# High energy collisions of black holes


**Vítor Cardoso (CENTRA/IST & PI) • Crete • 2014**

TÉCNICO LISBOA | More at http://blackholes.ist.utl.pt | erc supports this project

# BH dynamics

Brito, Fujita, Nerozzi, Okawa, Pani, Rocha (Sperhake, Witek, Zilhão)

Barausse, Berti, Emparan, Gualtieri, Khanna, Herdeiro, Mateos, Pretorius, Yunes

*(Cardoso, Gualtieri, Herdeiro, Sperhake, Living Reviews in Relativity (2014))*

* * *

*PRL 103:131102 (2009)*

*103:239001 (2009)*

*105:261102 (2010)*

*107:031101 (2011)*

*109:131102 (2012)*

*111:041101 (2013)*

*111:111101 (2013)*

*PRD 81:084052 (2010)*

*81:104048 (2010)*

*82:104014 (2010)*

*83:024037 (2011)*

*83:104048 (2011)*

*85:124062 (2012)*

*87:124020 (2013)*

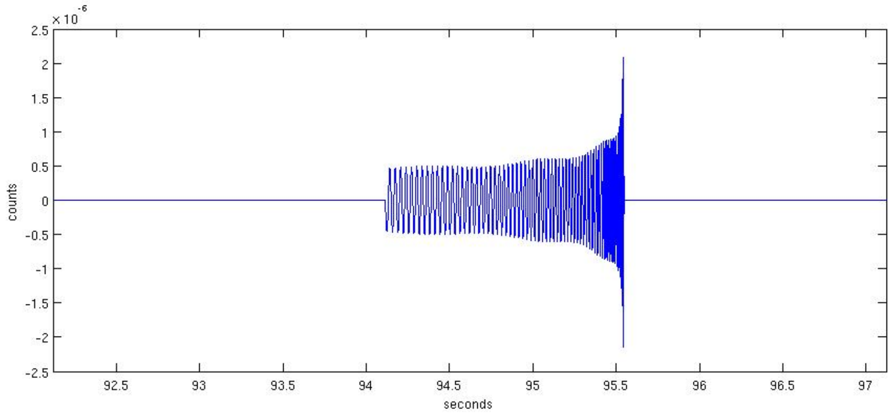
# Why study dynamics

Gravitational-wave detection, GW astrophysics

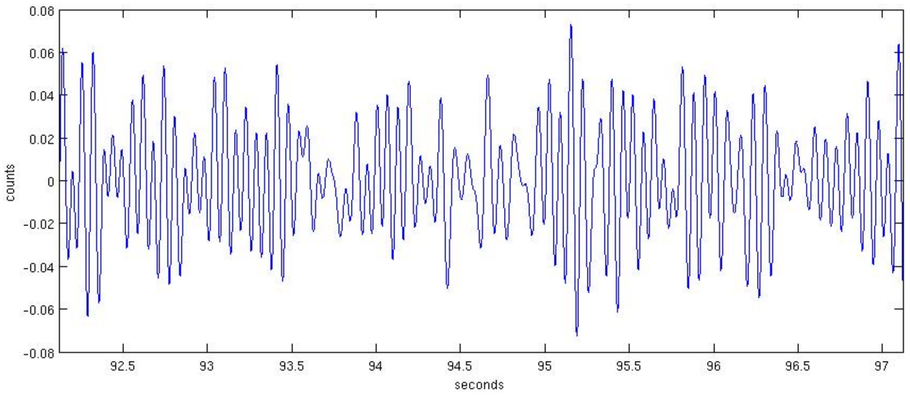
Mathematical physics

High-energy physics

# Typical signal for coalescing binaries




# Typical stretch of data



# A needle in a haystack!..burn the hay?

$$(h_1|h_2) \equiv 2\int_0^\infty \frac{\tilde{h}_1^*(f)\tilde{h}_2(f) + \tilde{h}_1(f)\tilde{h}_2^*(f)}{S_h(f)}\,df$$

$$\rho_{\mathrm{MF}} = \max_{\vec{\lambda}} \frac{(T\{\vec{\lambda}\}|h)}{\sqrt{(T\{\vec{\lambda}\}|T\{\vec{\lambda}\})}}$$

# Why study dynamics

Gravitational-wave detection, GW astrophysics

Mathematical physics

High-energy physics

Cosmic Censorship: do horizons always form?

✷

Are black objects always stable? Phase diagrams...

✷

Universal limit on maximum luminosity c^5/G *(Dyson ‘63)*

✷

Critical behavior,  resonant excitation of QNMs?

✷

Test analytical techniques, their predictions and power

*(Penrose ’74, D’Eath & Payne ’93, Eardley & Giddings ’02)*

# Why study dynamics

Gravitational-wave detection, GW astrophysics

Mathematical physics

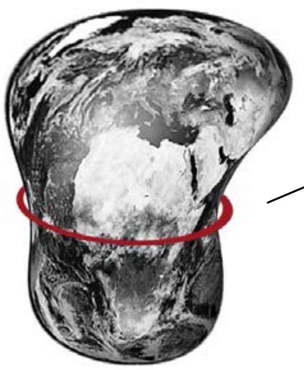
High-energy physics

# Hoop Conjecture

*(Thorne 1972)*

"An imploding object forms a BH when, and only when, a circular hoop with circumference $2\pi$ the Schwarzschild radius of the object can be made that encloses the object in all directions."

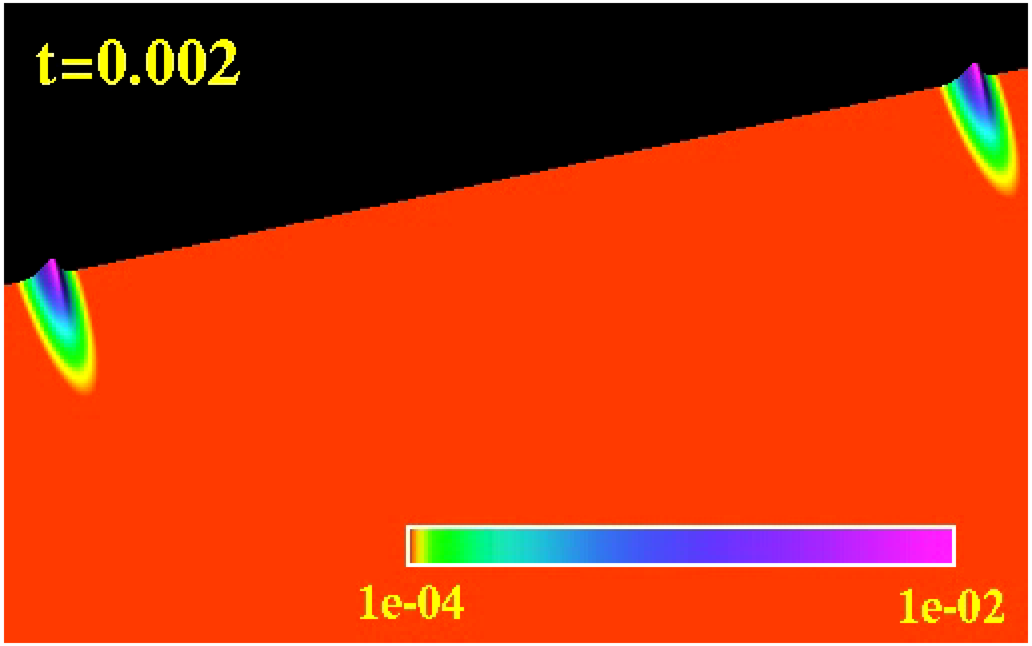
**Large amount of energy in small region**



This is the hoop
$R=2GM/c^2$

Size of electron: 10^(-17) cm
Schwarzschild radius: 10^(-55) cm

$$2M/R = 1/20 \Longrightarrow \gamma_{\rm crit} \sim 10$$




(*Choptuik & Pretorius 2010, East & Pretorius 2012, Rezzolla & Takami 2012*)

Black holes do form in high energy collisions

*also Choptuik & Pretorius, 2010*

Transplanckian scattering well described by BH collisions! (?)

How to go around and study BH collisions?

# ZFL

*(Weinberg '64; Smarr '77)*

Take two free particles, changing abruptly at t=0

$$T^{\mu\nu} = \sum_{i=1,2} \frac{P_i^\mu P_i^\nu}{E_i} \delta^3(x - v_i t)\theta(-t) + \frac{P_i^{'\mu} P_i^{'\nu}}{E_i'} \delta^3(x - v_i' t)\theta(-t)$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}\,, \qquad h_{\mu\nu} = 4\int \frac{T_{\mu\nu}(t_r, x') - 1/2\eta_{\mu\nu}(t_r, x')}{|x' - x|} d^3x'$$

