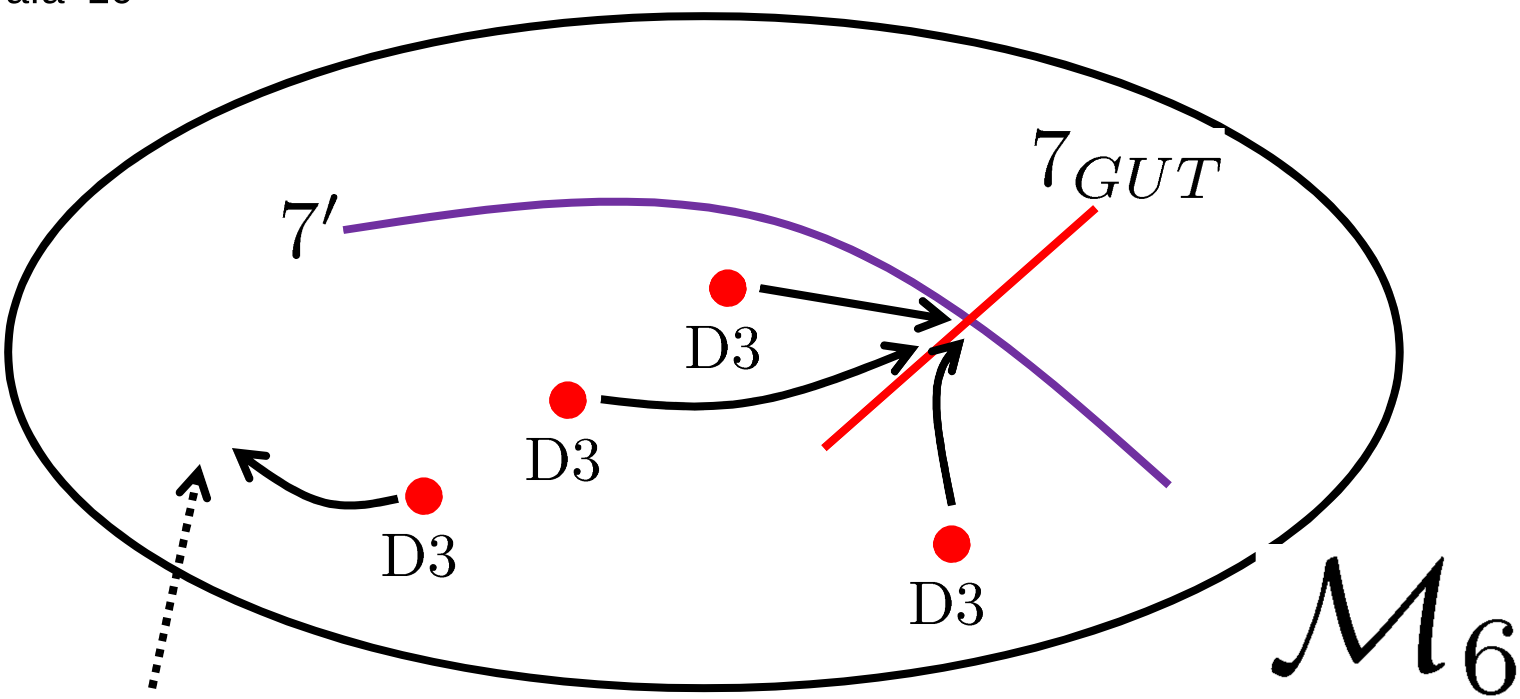
Bulk Fluxes and D3-Branes

Flux deforms D3-brane superpotential

Martucci '06; Cecotti, Cheng, JJH, Vafa '09

Flavor Physics $\Rightarrow$ Attraction to Yukawa points

JJH, Vafa '10



Generically $\exists$ other minima

