Holographic interaction effects on transport in Dirac semimetals

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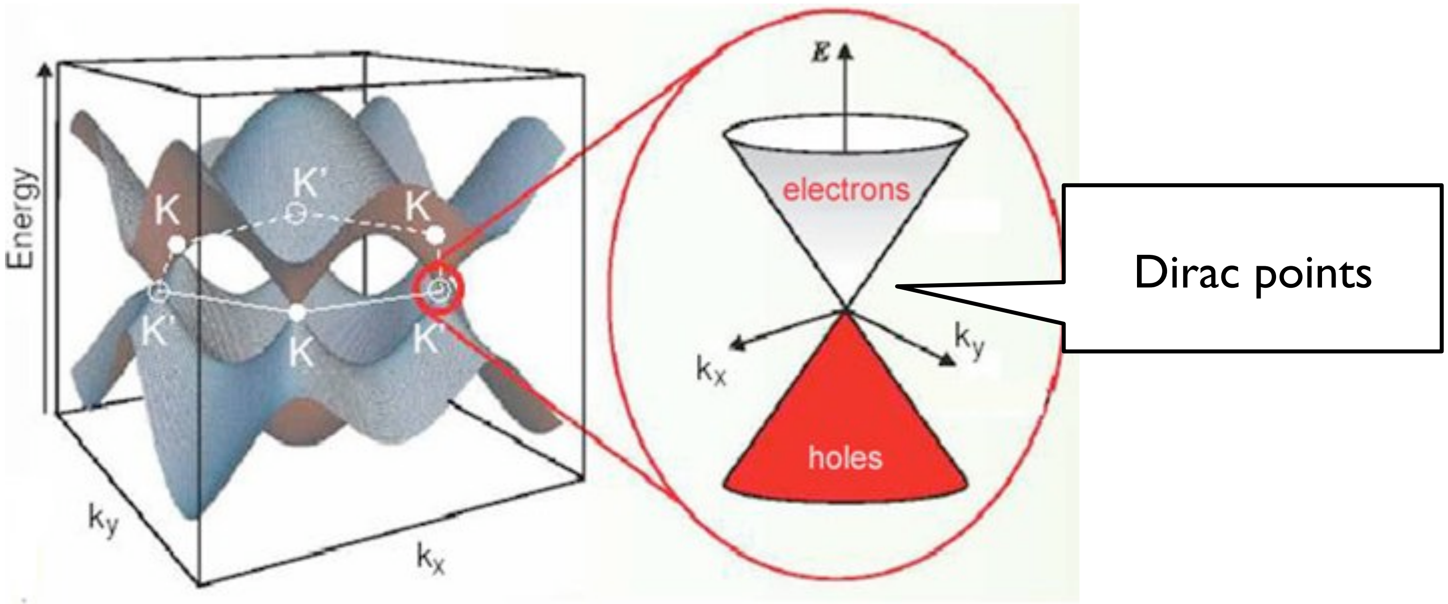
arXiv: 1403.3608

Semimetals

- Semimetal = gapless semiconductor
- Well-known example in 2+1 dim: graphene



- Effective description in terms of "relativistic" massless 2-component Dirac fermions.

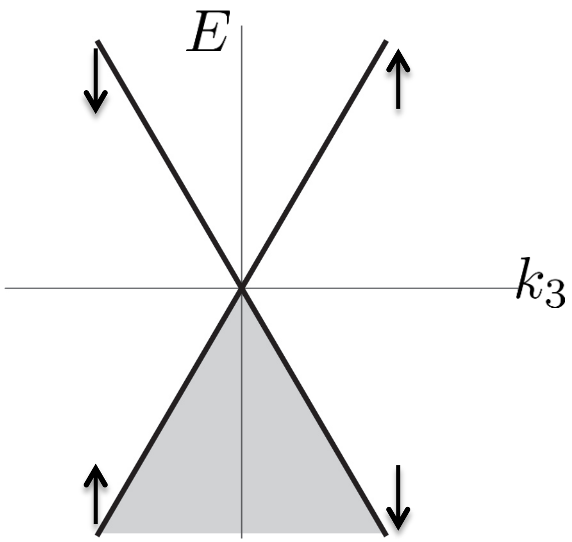
3+1 dim analog: Weyl & Dirac semimetals

- Based on chiral 2-component fermions, satisfying the Weyl equation.

$$H = \sigma_3 \otimes \vec{\sigma} \cdot c\hbar\vec{k}$$

(in the non-interacting and low-energy limit)

Weyl point of
+ chirality:



- Weyl points are topologically stable: no mass term for Weyl fermions.

- Dirac semimetal contains two Weyl fermions of opposite chirality

Three Experiments on 3D Dirac SM

Observation of a topological 3D Dirac semimetal phase in high-mobility Cd_3As_2

M. Neupane*,[1,2] S.-Y. Xu*,[1] R. Sankar*,[3] N. Alidoust,[1] G. Bian,[1] Chang Liu,[1] I. Belopolski,[1]
T.-R. Chang,[4] H.-T. Jeng,[4,5] H. Lin,[6] A. Bansil,[7] Fangcheng Chou,[3] and M. Z. Hasan[1,8]

(25 Sept 2013)

Experimental Realization of a Three-Dimensional Dirac Semimetal

Sergey Borisenko[1], Quinn Gibson[2], Danil Evtushinsky[1], Volodymyr Zabolotnyy[1*], Bernd Büchner[1,3],
Robert J. Cava[2]

(27 Sept 2013)

Discovery of a Three-dimensional Topological Dirac Semimetal, Na_3Bi

Z. K. Liu[1*], B. Zhou[2,3*], Z. J. Wang[4], H. M. Weng[4], D. Prabhakaran[2], S. -K. Mo[3], Y. Zhang[3],
Z. X. Shen[1], Z. Fang[4], X. Dai[4], Z. Hussain[3], and Y. L. Chen[2,3§]