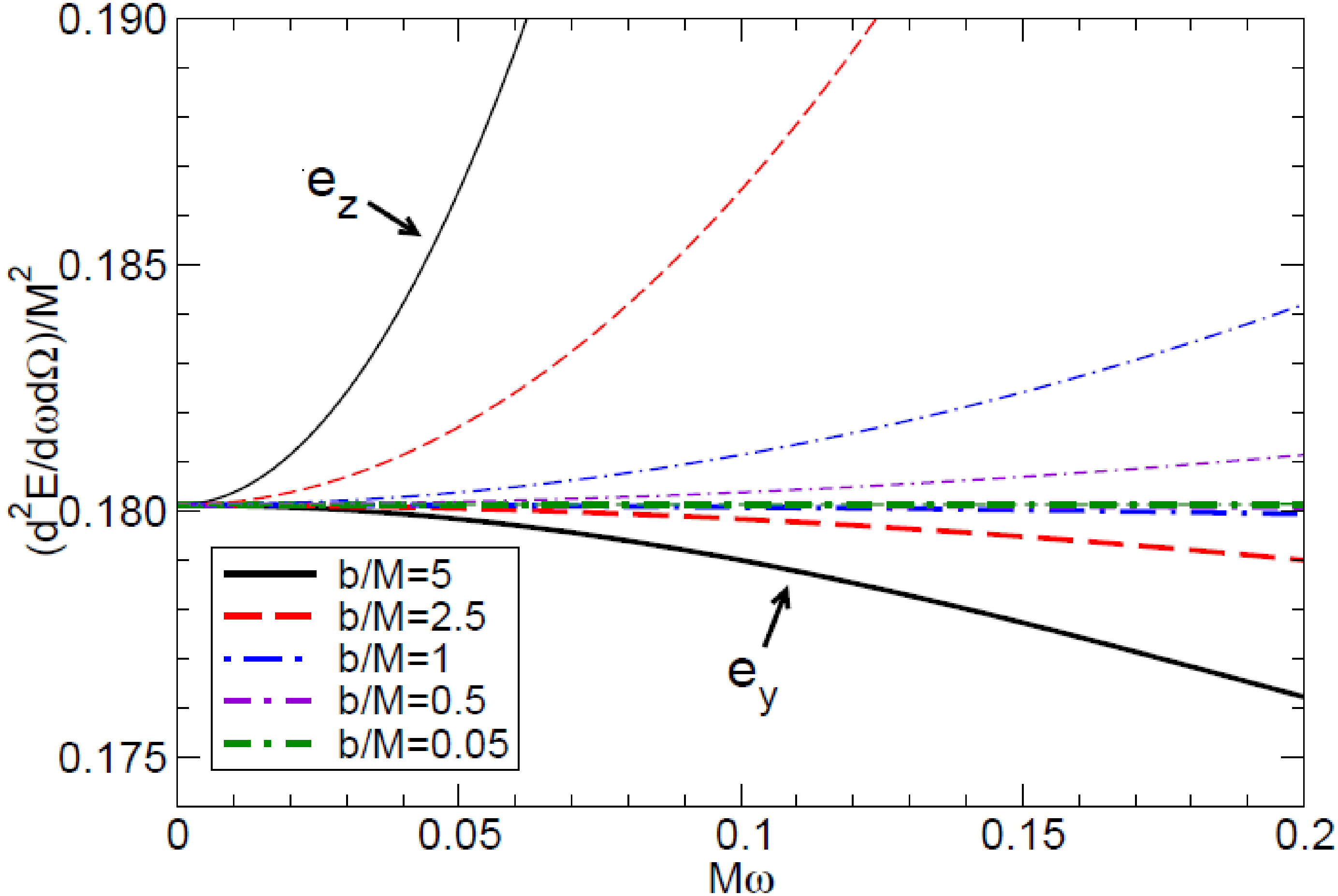
$$\frac{dE}{d\omega d\Omega} = \frac{M^2\gamma^2 v^4}{\pi^2} \frac{\sin^4\theta}{(1 - v^2\cos^2\theta)^2}$$

Radiation isotropic in the UR limit, multipole structure $E_l \propto \frac{1}{l^2}$

Functional relation $E_{rad}(\gamma)$, flat spectrum

Roughly 65% of maximum possible at $\gamma=3$

With cutoff M $\omega \approx 0.4$ we get 25% efficiency for conversion of gws




*(M. Lemos, MSc 2010; Berti et al 2010)*

# Perturbation theory

(*Regge & Wheeler '57; DRPP '71; Cardoso & Lemos '02*)

Metric=Schwarzschild + small perturbation due to infalling particle

$$\tilde{h}_{\mu\nu} = \begin{bmatrix} H_0(r)f & H_1(r) & 0 & 0 \\ H_1(r) & H_2(r)/f & 0 & 0 \\ 0 & 0 & r^2K(r) & 0 \\ 0 & 0 & 0 & r^2K(r)\sin^2\theta \end{bmatrix} Y_{l0}(\theta)$$

$$\frac{d^2\Psi}{dr_*^2} + \left(w^2 - V\right)\Psi = \left(1 - \frac{2M}{r}\right) S(\omega, r)$$

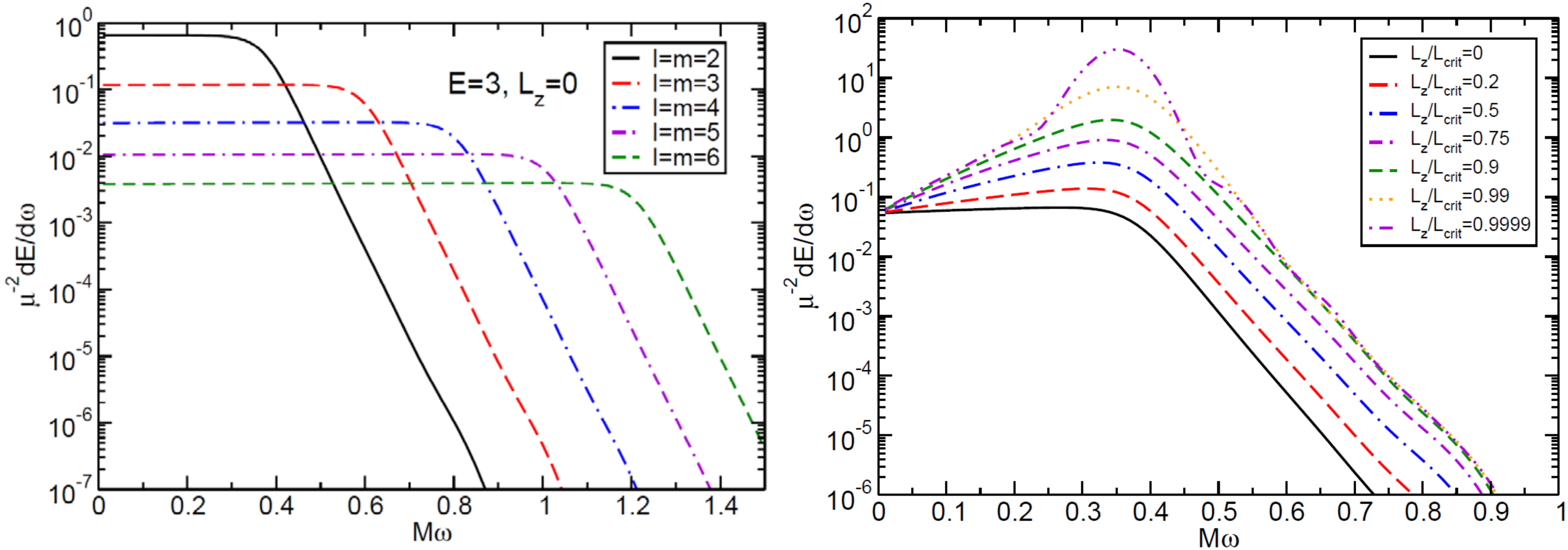
$$\frac{V}{1-2M/r} = \frac{2\lambda^2(\lambda+1)r^3 + 6\lambda^2 r^2 M + 18\lambda r M^2 + 18M^3}{r^3(3M+\lambda r)^2}$$

$$S(\omega, r) = \frac{4i\mu\lambda\sqrt{4\ell+2}}{\omega(3M+\lambda r)^2} e^{-i\omega r_*}\,, \qquad \lambda \equiv \frac{(\ell-1)(\ell+2)}{2}$$

| Velocity | | Radiated energy | Area theorem |
|---|---|---|---|
| 0 | Point particle | $E_{\rm rad} = 0.010\frac{\mu^2}{M}$ | $E_{\rm rad} < \frac{\mu^2}{M}$ |
| | Equal mass | $E_{\rm rad} = 0.00065M$ | $E_{\rm rad} < 0.29M$ |
| 1 | Point particle | $E_{\rm rad} = 0.26\frac{(\mu\gamma)^2}{M}$ | $E_{\rm rad} < \frac{(\mu\gamma)^2}{M}$ |
| | Equal mass | $E_{\rm rad} = 0.13M$ | $E_{\rm rad} < M$ |

*dE/dω flat at sufficiently low ω, multipole* $E_l \propto \frac{1}{l^2}$

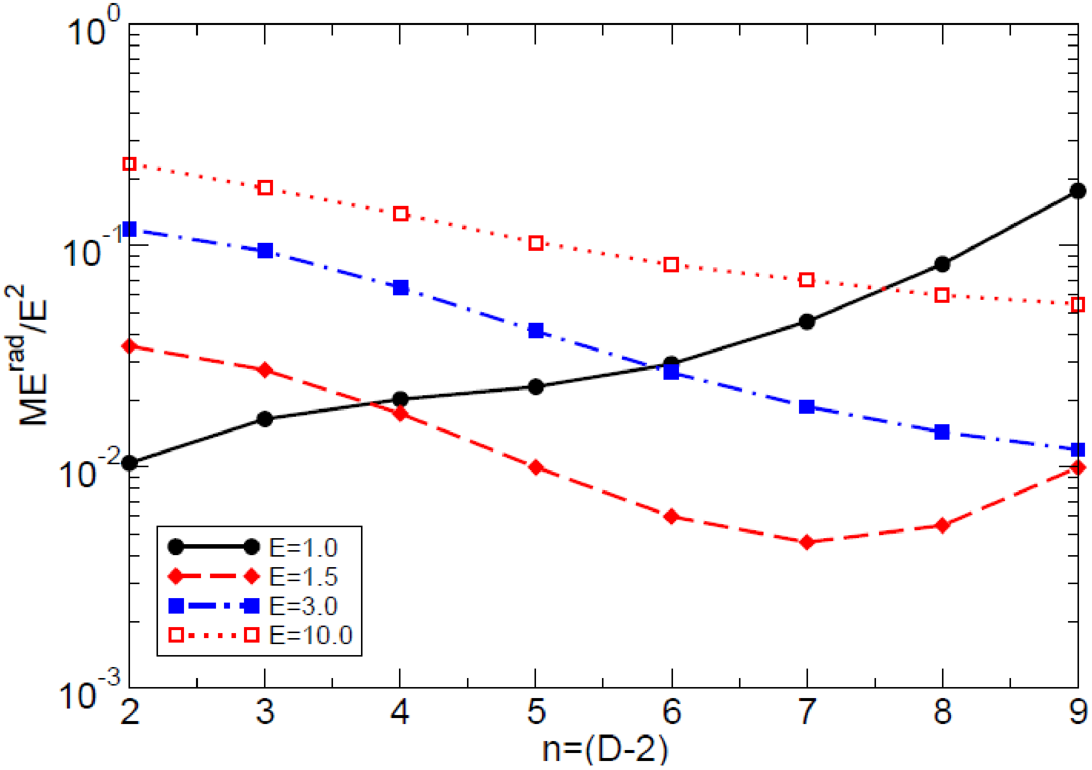
*Roughly 65% of maximum possible at γ=3*