D3-Branes Probing F-theory

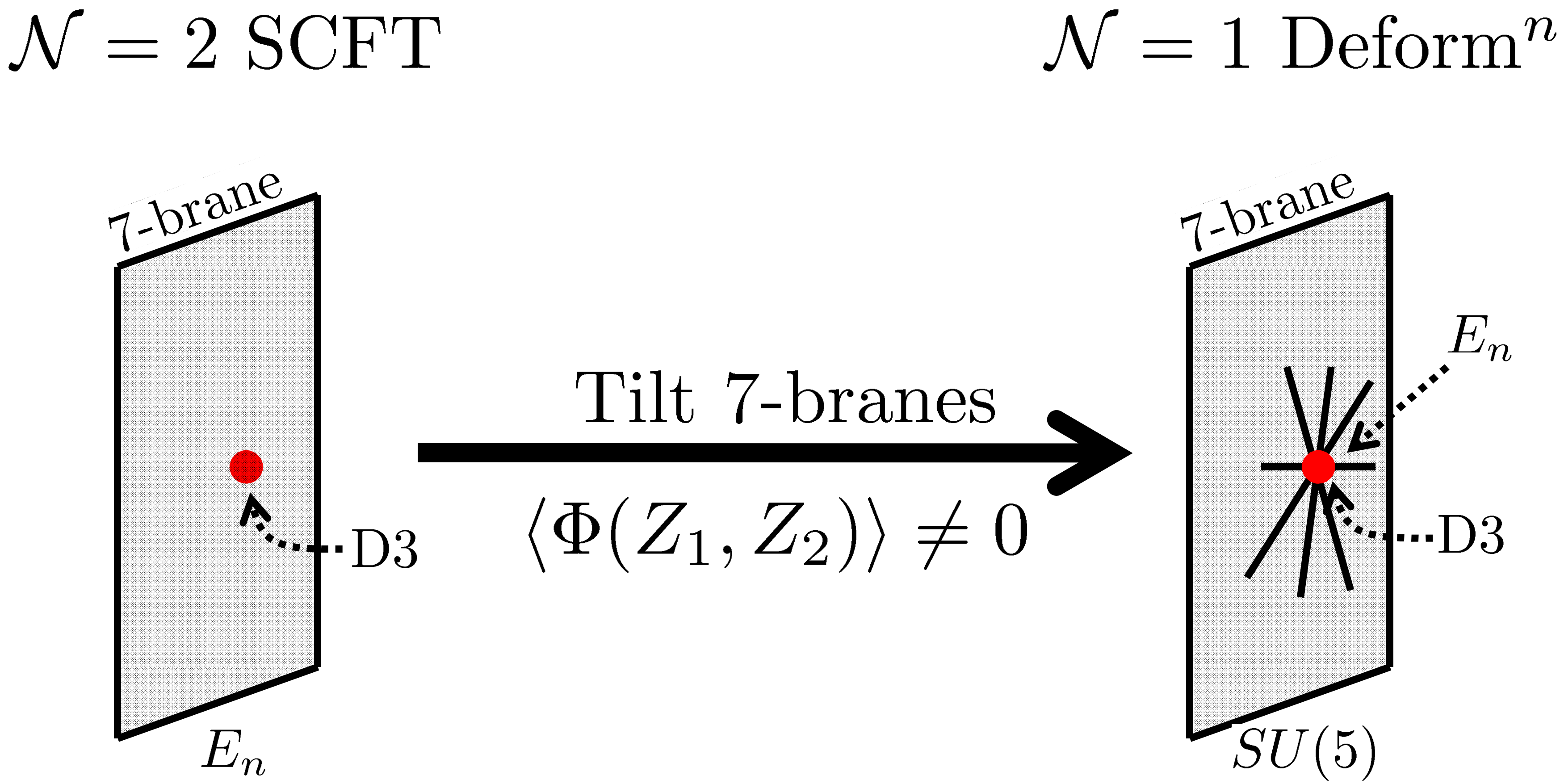
Fluxes attract D3-branes to Yukawa points

$\tau_{D3} = \tau_{IIB} \sim O(1)$ at E-points

$\Rightarrow$ Strongly coupled sector nearby SM?

What is the probe theory?