(1 Oct 2013)

Observation of a topological 3D Dirac semimetal phase in high-mobility Cd_3As_2

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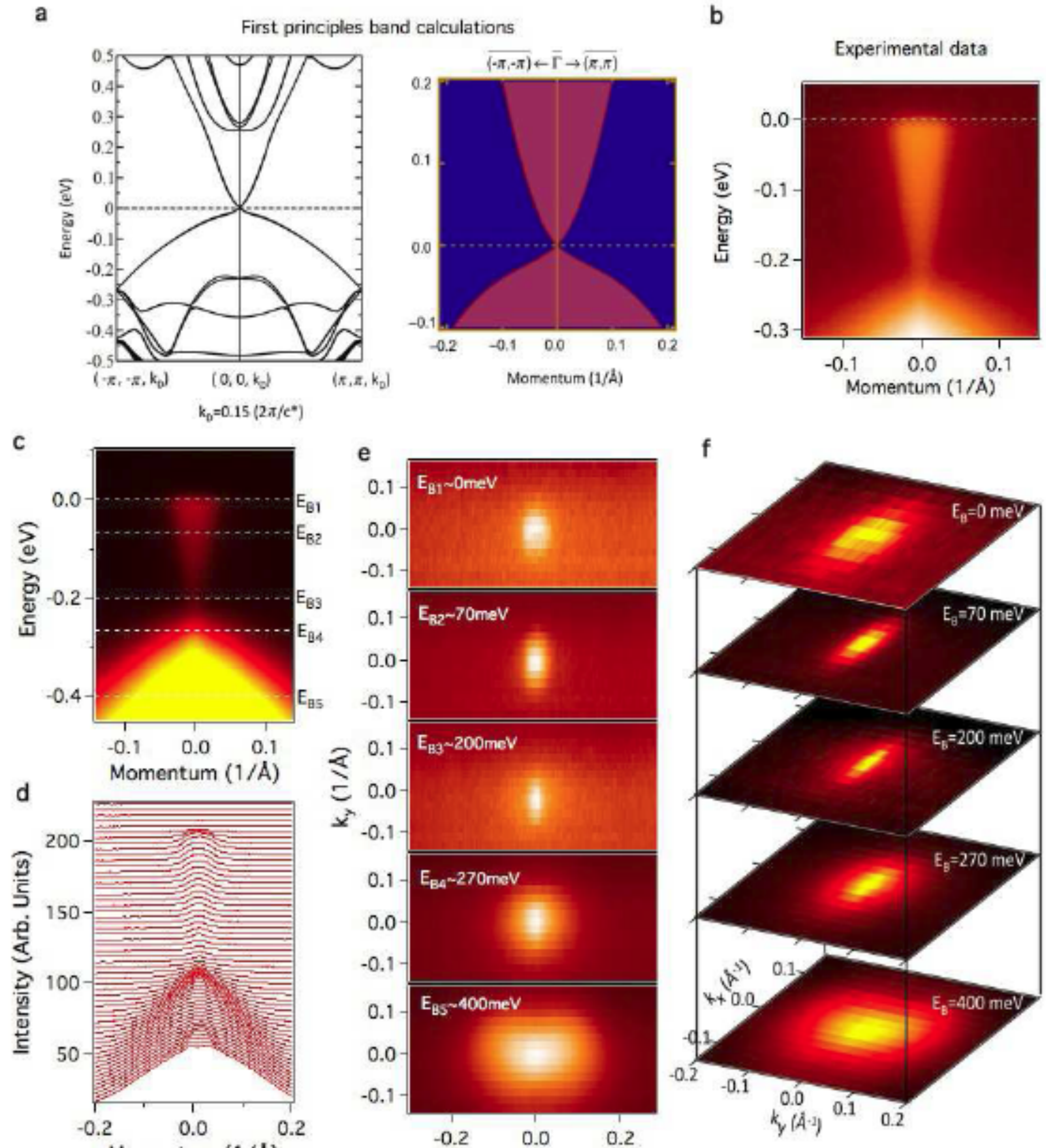


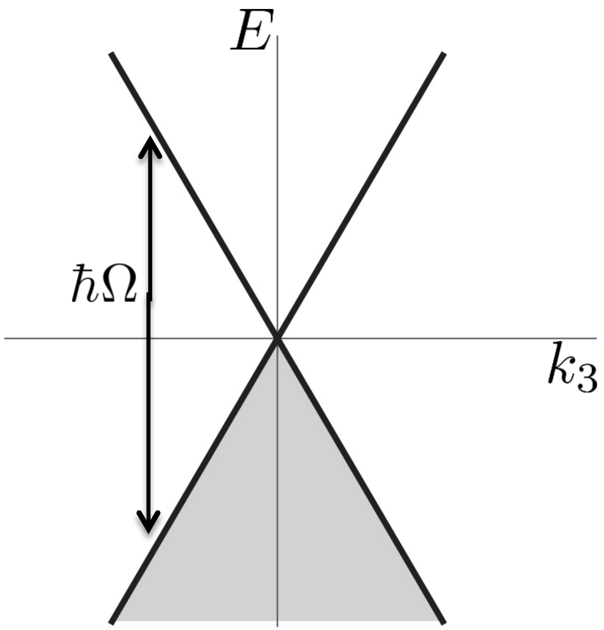
Outline

- Charge transport in free Weyl/Dirac semimetals (QFT)
- The strongly interacting case (holography)

Optical conductivity of ideal Dirac SM (1)