*(Berti et al, 2010)*

# Generic D

(*Kodama & Ishibashi '03; Berti, Cardoso & Kipapa, 2010*)




v=1: $dE/d\omega \propto \omega^{D-4}$ *agrees with ZFL*

# Trapped surface formation

*(Penrose '74; Eardley & Giddings '02; Kohlprath & Veneziano '03)*

Superpose two Aichelburg-Sexl metrics, find future trapped surface

Upper limit on gravitational radiation: 29% M

Perturb superposed A-S metric, correction: 16% M

*(D'Eath & Payne '90s)*

# Numerical evolution

GR: “Space and time exist together as Spacetime”

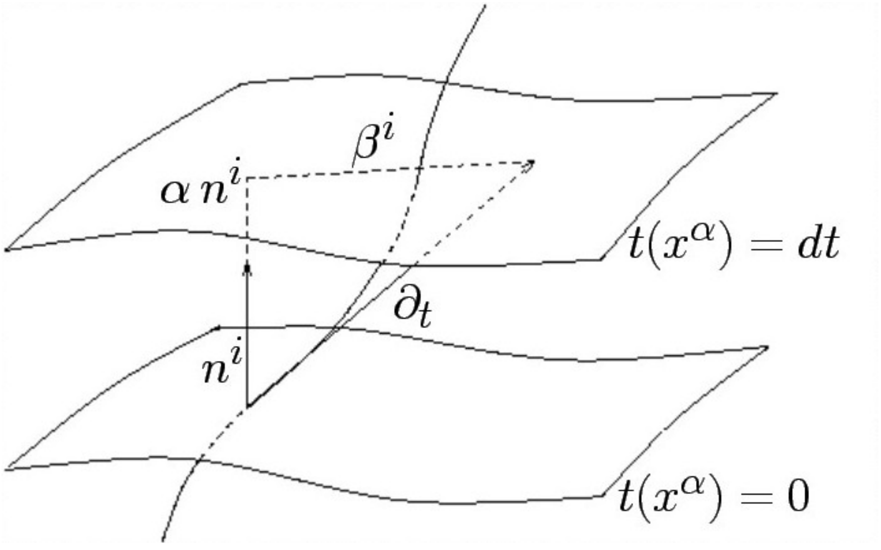
Numerical relativity: reverse this process! (*Okawa’s talk)*

ADM 3+1 decomposition

*Arnowitt, Deser, Misner (1962); York (1979); Choquet-Bruhat, York (1980)*

| | |
|---|---|
| 3-metric | $\gamma_{ij}$ |
| lapse | $\alpha$ |
| shift | $\beta^i$ |

lapse, shift $\Rightarrow$ Gauge



$$
\begin{aligned}
\partial_t \gamma_{IJ} &= \beta^M \partial_M \gamma_{IJ} + \gamma_{MJ}\partial_I \beta^M + \gamma_{IM}\partial_J \beta^M - 2\alpha K_{IJ}\,, \\
\partial_t K_{IJ} &= \beta^M \partial_M K_{IJ} + K_{MJ}\partial_I \beta^M + K_{IM}\partial_J \beta^M - D_I D_J \alpha + \alpha\left(\mathcal{R}_{IJ} + K K_{IJ} - 2K_{IM}K^M{}_J\right) \\
&\quad + 8\pi\alpha\left(\frac{S-\rho}{D-2}\gamma_{IJ} - S_{IJ}\right) - \frac{2}{D-2}\alpha\Lambda\gamma_{IJ}, \\
0 &= \mathcal{R} + K^2 - K^{MN}K_{MN} - 2\Lambda - 16\pi\rho, \\
0 &= D_I K - D_M K^M{}_I + 8\pi j_I\,.
\end{aligned}
$$

## Brill-Lindquist:

$$
K_{IJ} = 0\,, \quad \gamma_{IJ}dx^I dx^J = \psi^{4/(D-3)}\delta_{IJ}\,, \quad \psi = 1 + \sum_A \frac{\mu_A^{D-3}}{4\left[\sum_{K=1}^{D-1}(x^K - x_0^K)^2\right]^{(D-3)/2}}
$$

## Bowen-York:

$$
\gamma_{ij} = \psi^4 \bar{\gamma}_{ij}
$$

$$
K_{ij} = A_{ij} + \frac{1}{3}\gamma_{ij}K\,, \quad A^{ij} = \psi^{-10}\bar{A}_{ij} \quad \Leftrightarrow \quad A_{ij} = \psi^{-2}\bar{A}_{ij}
$$

$$
\bar{A}_{ij} = \frac{3}{2r^2}\left[P_i n_j + P_j n_i - (f_{ij} - n_i n_j)P^k n_k\right] + \frac{3}{r^3}\left(\epsilon_{kil}S^l n^k n_j + \epsilon_{kjl}S^l n^k n_i\right)
$$

## LEAN code (*Sperhake '07)*

Based on the Cactus computational toolkit

BSSN formulation (ADM-like, but strongly hyperbolic)

Puncture initial data (*Brandt & Brügmann 1996)*

Elliptic solver: TwoPunctures (*Ansorg 2005)*

Mesh refinement: Carpet (*Schnetter '04)*

## Numerically very challenging!

Length scales: $M_{ADM} \propto \gamma M_0$

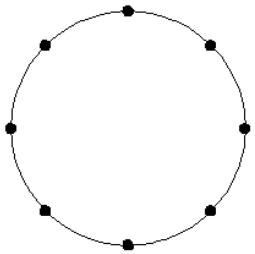
Horizon Lorentz-contracted "Pancake"

Mergers extremely violent

Substantial amounts of unphysical "junk" radiation

Most important result: Emitted gravitational waves (GWs)

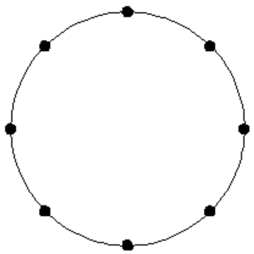
Newman-Penrose scalar:



$\Psi_4 = C_{\alpha\beta\gamma\delta}\, n^\alpha\, m^\beta\, n^\gamma\, m^\delta$

Complex ⟶ 2 free functions

GWs allow us to measure:



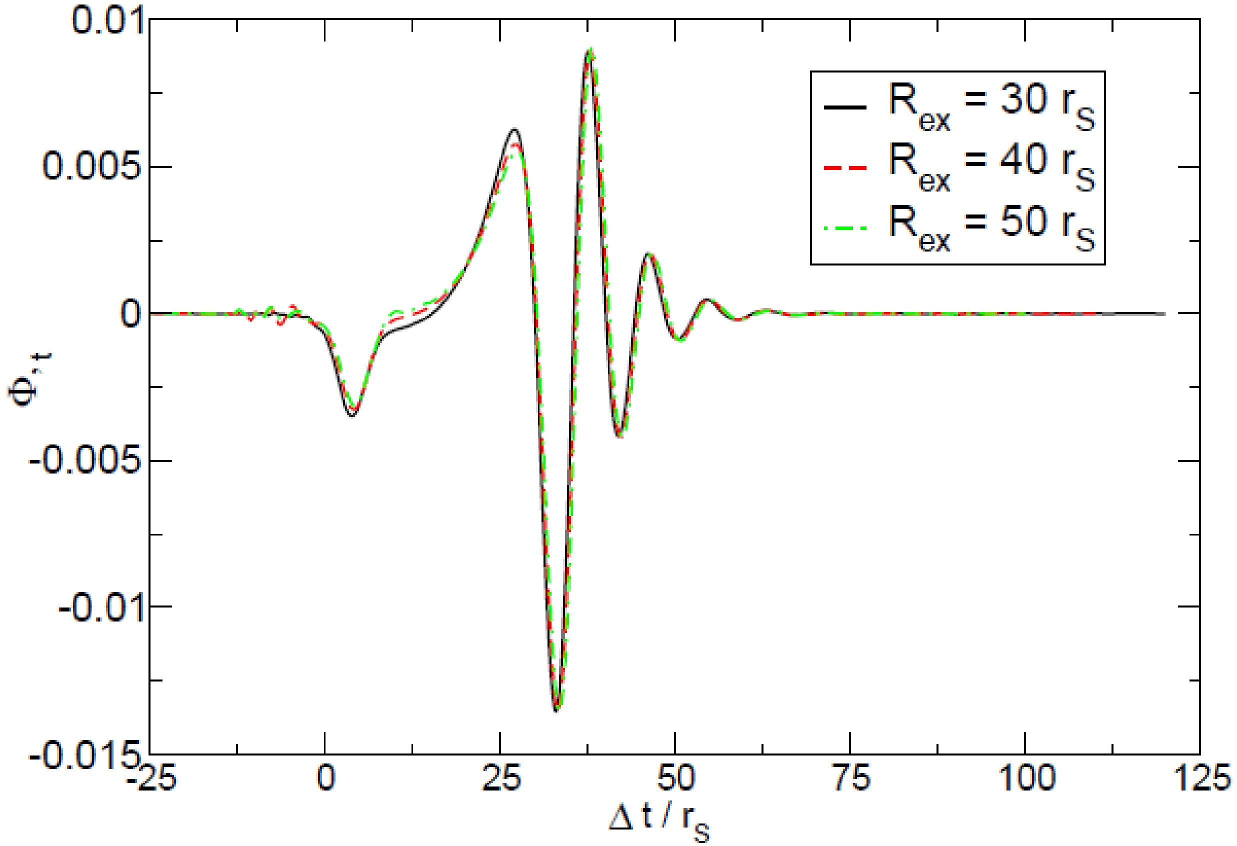
Radiated energy $E_{rad}$

Radiated momenta $P_{rad}$, $J_{rad}$

Angular dependence of radiation

Predicted strain $h_+$, $h_x$

# Rest

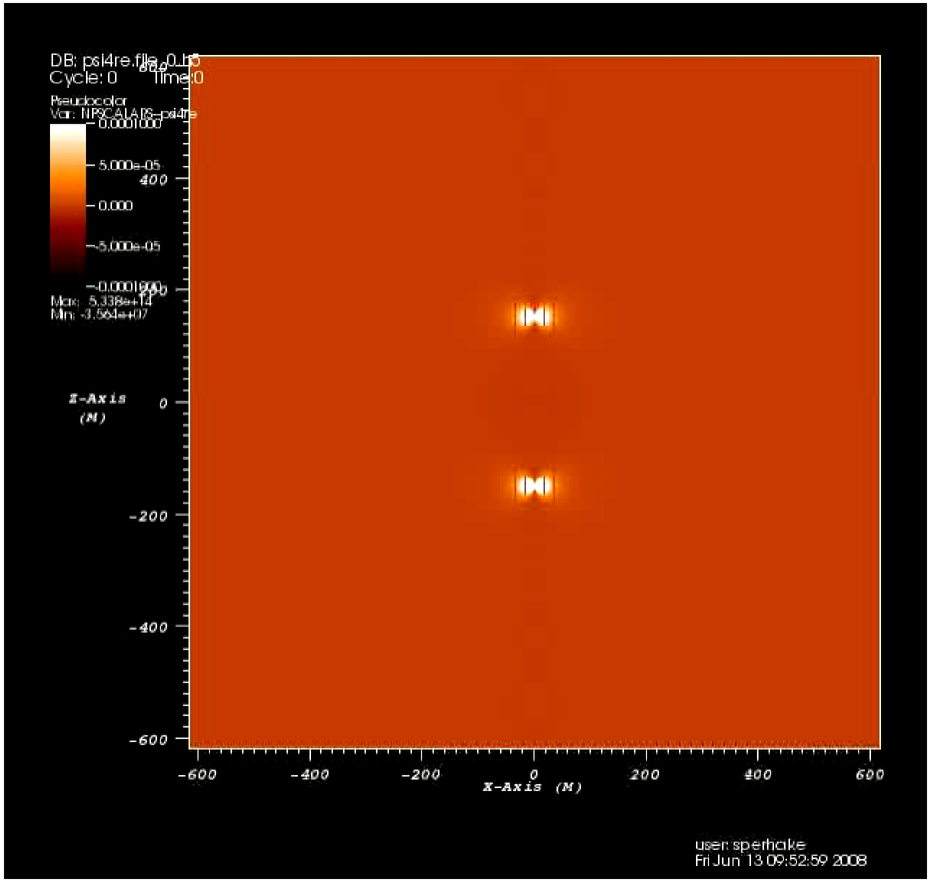



$$E_{\rm rad} = 0.00057M$$

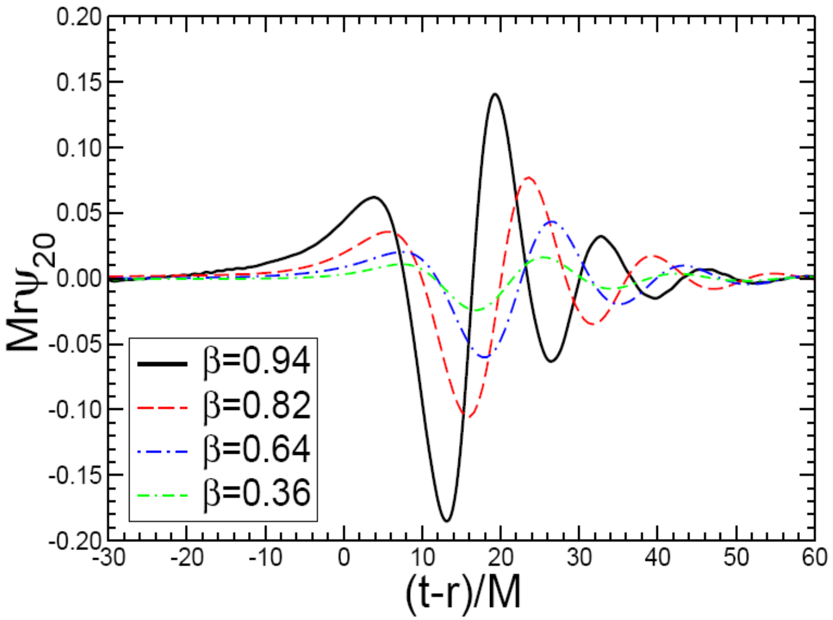
*(Witek et al, arXiv:1006.3081 [gr-qc])*