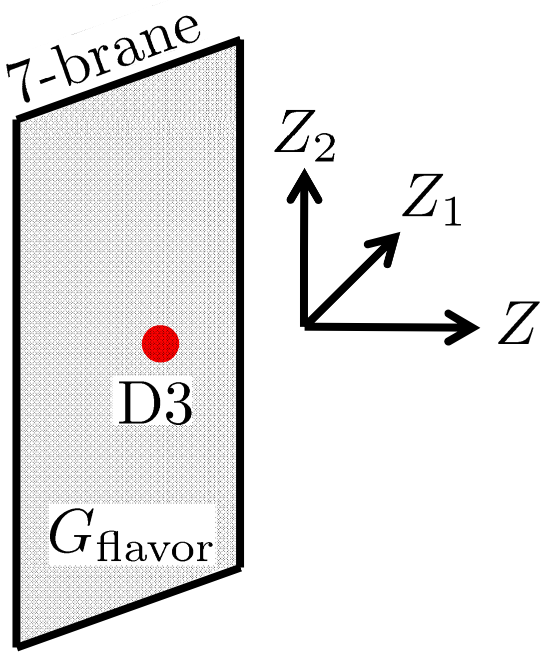
Probing an E-point



Warmup: $\mathcal{N} = 2$ Probes

D3-brane probing parallel stack of 7-branes

Banks Douglas Seiberg '96,
Douglas Lowe Schwarz '96, …



7-brane gauge group $= G_{\text{flavor}}$

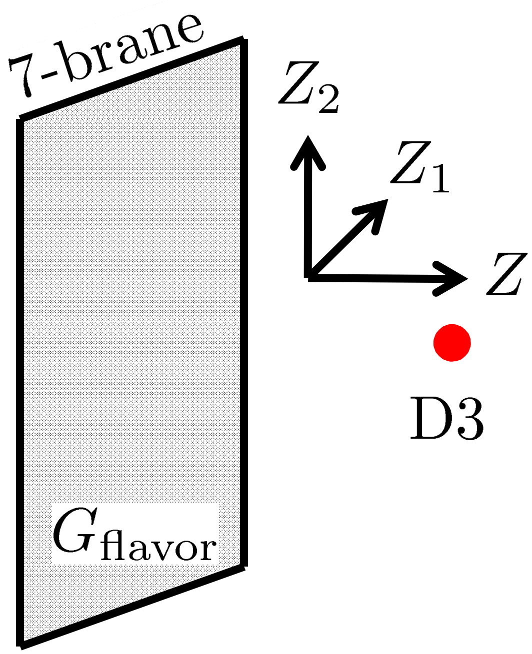
$3-3$ strings: Z_i and Z

$3-7$ string composite operators $\mathcal{O}_{\text{adj}}$

(analogue at weak coupling: $\mathcal{O} \sim Q\widetilde{Q}$)

$\mathcal{N} = 2$ Moduli Space

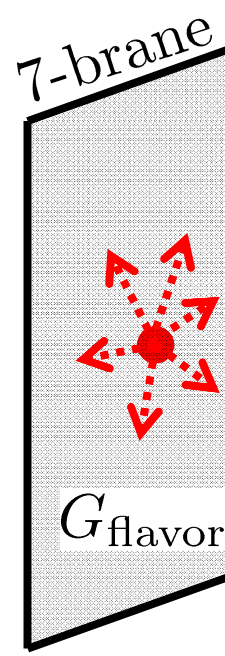
Coulomb Branch:



Move D3-brane off of 7-brane

$\langle Z \rangle \neq 0$

Higgs Branch:



Operators $\mathcal{O}$

adj. of G_{flavor}

Dissolve D3-brane as flux

$\langle \mathcal{O} \rangle \neq 0$

$\mathcal{N} = 2$ E_n Probes

Minahan-Nemeschansky: Introduce $\mathcal{N} = 2$ SCFT

Minahan Nemeschansky '96

Seiberg Witten Curves:

$$E_8 : y^2 = x^3 + z^5$$

$$E_7 : y^2 = x^3 + xz^3$$

$$E_6 : y^2 = x^3 + z^4$$

$\mathcal{N} = 2$ Seiberg-Witten Curve = F-th geometry!