- Linear response

$$\langle j_i(\Omega)\rangle = \sigma_{ij}(\Omega)\, E_j(\Omega)$$



- Fermi's golden rule: transition rate

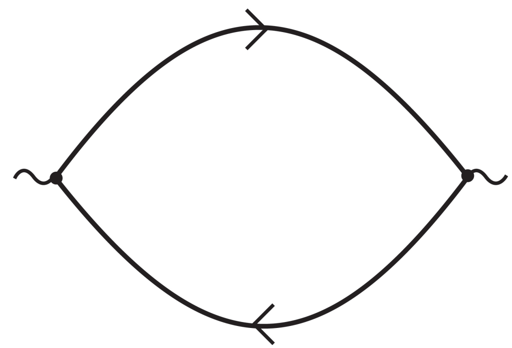
- LINEAR optical conductivity at zero temperature

$$\mathrm{Re}\,\sigma(\Omega) = \sum_{i=1}^{3}\sum_{\text{cones}} \frac{|\Omega|}{24\pi\hbar c}$$

Optical conductivity of ideal Dirac SM (2)

- Diagrammatic approach (Kubo)

$$\mathrm{Re}\,\sigma_{xx}(\Omega) = -\frac{\mathrm{Im}\,\Pi_{xx}(\vec{0},\Omega^+)}{\Omega}$$



with Matsubara current-current correlation function

$$\Pi^{\mu\nu}(\vec{q}, i\omega_p) = \frac{1}{\hbar^2\beta}\int\frac{d^3k}{(2\pi)^3}\sum_n \mathrm{Tr}\left[G_0(\vec{k}+\vec{q}, i\omega_n + i\omega_p)\,\gamma^0\gamma^\mu\, G_0(\vec{k}, i\omega_n)\,\gamma^0\gamma^\nu\right]$$

and Green's function

$$G_0^\chi(\vec{k}, i\omega_n) = \frac{i\omega_n \mathbb{1}_2 + \chi\vec{\sigma}\cdot c\vec{k}}{(c|\vec{k}| - i\omega_n)(c|\vec{k}| + i\omega_n)} \qquad \chi = \pm 1$$

The strongly interacting case

- What is the effect of strong interactions on the system's transport coefficients?
- Dirac semimetal ($\mu = 0\, , T = 0$) is scale invariant: holographic description? Massless Dirac = 2xWeyl.
- …but keeping the elementary Weyl fermion picture?
  - holographic model for single-particle correlation functions

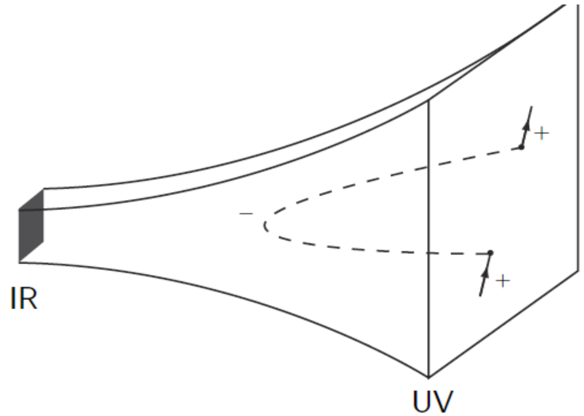
Holographic model for fermions (1)

Dirac spinor in five bulk dimensions has four (complex) components.

$$\Psi \equiv \begin{pmatrix} \Psi_+ \\ \Psi_- \end{pmatrix}, \qquad \Gamma^r \, \Psi = \begin{pmatrix} +\Psi_+ \\ -\Psi_- \end{pmatrix}$$

Fourier transform to two Weyl spinors on the boundary

$$\Psi_\pm(r,x) = \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \, \psi_\pm(r,p) \, e^{ip_\mu x^\mu}, \qquad p_\mu = (-\omega, \vec{k})$$



Holographic model for fermions (2)

- 5D asymp. Anti-de-Sitter spacetime with 5D Dirac fermions

$$(\not{D} - M)\,\Psi = 0 \qquad \Psi = \Psi_R + \Psi_L \qquad -\frac{1}{2} < M < \frac{1}{2}$$

- Boundary conditions in IR: infalling

- Boundary conditions in UV:
  - Dirichlet on 4D bdy. $\delta\Psi_R = 0$
  - Ψ_R is boundary source, a Weyl fermion
  - Make source dynamical!

$$S_\delta = -\int d^4x\sqrt{-h}\left(\Psi_+^\dagger \not{D}_z \Psi_+ + \Psi_+^\dagger \Psi_-\right)$$

U. Gursoy, E. Plauschinn, H. Stoof, S. Vandoren, JHEP **5**, 18 (2012)

Holographic model for fermions (3)

- Dirac eqn. in grav. background:

$$\Psi_L = \Sigma\, \Psi_R$$
$$\Sigma = \gamma^\mu \Sigma_\mu(\vec{k}, \omega, T)$$

- Result: effective action for 4D Weyl fermions on the boundary:

$$S_{\text{eff}}[\Psi_+] = -\int d^4x\, \bar{\Psi}_+ (\not{D}_4 - \not{\Sigma}) \Psi_+$$

$$\Psi_R = \begin{pmatrix} \Psi_+ \\ 0 \end{pmatrix}$$

- By construction effective description of strong interactions between the boundary Weyl fermions, via CFT.

Holographic model for fermions (4)

- Sum rule for single particle spectral density:

$$\frac{1}{\pi}\int_{-\infty}^{+\infty} \mathrm{d}\omega \ \mathrm{Im}\left[G_{R;\alpha,\alpha'}(\vec{k},\omega)\right] = \delta_{\alpha,\alpha'}$$

