AdS/CFT and Landau Fermi liquids

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Introduction

- Landau Fermi liquid theory is a well understood low-energy effective theory of fermionic matter at finite density
- AdS/CFT correspondence presents a powerful approach to finite-density systems in the strong coupling regime
- As we will see generic predictions of AdS/CFT are quite different from the features of generic Landau Fermi liquid
- We will discuss how one can compare and match AdS/CFT dual of a two-charge black hole and Landau Fermi liquid

Outline of the talk

- Landau Fermi liquid: quasiparticles, zero sound, hydrodynamics
- AdS/CFT: Probe branes and AdS-RN, zero sound, hydrodynamics
- AdS/CFT: Two-charge black hole, zero sound, hydrodynamics
- Compare dual of two-charge BH and Landau Fermi liquid
- Landau Fermi liquid: fine tuning
- Higher-derivative gravity

Landau Fermi liquid: quasiparticles

- Fermions at zero temperature fill up a ball in momentum space. The boundary of this ball is Fermi surface
- LFL is the theory of weakly-interacting qusaiparticles excited on top of the Fermi surface
- It is defined at small temperatures, $T/\mu \ll 1$, for fluctuations around the Fermi surface, $\omega/\mu\,,\,q/\mu \ll 1$. Here μ is a chemical potential
- Quasiparticle life-time is $\tau \simeq \mu/T^2$
- To have long-lived weakly-interacting quasiparticles one imposes the condition $\omega\tau \gg 1$

Landau Fermi liquid: quasiparticles

- Fix frequency ω and chemical potential μ
- The quasiparticle stability condition $\omega\tau \gg 1$ says that temperature is bounded from above, $T < \sqrt{\omega\mu}$
- Because $\omega/\mu \ll 1$ as the temperature is increased from zero, $T = 0$, we first pass the point $T = \omega$ and then approach the stability threshold $T = \sqrt{\mu\,\omega}$
- The regime $0 \leq T \ll \omega$ is quantum collisionless
- The regime $\omega \ll T \ll \sqrt{\mu\,\omega}$ is thermal collisionless

Landau Fermi liquid: zero sound

- Let us start at small temperatures in the collisionless quantum regime $T/\omega \ll 1$. LFL exhibits a gapless excitation
$$\frac{\omega}{\mu} = u\frac{q}{\mu} - i\frac{\omega^2}{\mu^2}$$
called zero sound. Here u is the speed of zero sound
- As we increase the temperature, we move to collisionless thermal regime, $\omega \ll T \ll \sqrt{\mu\,\omega}$, and the sound attenuation gets temperature-dependent piece
$$\frac{\omega}{\mu} = u\frac{q}{\mu} - i\frac{\omega^2 + T^2}{\mu^2}$$

Landau Fermi liquid: hydrodynamics

- As the temperature continues to increase, crossover to hydrodynamic description of Fermi liquid takes place
- Hydrodynamics is a theory of excitations of wavelengths much larger than mean free path in the system
- For LFL this mean free path is quasiparticle life-time $\tau = \mu / T^2$
- In hydrodynamic regime Fermi liquid supports sound excitation

$$\omega = c_1 q - i \frac{2\eta}{3(\epsilon + P)} q^2$$

with $c_1 = \sqrt{dP/d\epsilon} = 1/\sqrt{3}$ in conformal theory. In LFL the speed of zero sound is not restricted to $u = 1/\sqrt{3}$

Probe branes and zero sound

- One way to describe finite density matter via AdS/CFT is to consider brane intersection models, namely $D3/D7$ and $D3/D5$ systems
- Fluctuations of $U(1)$ gauge field on the probe brane world-volume describe fluctuations of density of matter in the field theory
- Study of sound excitations in such systems shows behavior similar to that of LFL (Davison, Starinets 2011). There are also collisonless quantum and collisionless thermal regimes where zero sound mode exists with LFL kind of attenuation. There exists a crossover to hydrodynamic regime, although with vanishing speed of first sound
- This is despite the fact that brane intersection systems definitely are not duals of LFL, since the heat capacity behaves as $C \simeq T^6$, contrary to $C \simeq T$ in LFL

AdS-RN, zero sound and hydrodynamics

- The fate of sound mode at various temperatures was studied for AdS-RN
- It has been found that $D3/Dp$ properties of LFL-kind of sound mode are not generic and are not shared by excitations in charged black hole background (Davison, Kaplis, 2011)
- Hydrodynamic sound mode persists up to zero temperature (Davison, Parnachev 2013)