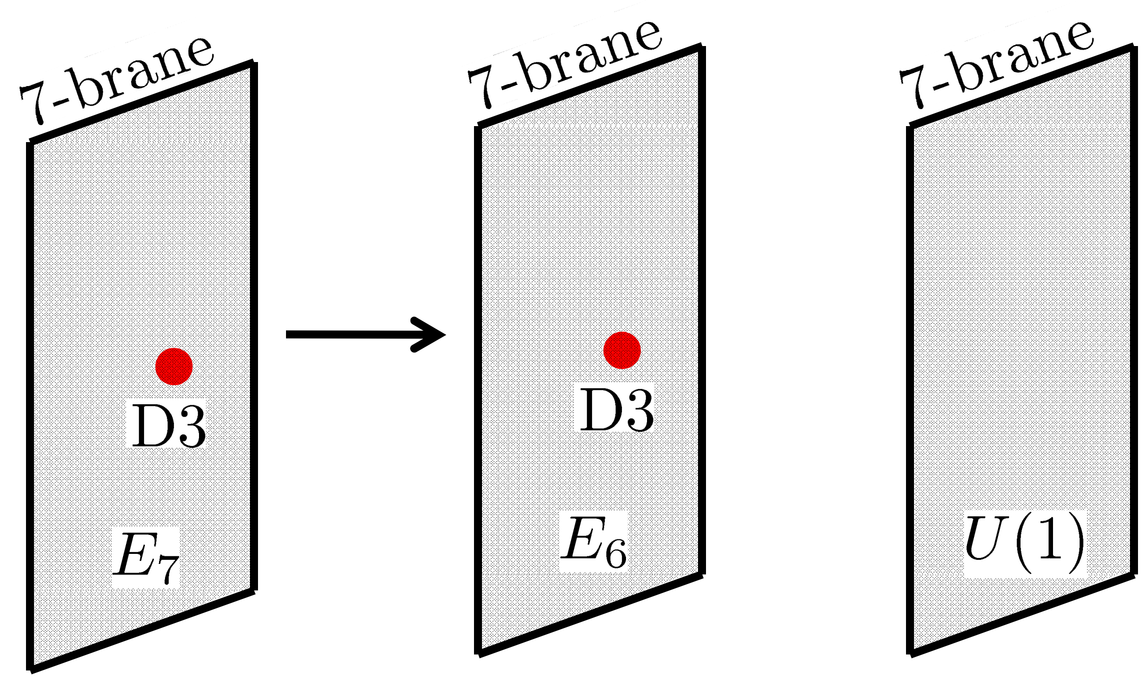
$\mathcal{N} = 2$ D3-probe = $\mathrm{MN}_{\mathcal{N}=2} \oplus (Z_1 \oplus Z_2)$

$\mathcal{N} = 2$ Deformations

Deformations: $\delta\mathcal{L} = \int d^2\theta\ \mathrm{Tr}_{E_n}(\Phi \cdot \mathcal{O}_{\mathrm{adj}}) + \mathrm{h.c.}$

Φ constant and $[\Phi, \Phi^\dagger] = 0$

Moves 7-branes:



$$\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$$

$$\delta\mathcal{L} = \int d^2\theta \ \mathrm{Tr}_{E_n}(\Phi(Z_1, Z_2) \cdot \mathcal{O}_{\mathrm{adj}}) + \mathrm{h.c.}$$

JJH Vafa ’10 (see also Aharony Kachru Silverstein ’96)