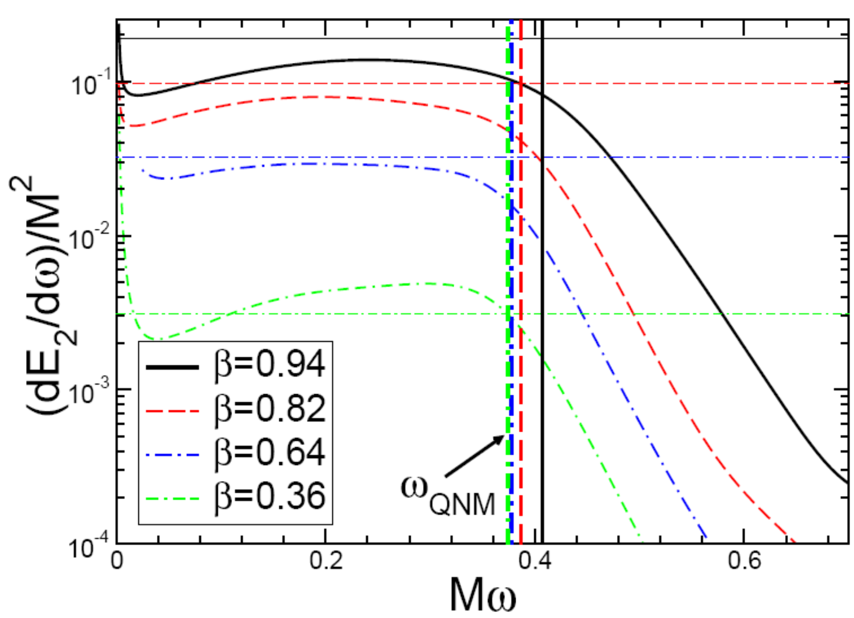
# High energy head-ons

$\beta=0.93$




*(Sperhake et al 2008)*

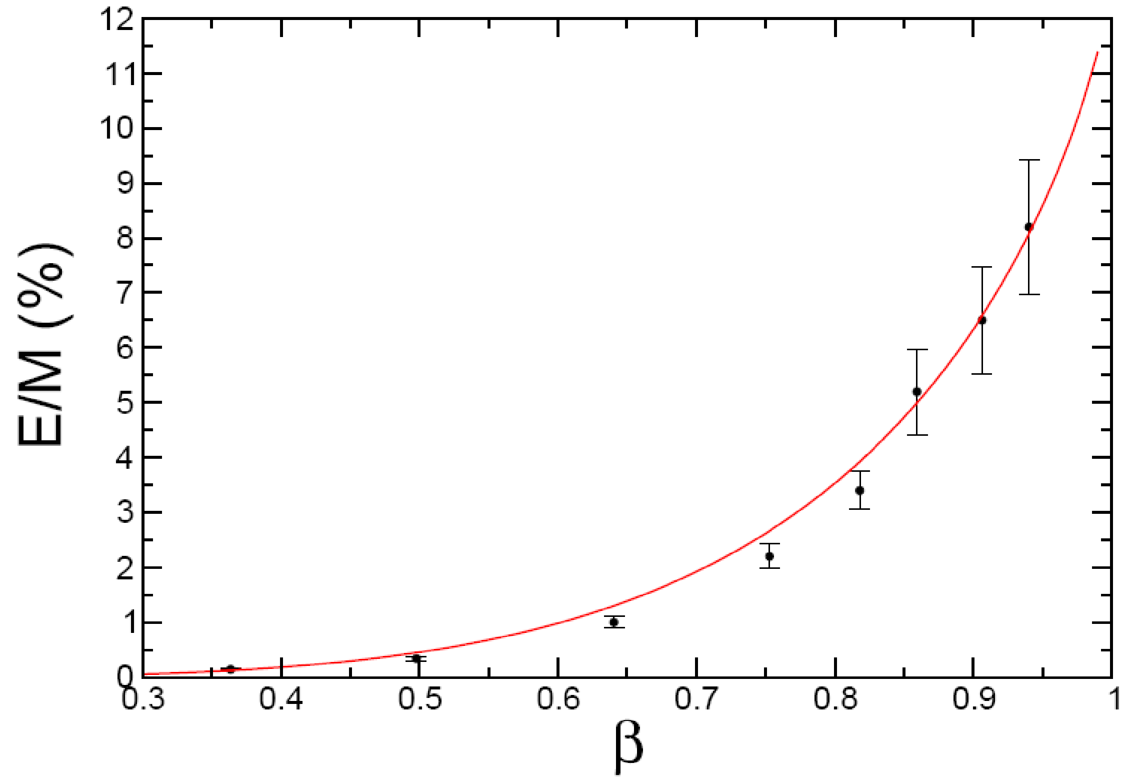



Waveform is almost just ringdown

Spectrum is flat, in good agreement with ZFL

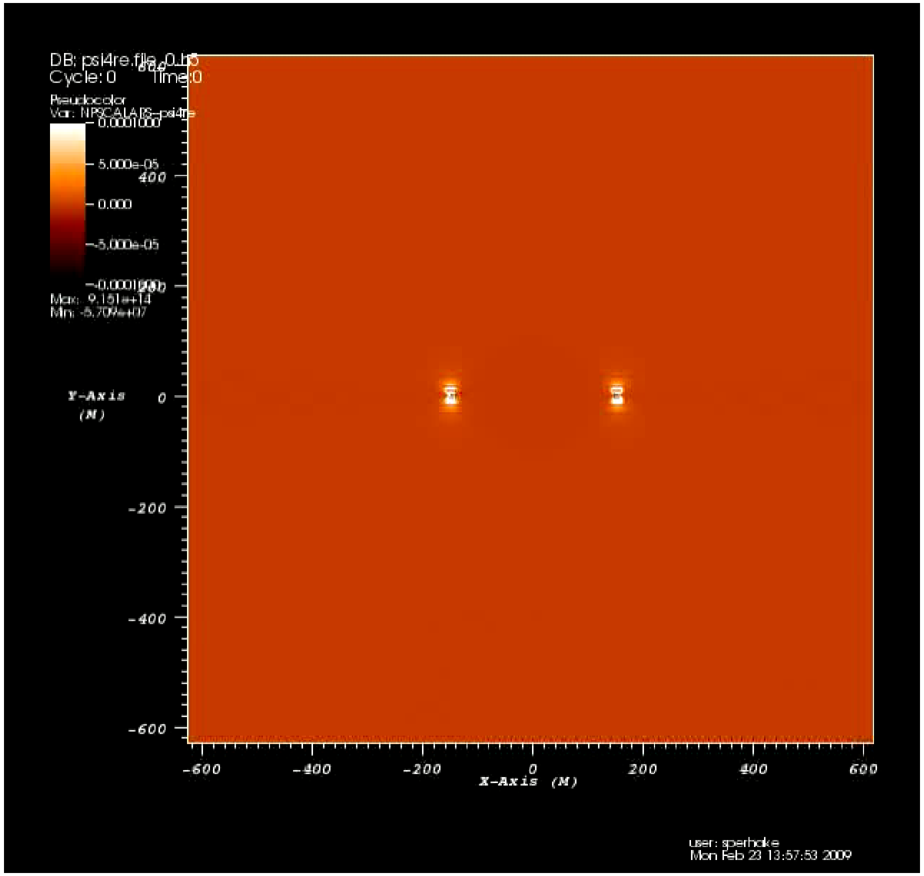
Cutoff frequency at the lowest quasinormal frequency




14%

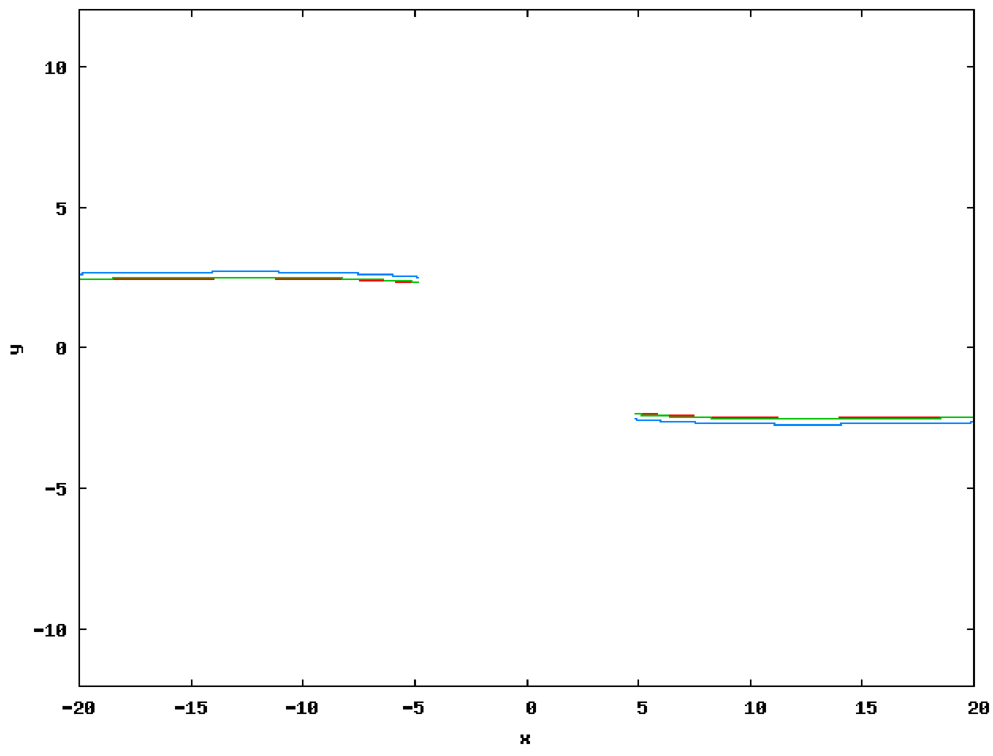
$$\frac{E}{M} = E_\infty \left( \frac{1+2\gamma^2}{2\gamma^2} + \frac{(1-4\gamma^2)\log\left(\gamma+\sqrt{\gamma^2-1}\right)}{2\gamma^3\sqrt{\gamma^2-1}} \right)$$