- Kramers-Kronig relations (no poles in upper half plane) satisfied for

$$-\frac{1}{2} < M < \frac{1}{2}$$

- Particle-Hole and Chiral symmetry

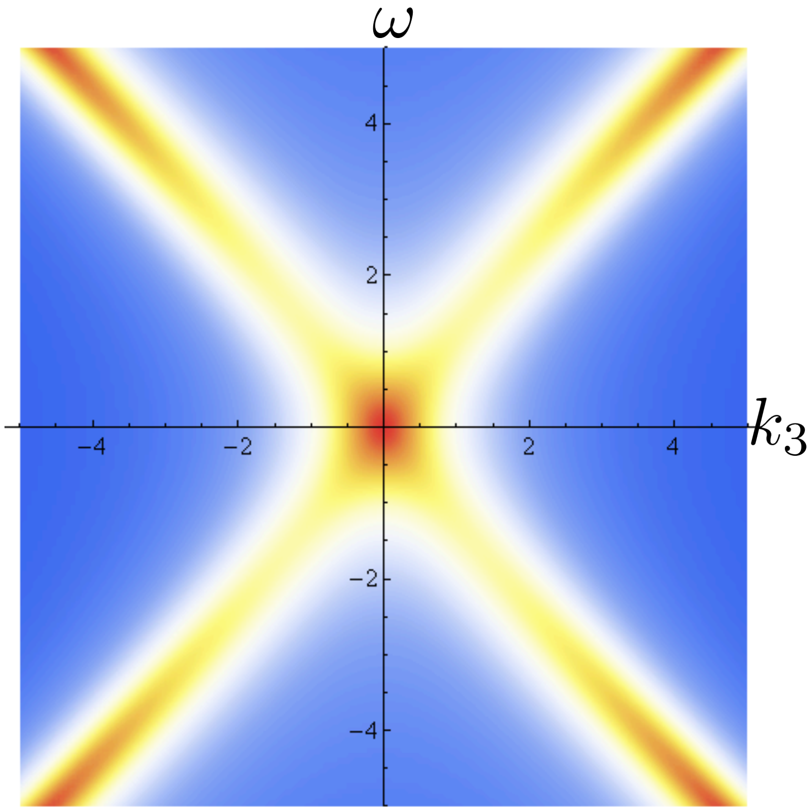
$$G_R^{\pm}(\vec{k},\omega) = -\left(G_R^{\pm}(-\vec{k},-\omega)\right)^* = -\left(G_R^{\mp}(\vec{k},-\omega)\right)^*$$

Single-particle Green's function

- Interacting Dirac semimetal:

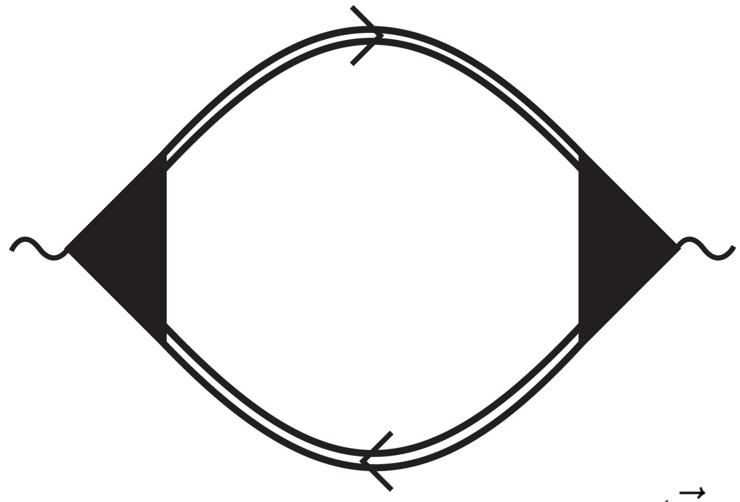
$$G_R(k) = \frac{ck_\mu + \Sigma_\mu}{(ck+\Sigma)^2}\gamma^\mu\gamma^0$$

$$k^\mu = (\omega/c, \vec{k})$$



Conductivity in interacting case

$$\Pi^{\mu\nu} \propto$$



- Conductivity expressed in terms of a function $\Sigma_\mu(\vec{k}, \omega, T)$, solution of a 1[st] order ODE.

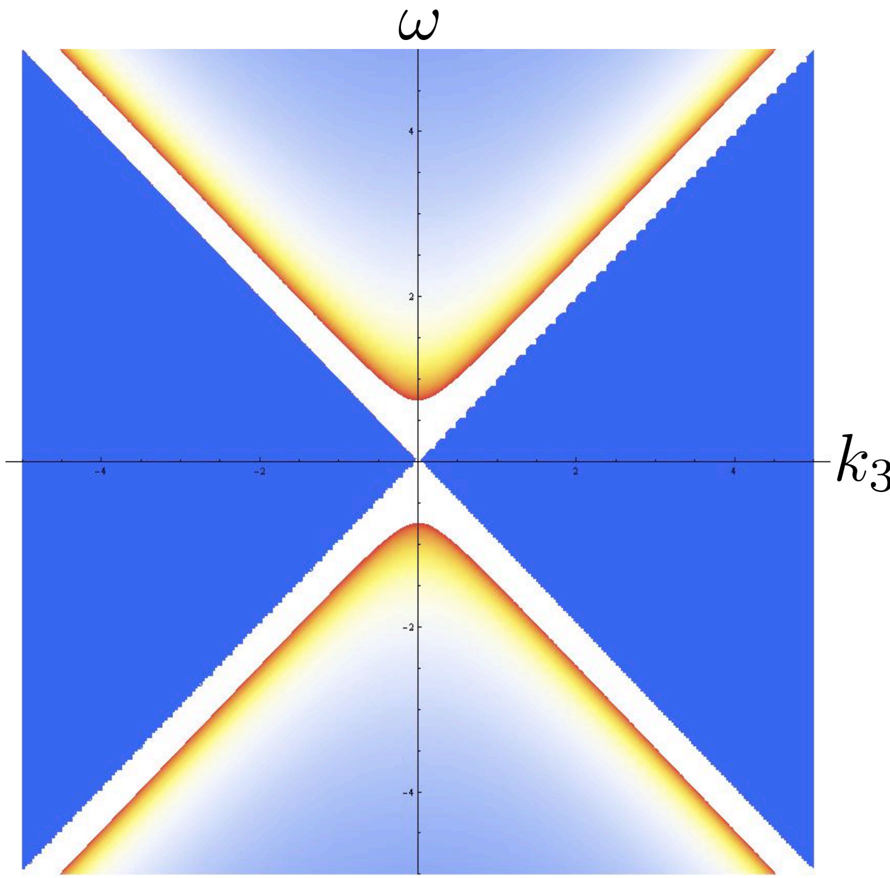
- Unfortunately, Dirac equation in curved background only analytically solvable in simple cases: e.g. *T=0*.