$$\text{Monodromy} \Rightarrow [\Phi, \Phi^\dagger] \neq 0$$

With Monodromy, Eigen(Φ) has branch cuts

No Branch Cuts: Flows back to $\mathcal{N} = 2$ theory

Follows from Green et al. ’10

Branch Cuts: Can flow to new $\mathcal{N} = 1$ SCFTs

JJH Tachikawa Vafa Wecht ’10

Visible Sector Couplings

$\exists$ CFT states charged under SM gauge group

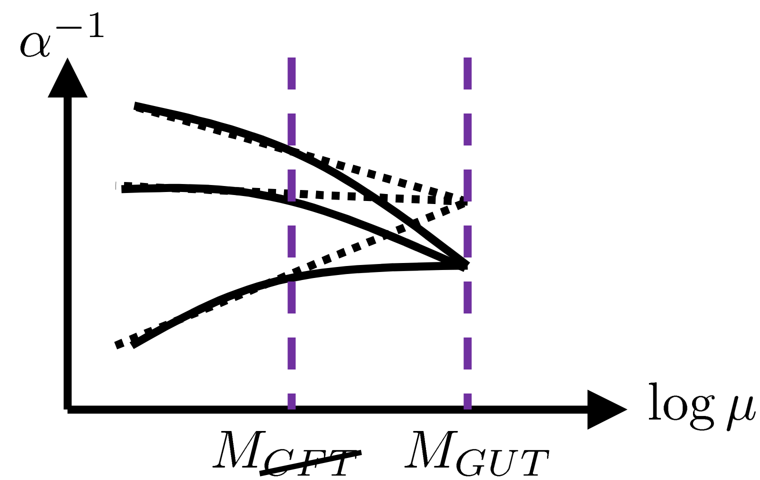
$\Rightarrow$ CFT must be broken at scale $M_{\cancel{CFT}} > M_{\text{weak}}$

Coupling to matter: $\int d^2\theta\ \Psi_R \mathcal{O}_{R^*}$

Also couples to gauge fields

irrat^l # of "particles"

$\sim$ two $5 \oplus \overline{5}$'s



Applications?

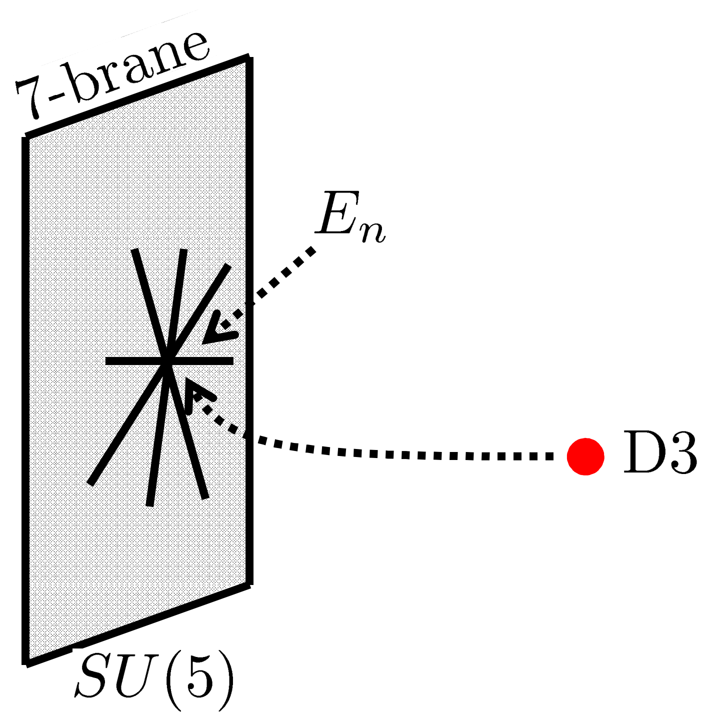
Phenomenology looks quite rich (and unexplored)

As an inflaton?

~~SUSY~~? Dark Matter?

Collider Signatures?

$+\cdots ?$



A Broader Question

D3-Brane sensitive to $\Phi(Z_1, Z_2)$, *not* $\text{Eigen}(\Phi(Z_1, Z_2))$

Locally construct CY_4 from $\text{Eigen}(\Phi(Z_1, Z_2))$

Need to keep track of non-commutativity: $[\Phi, \Phi^\dagger] \neq 0$

What is the global description of this extra data?