# Grazing collisions

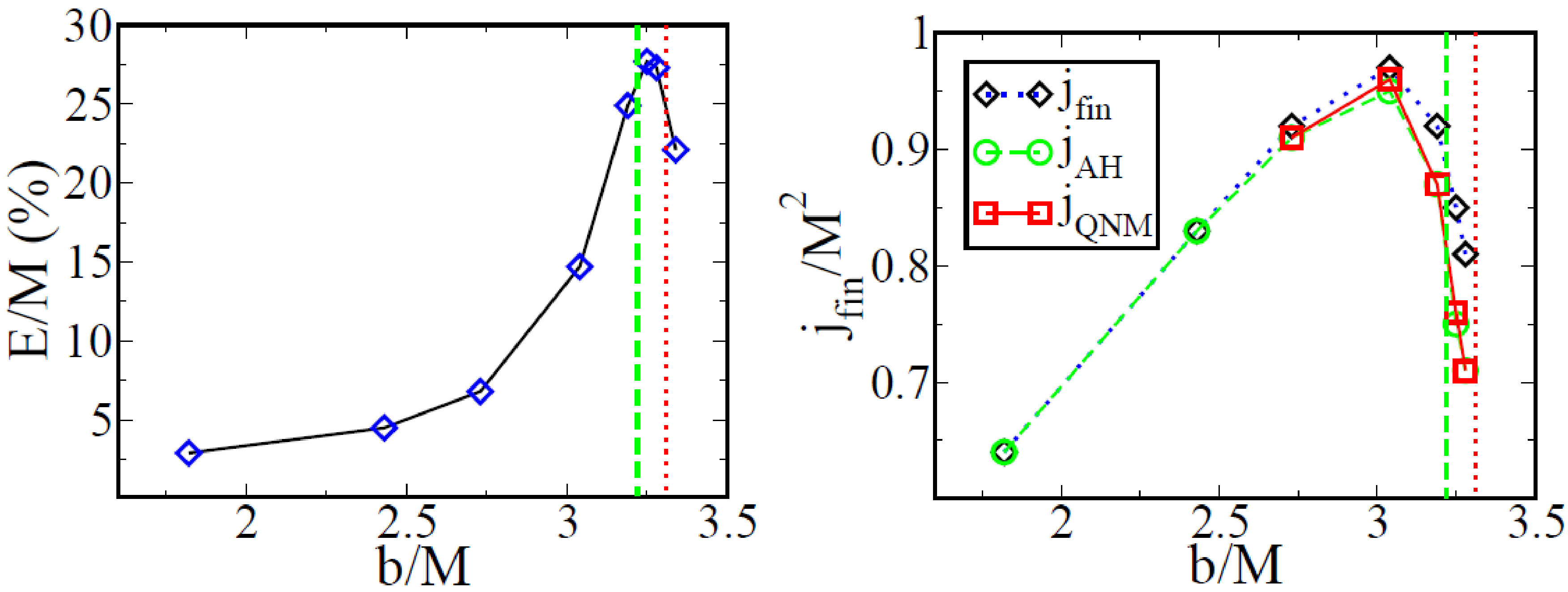



*(Sperhake et al 2009)*

# Plunge, zoom-whirl and scattering

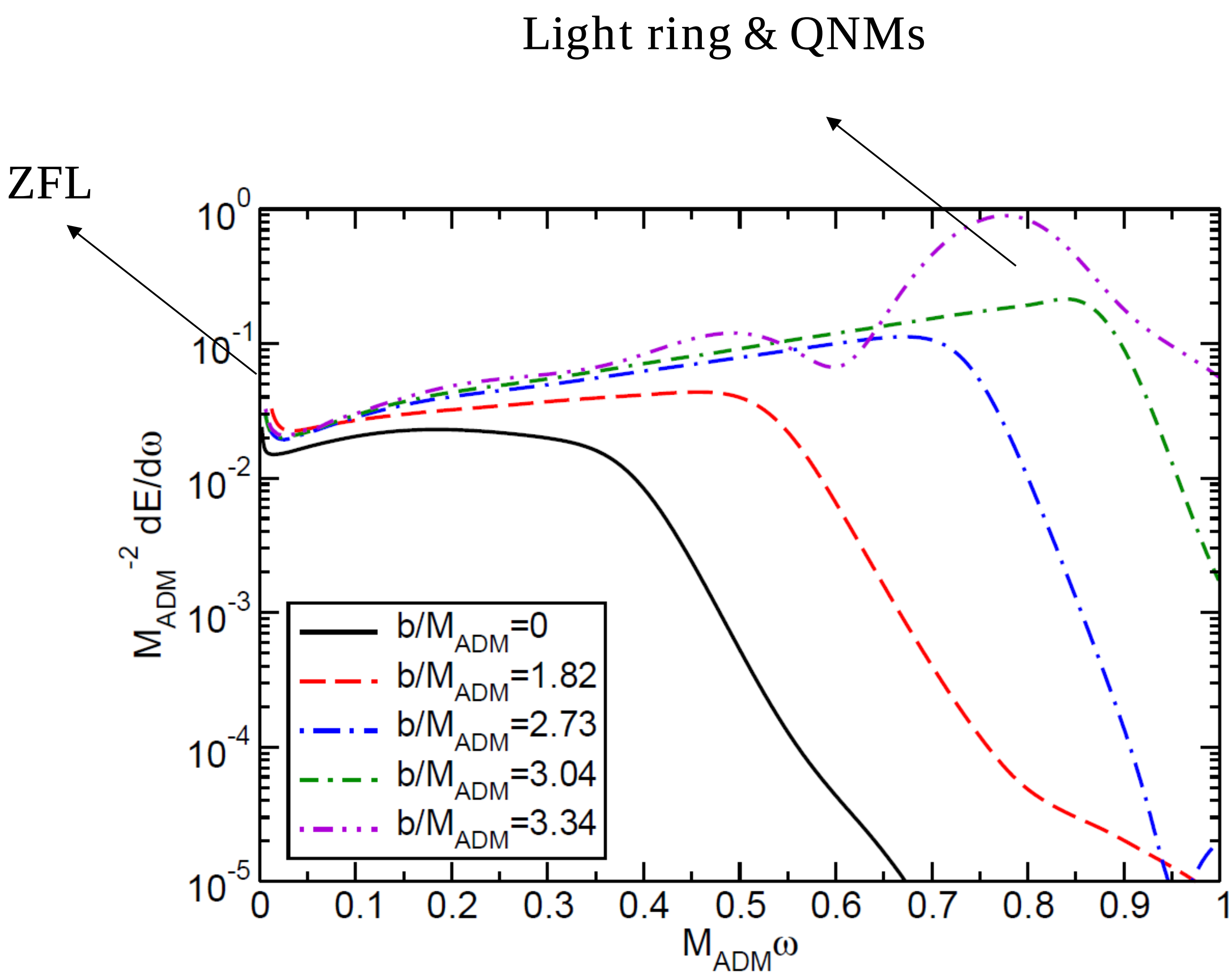



*(Sperhake et al 2009)*



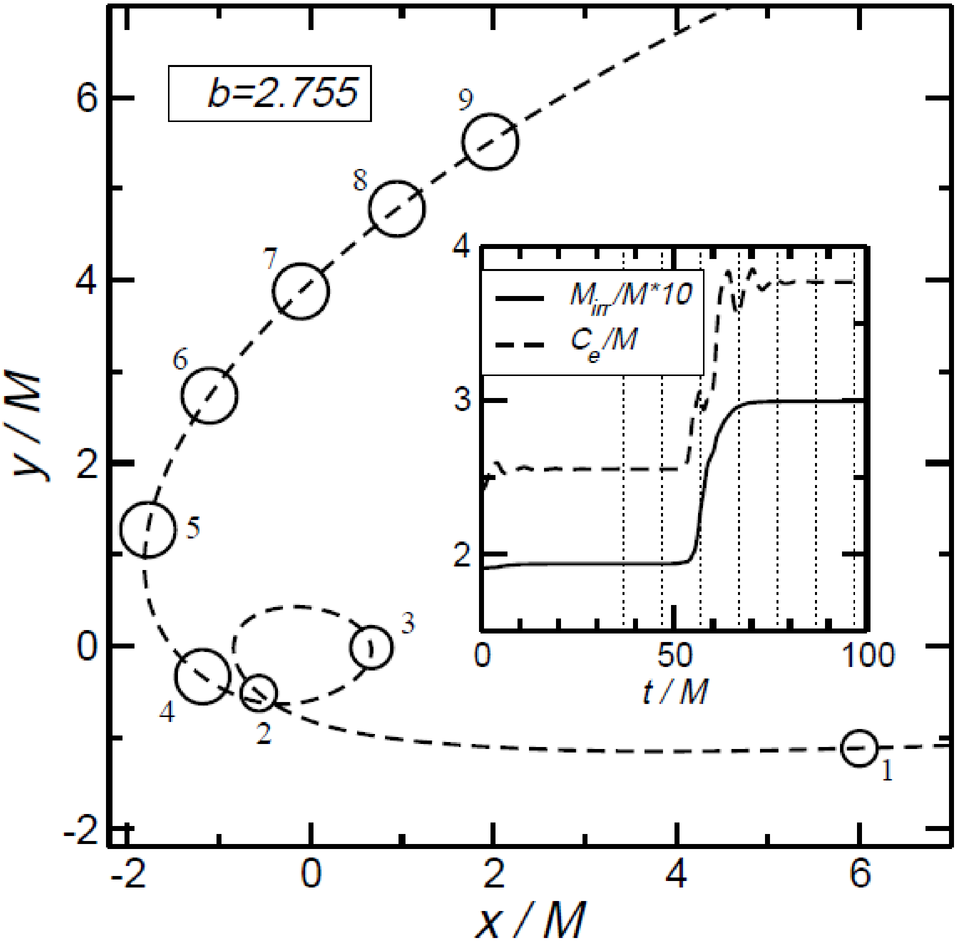

More than 25% CM energy radiated for v=0.75 c!

Final BH rapidly spinning



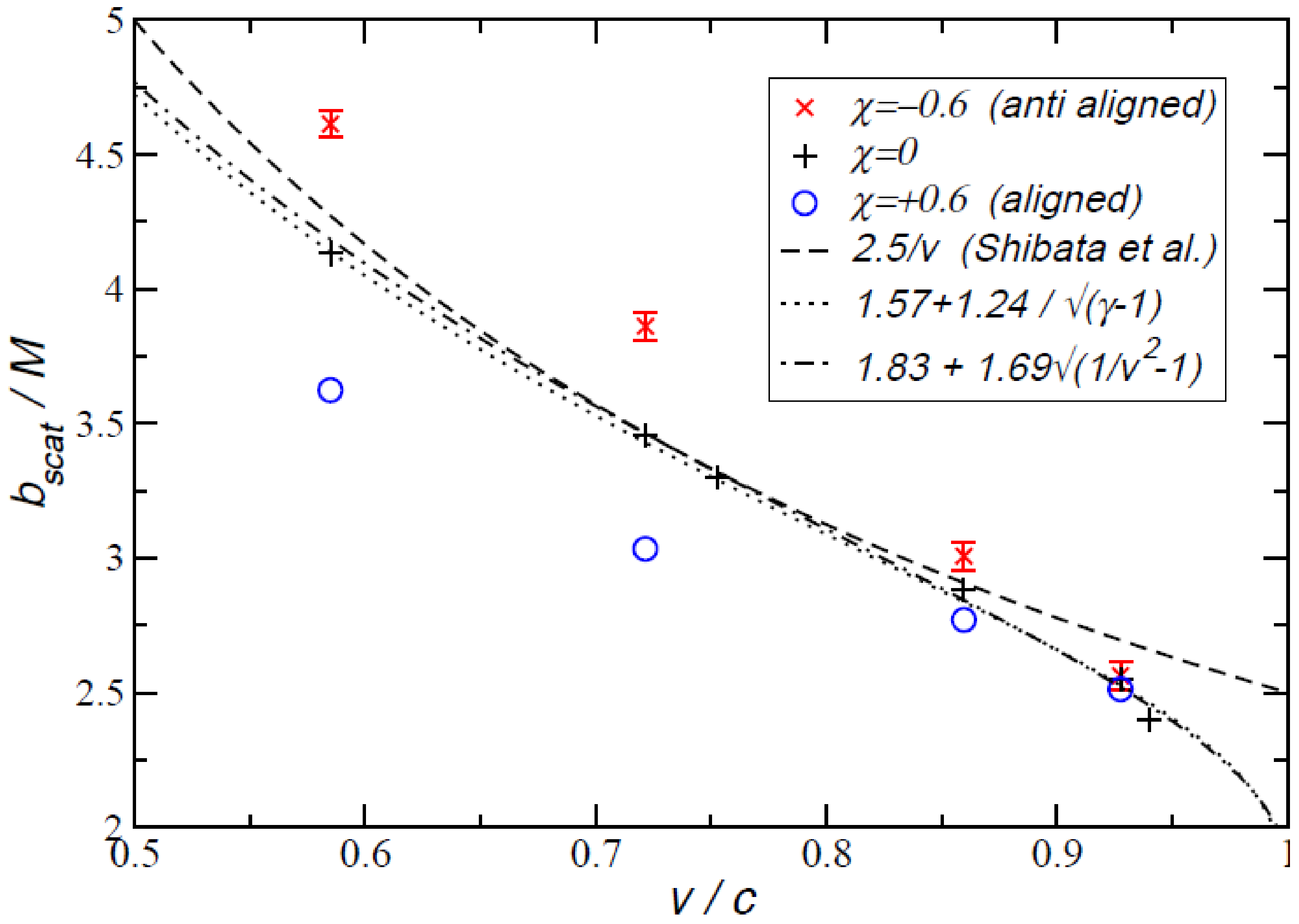
Light ring & QNMs
ZFL
$10^0$
$10^{-1}$
$10^{-2}$
$10^{-3}$
$10^{-4}$
$10^{-5}$
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
$M_{ADM}^{-2}\, dE/d\omega$
$M_{ADM}\omega$
$b/M_{ADM}=0$
$b/M_{ADM}=1.82$
$b/M_{ADM}=2.73$
$b/M_{ADM}=3.04$
$b/M_{ADM}=3.34$