- Ignore vertex corrections…

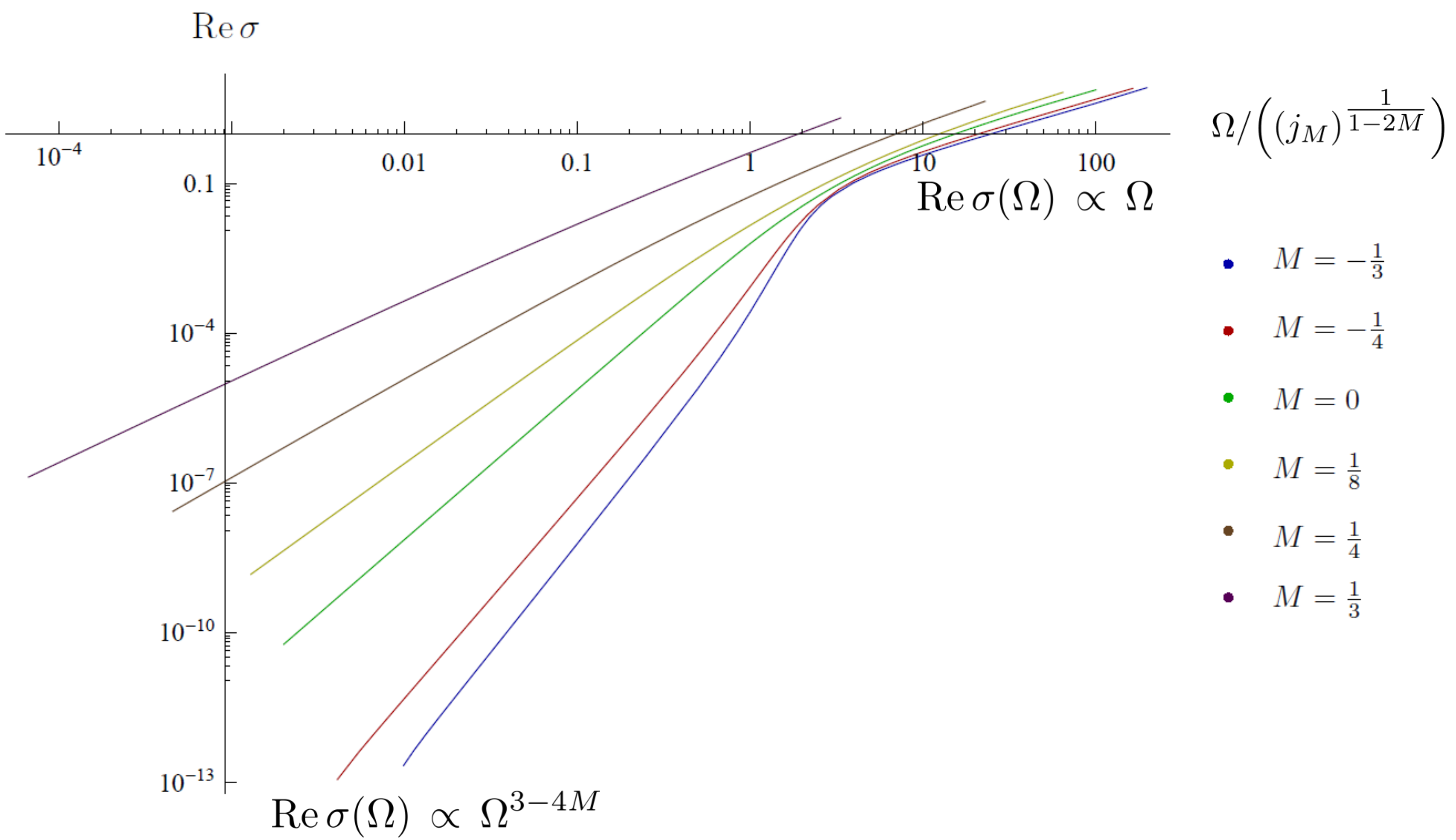
Results zero temperature

$$G(k) = \frac{-ck_\mu}{c^2k^2 + j_M(c^2k^2)^{M+\frac{1}{2}}}\gamma^\mu\gamma^0$$