Roadmap

- Extra Sectors
- Fuzzy Local Models

More Non-Commutativity

Monodromy suggests a natural role

for non-commutativity: $[\Phi, \Phi^\dagger] \neq 0$

Even defining a 4D decoupling limit

with 7-branes requires $[Z_i, Z_i^\dagger] \neq 0$

JJH Verlinde '10

Revisiting Local Models

Main Premise: Geometry $\rightarrow$ 4D Field Theory

Works well for D3-brane probes (it's 4D)

But 7-Brane is 8D Theory $\Rightarrow \infty$ KK Modes?

Local Models and 7-Branes

Decoupling Limit: i) $\mathrm{Vol}(\mathcal{M}_4)_{\mathrm{closed}} \to 0$ (4D theory)

ii) $\mathrm{Vol}(\mathcal{M}_4)_{\mathrm{open}} \gg \mathrm{l}_*^4$ (weak coupling)

Condition i) $\Rightarrow$ $\mathcal{M}_4 =$ del Pezzo surface ($\mathcal{R}_{\mathcal{M}_4} > 0$)

Also requires $\mathcal{M}_6$ non-Fano ($\mathcal{R}_{\mathcal{M}_6} \not> 0$)

Cordova '09, see also Donagi Wijnholt '09; Grimm, Krause, Weigand '09

Conditions i) + ii)?

Decoupling Limit

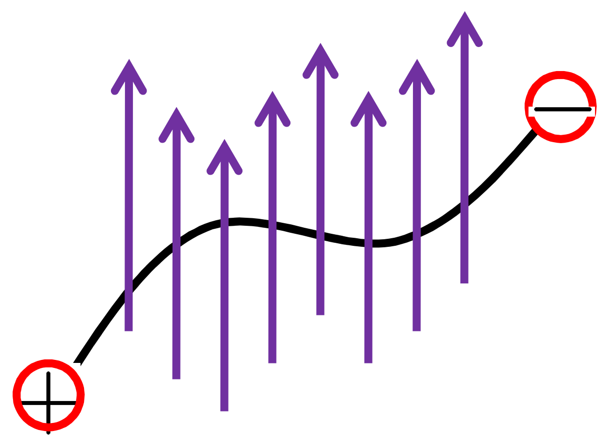
Seiberg-Witten limit:

Seiberg Witten '99

$$g_{ij}^{\text{closed}} \longrightarrow 0$$

Large $\mathcal{B} = F + B$

$\mathcal{B}$-flux spreads out string ends



Open string geometry non-commutative:

$$[Z_i, Z_i^\dagger] = \hbar_{NC}$$

c.f. Connes…

Non-Commutative Geometry

$$[Z_i, Z_i^\dagger] = \hbar_{NC}$$

	Comm. Theory	Fuzzy Theory
Points:	$p \in \mathcal{M}_4$	$\vert p\rangle \in \mathcal{H}(\mathcal{M}_4)$ Hilbert space of pts.
Curves:	$f(z_i) = 0$	$f(Z_i)\vert p\rangle = 0$
KK Modes:	Infinite	$N \times N'$ matrices