# How far can we go?




*Sperhake, Berti, Cardoso and Pretorius 2013*

# *"Matter does not matter"*




*Sperhake, Berti, Cardoso & Pretorius 2013*

Cosmic Censor: as strong as ever

Production Cross-section: b/M=2.5/v

Peak luminosity: Close to Dyson limit c^5/G

Maximum spin: >0.95

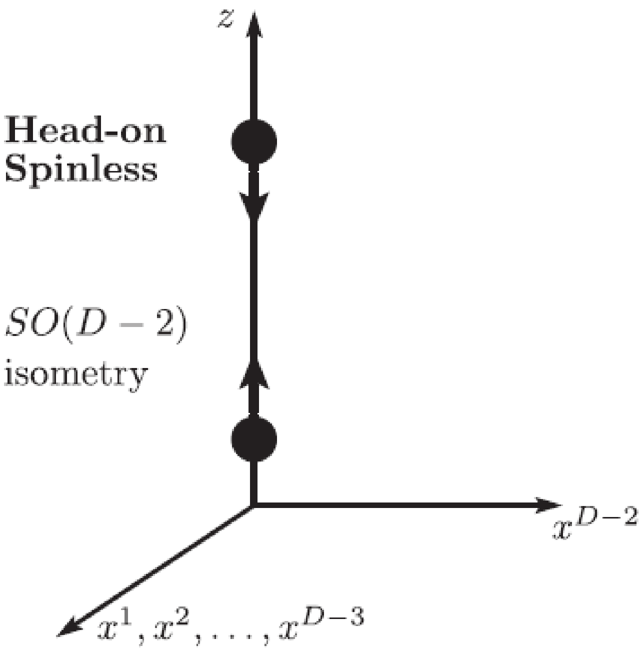
Radiated energy: >40% CM (but <50%)

Junk: ~2 Erad, interesting topic for further study

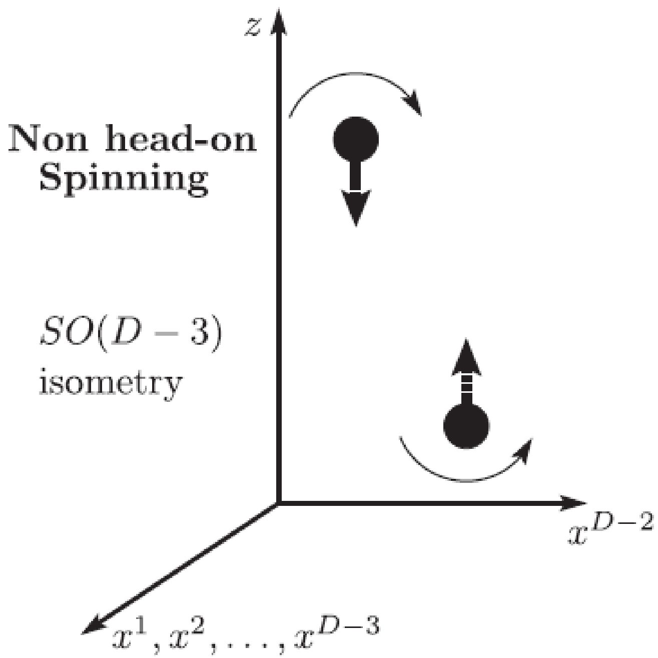
Radiation: Almost just ringdown, relation with ZFL...

# Extensions

# Axial symmetry




Head-on in D>4




Grazing in D>5

Can be reduced to effective 3+1

*(Zilhão et al, 2010)*

$$
\begin{aligned}
ds^2 &= g_{\mu\nu} dx^\mu dx^\nu + e^{2\phi} d\Omega^2_{D-4} \\
\mu &= 0, 1, 2, 3
\end{aligned}
$$

D-dimensional Einstein equations imply

$$
e^{2\phi} \left[ (D-4) \partial^\alpha \phi \partial_\alpha \phi + \nabla^\alpha \partial_\alpha \phi \right] = D - 5
$$

$$
R_{\mu\nu} = (D-4) \left[ \nabla_\nu \partial_\mu \phi + \partial_\mu \phi \partial_\nu \phi \right]
$$

*(Zilhão et al, 2010)*

# Effective 3+1 system with sources

$$(\partial_t - \mathcal{L}_\beta)\,\gamma_{ij} = -2\alpha K_{ij}$$

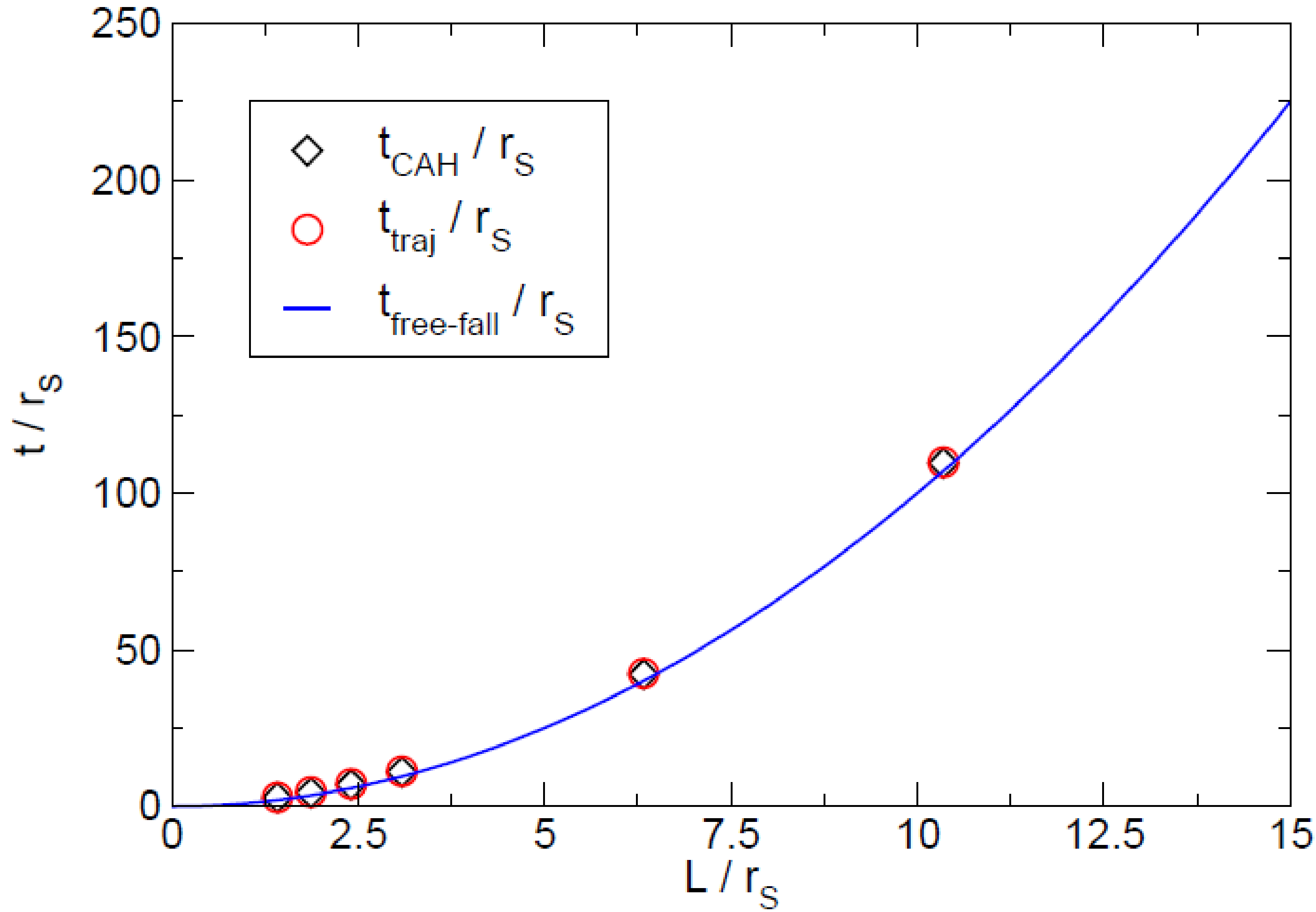
$$(\partial_t - \mathcal{L}_\beta)\,K_{ij} = -D_i\partial_j\alpha + \alpha\left[{}^{(3)}R_{ij} + KK_{ij} - 2K_{ik}K^k_j\right]$$

$$-\alpha(D-4)\left(D_i\partial_j\phi - K_{ij}K_\phi + \partial_i\phi\partial_j\phi\right)$$

$$(\partial_t - \mathcal{L}_\beta)\,\phi = -\alpha K_\phi$$

$$(\partial_t - \mathcal{L}_\beta)\,K_\phi = \alpha\left[(D-5)e^{-2\phi} - (D-4)\partial_i\phi\partial^i\phi\right]$$

$$+\alpha\left[(D-4)K_\phi^2 + KK_\phi - D^i\partial_i\phi\right] - \partial^i\alpha\partial_i\phi$$