$$j_M = g\,4^{-M}\frac{\Gamma(\frac{1}{2}-M)}{\Gamma(\frac{1}{2}+M)}$$

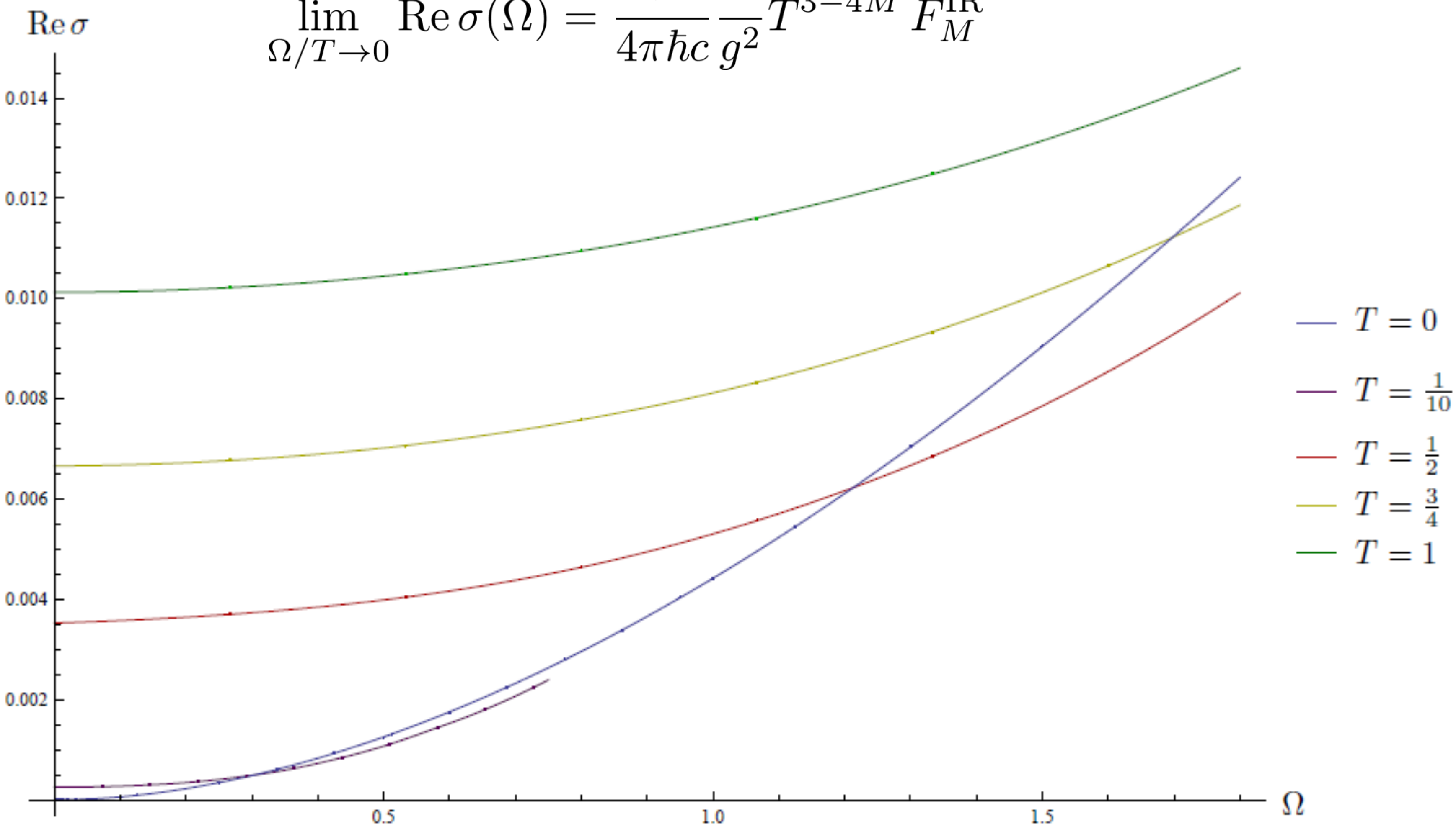


Results zero temperature (log-plot)



Results non-zero temperature

$$\lim_{\Omega/T\to 0} \mathrm{Re}\,\sigma(\Omega) = \frac{1}{4\pi\hbar c}\frac{1}{g^2} T^{3-4M}\, F_M^{\mathrm{IR}}$$



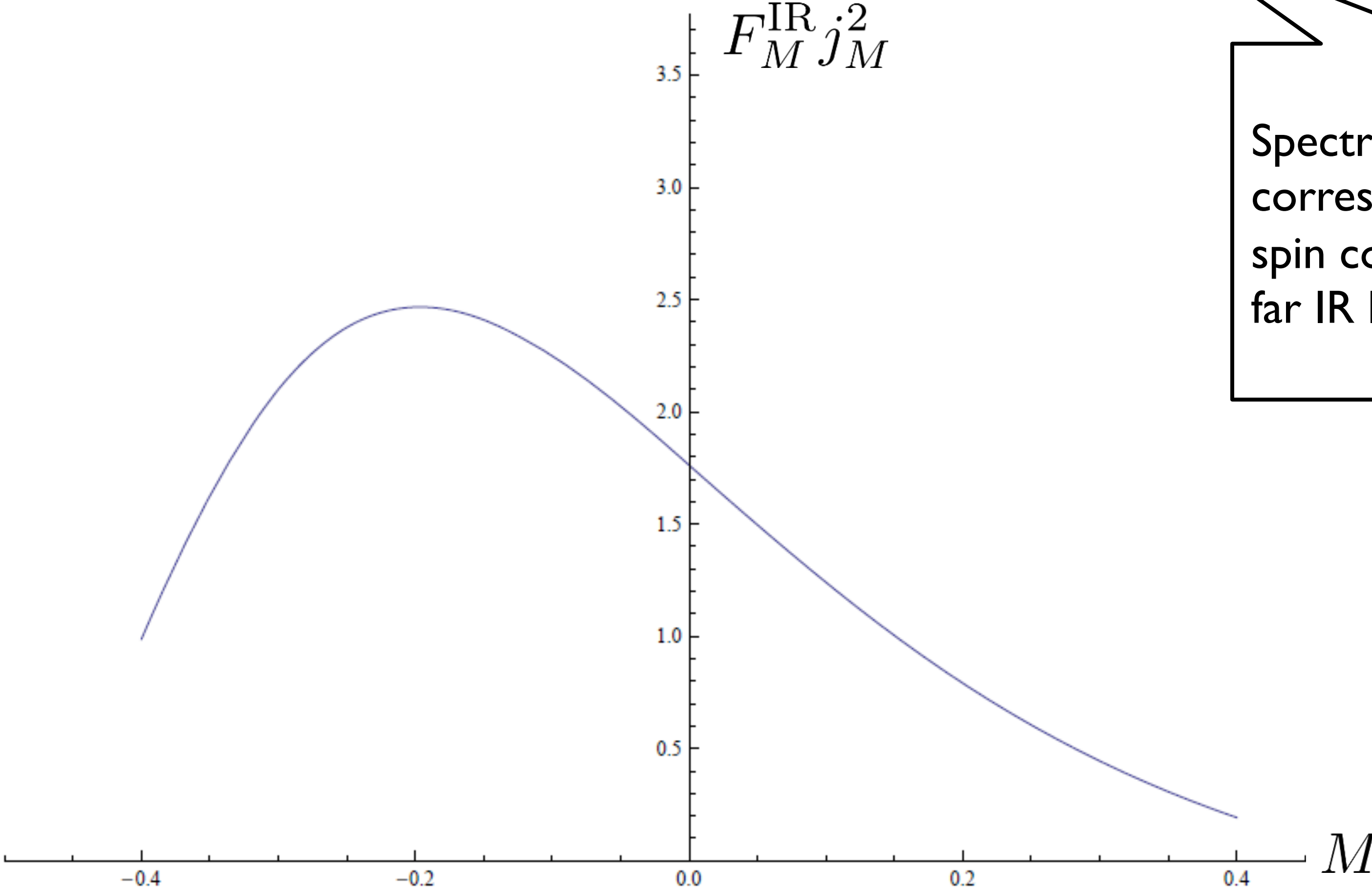
Plot for M=1/4.