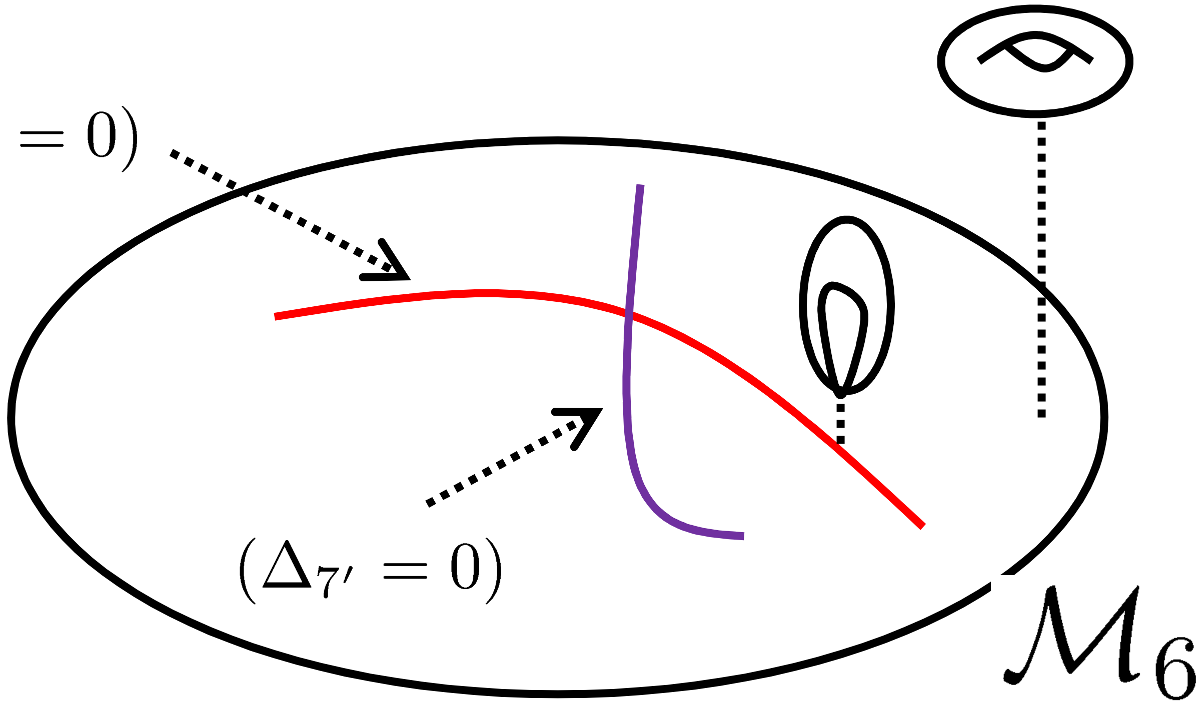
JJH
Verlinde '10

F(uzz) Theory

JJH Verlinde '10

$\mathcal{M}_4 = (\Delta_{GUT} = 0)$

Comm. Theory



Non-Comm

Quantize Coordinates: $[Z_i, Z_i^\dagger] = \hbar_{NC}$

Matter Curve inside $\mathcal{H}(\mathcal{M}_4) : \Delta_{7'}|p\rangle = 0$

4D Theory

JJH Verlinde '10

CY_4 provides template for defining 4D theories

Retains holom. data, modifies non-holom.

$$\phi(x_\mu, z, \overline{z}) \to \phi(x_\mu, Z^\dagger, Z)$$

Theory with *finite* # 4D fields $\sim N \times N$ matrices

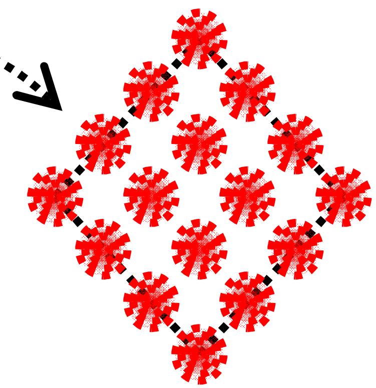
8D Lagrangian now an operator: $\mathcal{L}_{8D}(Z^\dagger, Z)$

$$\mathcal{L}_{4D} = \sum_{|p\rangle} \langle p | \mathcal{L}_{8D}(Z^\dagger, Z) | p \rangle$$

Gauge Coupling

$$\mathrm{Vol}(\mathcal{M}_4)_{\mathrm{open}} = \int \mathcal{B} \wedge \mathcal{B} = \dim \mathcal{H}(\mathcal{M}_4) = N_{\mathrm{fuzz}}$$

$\int \mathcal{B} \wedge \mathcal{B} = N_{D3}$, one per fuzzy point:



At GUT scale: $\frac{1}{\alpha_{GUT}} = \frac{N_{\mathrm{fuzz}}}{g_s} \simeq N_{\mathrm{fuzz}}$