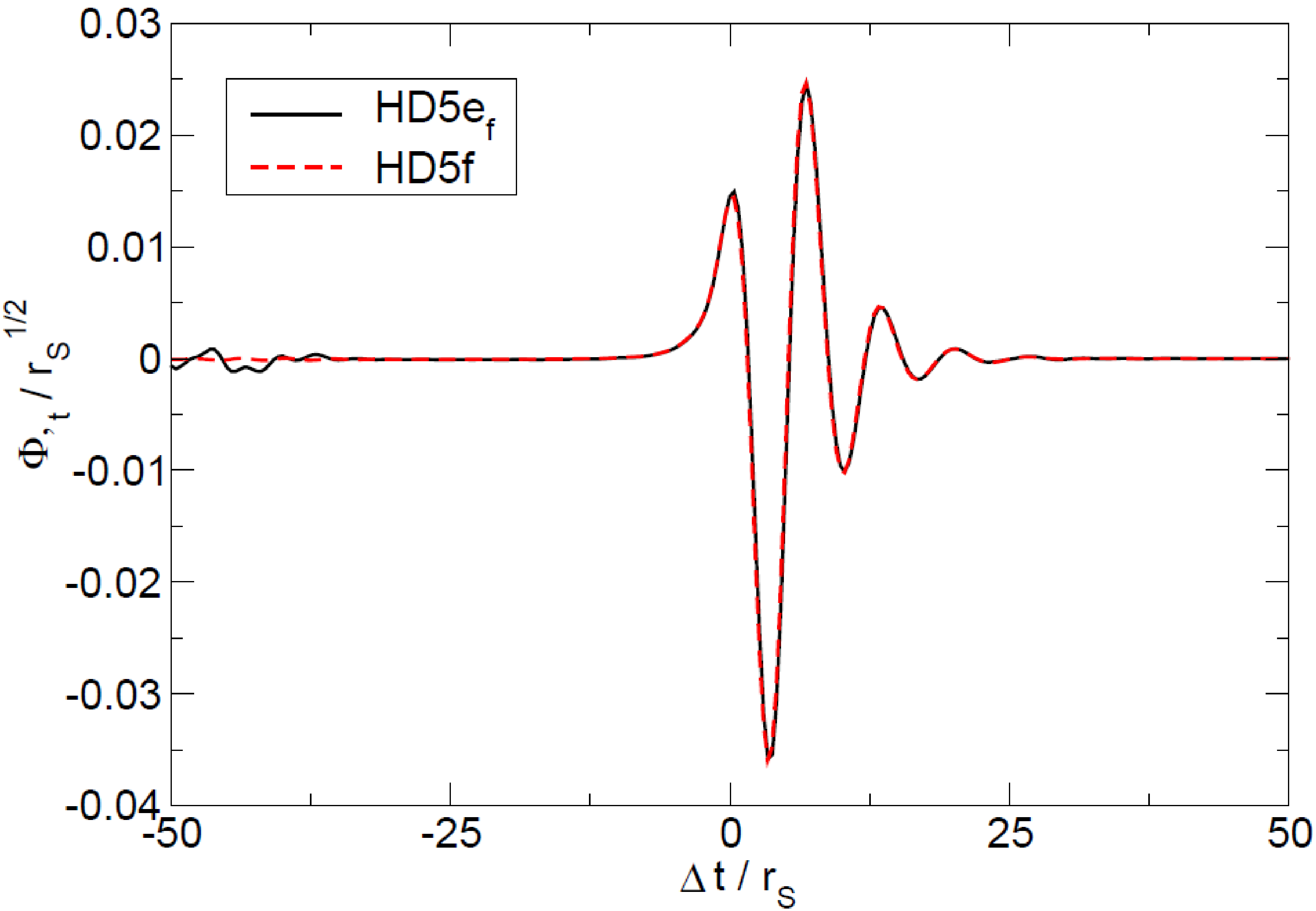

*(Witek et al, 2010)*

# Wave extraction

$$ds^2 = \text{Tangherlini} + \delta g_{AB}$$

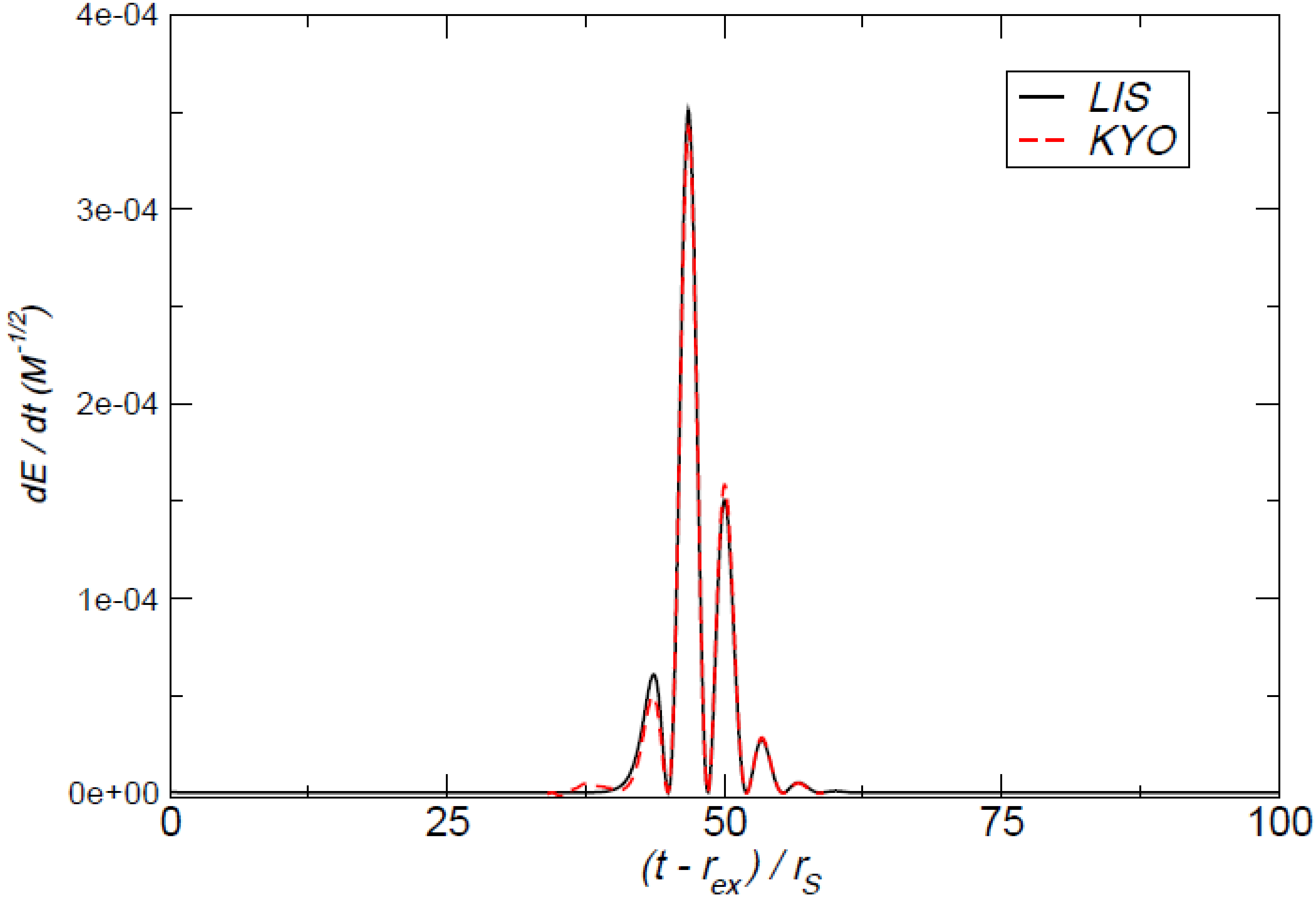
$$\begin{aligned}
\delta g_{ab} = h_{ab} &= f_{ab}\mathcal{S} \\
\delta g_{a\bar{i}} = h_{a\bar{i}} &= r f_a \mathcal{S}_{\bar{i}} \\
\delta g_{\bar{i}\bar{j}} = h_{\bar{i}\bar{j}} &= 2r^2 (H_L \gamma_{\bar{i}\bar{j}} \mathcal{S} + H_T \mathcal{S}_{\bar{i}\bar{j}})
\end{aligned}$$

$$\begin{aligned}
H_L(t,r) &= \frac{1}{2(D-2)r^2} \int d\Omega^{D-2} \left[ h_{\bar{\theta}\bar{\theta}} + \frac{D-3}{\sin^2\bar{\theta}} h_{\theta\theta} \right] \mathcal{S}_l \\
&= \frac{1}{2(D-2)r^2} \frac{\mathcal{A}_{D-3}}{\sqrt{K^{lD}}} \int_0^\pi d\bar{\theta} (\sin\bar{\theta})^{D-3} \left[ h_{\bar{\theta}\bar{\theta}} + \frac{D-3}{\sin^2\bar{\theta}} h_{\theta\theta} \right] C_l^{(D-3)/2} \\
H_T(t,r) &= \frac{1}{2r^2(k^2-D+2)} \int d\Omega^{D-2} \left[ h_{\bar{\theta}\bar{\theta}} - \frac{1}{\sin^2\bar{\theta}} h_{\theta\theta} \right] \mathcal{W}_l \\
&= \frac{1}{2r^2(k^2-D+2)} \frac{\mathcal{A}_{D-3}}{\sqrt{K^{lD}}} \int_0^\pi d\bar{\theta} (\sin\bar{\theta})^{D-3} \left[ h_{\bar{\theta}\bar{\theta}} - \frac{1}{\sin^2\bar{\theta}} h_{\theta\theta} \right] W_l
\end{aligned}$$



0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-50
-25
0
25
50
$\Phi_{,t} / r_S^{1/2}$
$\Delta t / r_S$
HD5e$_f$
HD5f

# Don’t trust our results?




*(Witek, Okawa et al, 2010)*

| $D$ | $E/M(\%)$ | $E^{\text{area}}/M(\%)$ | $E^{\text{PP}}_{\text{ext}}M/\mu^2$ | $P^{\text{PP}}_{\text{ext}}M/\mu^2$ |
|---|---|---|---|---|
| 4 | 0.055 | 29.3 | 0.0102 | 0.00083 |
| 5 | 0.089 | 20.6 | 0.0160 | 0.0024 |

*(Witek et al 2010; Berti et al 2010)*

BH collisions are a fascinating topic in GR

✷

Cosmic Censorship preserved

✷

Much remains to be done:

Understand initial data, add charge, go to higher boosts,

higher dimensional spacetimes, compactified EDs, anti-de Sitter

# Thank you