For M=1/2, linear behaviour, but with extra logarithmic corrections: Coulomb interactions

Results non-zero temperature

$$F_M^{\mathrm{IR}} = \int_0^\infty dy y^2 \int_{-\infty}^\infty dx \frac{1}{\cosh^2(\frac{x}{2})} \mathcal{A}_+^{\mathrm{IR}}(x,y)\mathcal{A}_-^{\mathrm{IR}}(x,y).$$

Spectral-weight functions corresponding to the two spin components in the far IR limit.

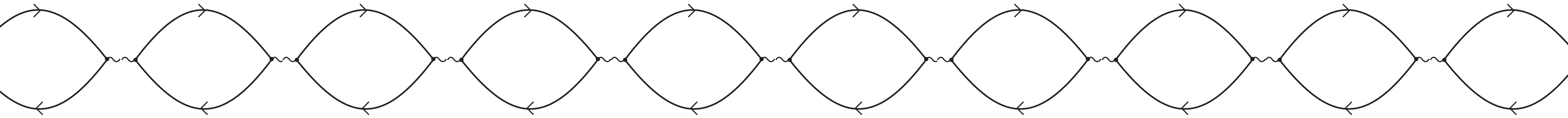


$$x = \hbar\omega / k_B T$$
$$y = \hbar c |\vec{k}| / k_B T$$

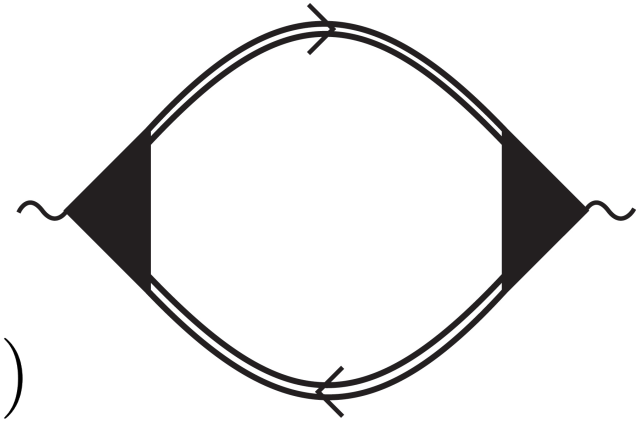
Conclusion and discussion

- Transport properties of free and interacting Dirac semimetals
  - Optical conductivity vanishes as $\sigma(\Omega) \propto \Omega^{3-4M}$ at zero temperature
  - Constant DC conductivity $\sigma(\Omega) \propto T^{3-4M} F_M^{\mathrm{IR}}$ at non-zero temperature
- Strongly interacting case: holographic single-particle model

- Thanks for your attention!



Vertex corrections



- Ward identity

$$q_\mu \Lambda^\mu = G^{-1}(k+q) - G^{-1}(k)$$

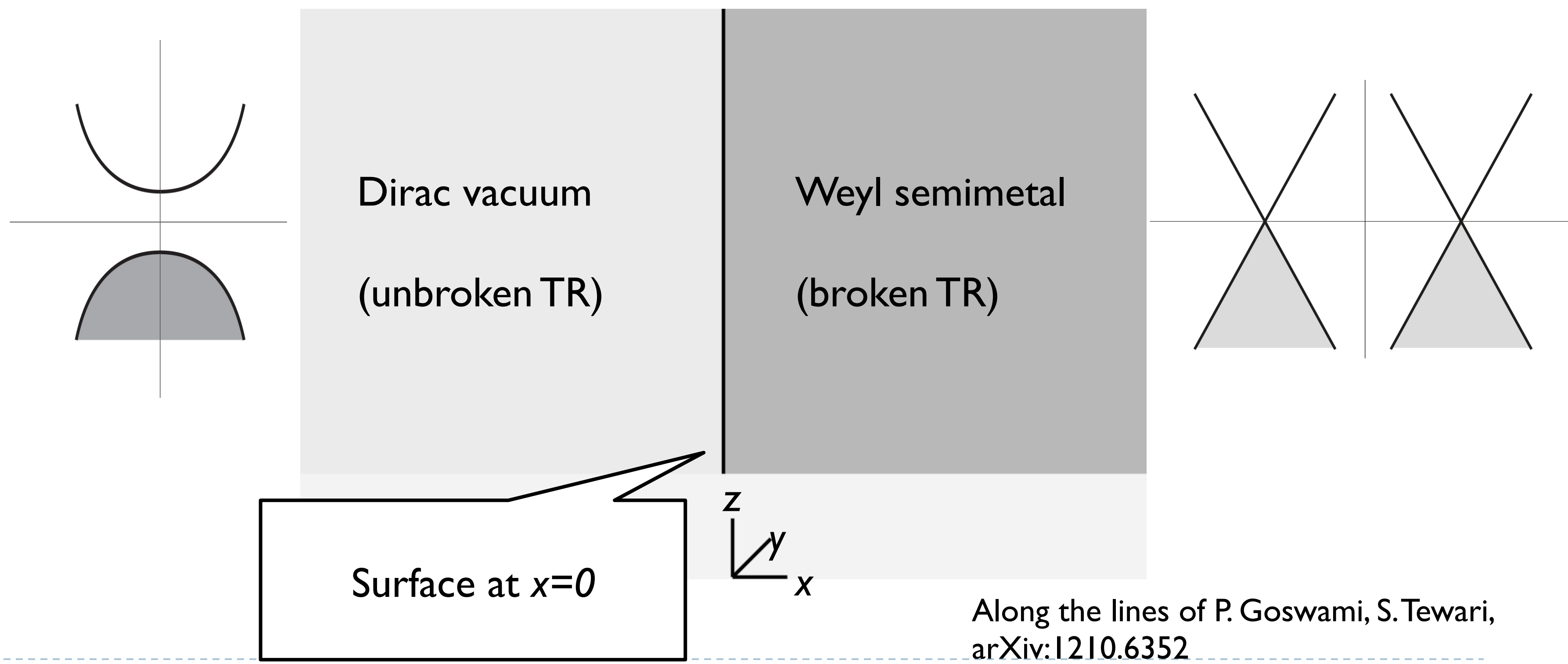
- Vertex constrained up to 6 unknown scalar functions