Thresholds

At high energies KK modes become dynamical

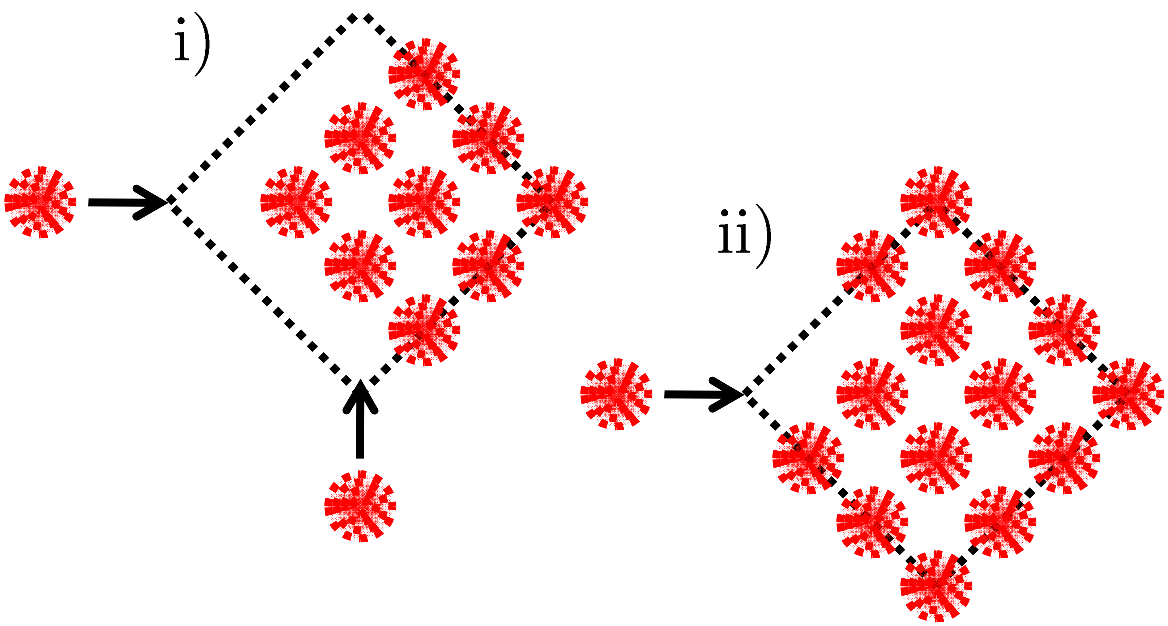
Bulk Modes: $\sim N_{\text{fuzz}} \times N_{\text{fuzz}}$ matrices

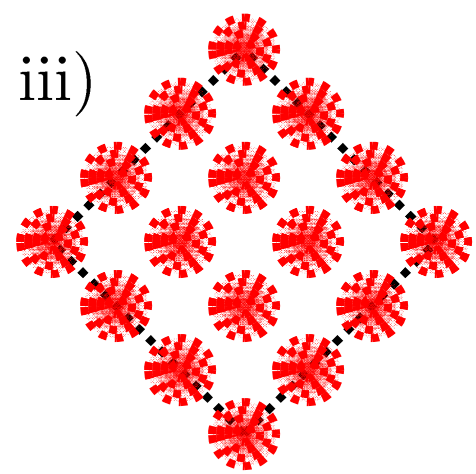
Localized Modes: $\sqrt{N_{\text{fuzz}}} \times \sqrt{N_{\text{fuzz}}}$

$1/N_{\text{fuzz}}$ expansion with $\lambda = g_{YM}^2 N_{\text{fuzz}}$

Fuzzy Unification

$\int \mathcal{B} \wedge \mathcal{B} = N_{D3} \Rightarrow$ Build up 7-brane





$SU(N_{\text{fuzz}} \times N_c) \to SU(N_c)_{\text{diag}}$

$\tau_{GUT} = \tau_{\text{diag}} = N_{\text{fuzz}} \times \tau_{D3}$

Roadmap

- Fuzzy Local Models
- Conclusions

Conclusions

- F-theory combines open strings with GUTs
- Geometric perspective on 4D Standard Model
- ¿Non-Commutativity: Uniform Description?

  $$[Z_i, Z_i^\dagger] \neq 0, \; [\Phi, \Phi^\dagger] \neq 0$$

- ¿Model building with D3-branes?