- Transversality of polarization tensor

Surface states

- Calculate surface states by solving 4x4 eigenvalue problem

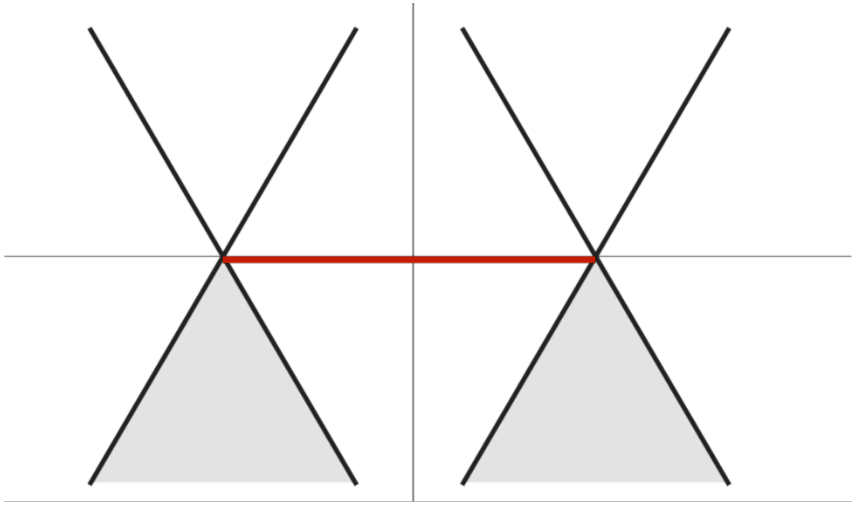
$$\hat{H}\psi = E\psi$$



Along the lines of P. Goswami, S. Tewari, arXiv:1210.6352

Surface states

- Look for bound states at $x=0$ in this setup.
- They exist! But… only between the Weyl points.
- Linear dispersion (red: gapless part)
- Give rise to Fermi arcs.



Anomalous Hall conductivity

- Momentum-space topology of Weyl points leads to non-zero Berry curvature

$$\vec{B}(\vec{k}) = \vec{\nabla}_{\vec{k}} \times \vec{A}(\vec{k})$$

Berry connection

$$\vec{A}(\vec{k}) = i\langle m, \vec{k}|\vec{\nabla}_{\vec{k}}|m, \vec{k}\rangle$$

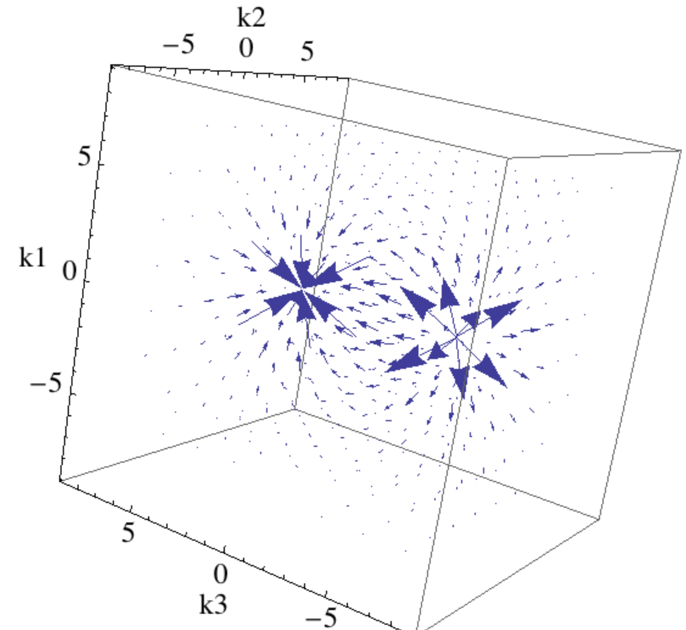
$$\oint_\gamma d\vec{k} \cdot \vec{A}(\vec{k}) = 2\pi n$$

- Magnetic (anti)monopoles in *k*-space

- Result:

$$\mathrm{Re}\,\sigma_{ij} = -\frac{e^2}{\hbar}\epsilon_{ij\ell}\int \frac{d^3k}{(2\pi)^3}\sum_m N_f(\epsilon_m)(B_m(\vec{k}))_\ell$$

$$= \frac{e^2}{2\pi^2\hbar}\epsilon_{ij\ell}(\Delta k)_\ell$$



Z.W. Sybesma (2012)