Two-charge black hole

- Let us consider different gravitational background
- Bulk background which we base on is the two-charge black hole solution of the $AdS_5 \times S^5$ supergravity (Cvetic et al, 1999; recent in AdS/CMT context: O. DeWolfe, S. S. Gubser, C. Rosen, 2012 and more)
- The truncated Einstein-Maxwell-dilaton action is

$$I_2 = \frac{1}{16\pi G} \int d^5x \sqrt{g} \left(R - \frac{1}{2}(\partial\phi)^2 - \frac{8}{L^2} e^{\phi/\sqrt{6}} - \frac{4}{L^2} e^{-2\phi/\sqrt{6}} \right.$$
$$\left. + 2e^{2\phi/\sqrt{6}} F_{ab} F^{ab} \right)$$

Gravitational background

- The two-charge black hole solution is

$$ds^2 = e^{2a(r)}\left(h(r)dt^2 - dx^2 - dy^2 - dz^2\right) - \frac{e^{2b(r)}}{h(r)}dr^2$$

$$a(r) = \log\left(\frac{r}{L}\left(1+\frac{Q^2}{r^2}\right)^{\frac{1}{3}}\right) \quad b(r) = -\log\left(\frac{r}{L}\left(1+\frac{Q^2}{r^2}\right)^{\frac{2}{3}}\right)$$

$$h(r) = 1 - \frac{(r_H^2+Q^2)^2}{(r^2+Q^2)^2} \quad \phi(r) = \sqrt{\frac{2}{3}}\log\left(1+\frac{Q^2}{r^2}\right)$$

$$A_t(r) = \frac{Q}{2L}\left(1 - \frac{r_H^2+Q^2}{r^2+Q^2}\right)$$

Thermodynamics

- The temperature, chemical potential, entropy density, charge density, energy density and pressure are given by

$$T = \frac{r_H}{\pi L^2} \qquad \mu = \frac{\sqrt{2}Q}{L^2} \qquad s = \frac{r_H}{4GL^3}(r_H^2 + Q^2)$$

$$\sigma = \frac{\sqrt{2}Qs}{2\pi r_H} \qquad \varepsilon = 3P = \frac{3\left(r_H^2 + Q^2\right)^2}{16\pi GL^5}$$

- When the charge density is large, $Q/r_H \simeq \mu/T \gg 1$, we obtain

$$s \simeq T$$

  the linear dependence of entropy on temperature, as in Landau Fermi liquid

Two-charge BH, (zero) sound and hydrodynamics

- In the large density limit hydrodynamic description is valid for quasinormal mode in the two-charge black hole background for any temperature
- Let's be more specific. Sound mode dispersion relation is

$$\omega = \sqrt{\frac{dP}{d\epsilon}}q - i\frac{2\eta}{3(\epsilon + P)}q^2 + \mathcal{O}(q^3)$$

Using $\eta = s/(4\pi)$ and thermodynamic relations for 2-charge BH we obtain

$$\omega = \frac{1}{\sqrt{3}}q - i\frac{\pi T}{3(\mu^2 + 2\pi^2 T^2)}q^2$$

Two-charge BH, (zero) sound and hydrodynamics

- From the sound mode dispersion relation we read off the mean free path
$$l = \frac{T}{\mu^2 + 2\pi^2 T^2}$$
- The hydrodynamic description is valid while $\omega l \ll 1$. Which means
$$\frac{\omega}{\mu} \ll \frac{\mu}{T} + 2\pi^2 \frac{T}{\mu}$$
- Therefore in the large density limit $\mu/T \gg 1$ hydrodynamic description of two-charge black hole sound quasinormal mode holds all the way to zero temperatures
- We have sound mode at small temperatures, including zero temperature, but it cannot be identified with the zero sound of LFL for this reason

AdS/CFT, (zero) sound and hydrodynamics

- We have verified this numerically by computing dispersion relation of sound mode and matching it with hydrodynamic formula
- We see that the field theory dual of two-charge black hole in AdS space exhibits rather different properties than predicted by Landau Fermi liquid theory
- Hydrodynamic behavior at large densities is valid even at zero temperature, i.e. there is no crossover to the collisionless thermal/quantum regime.
- Which is to be contrasted with the results in the probe brane models

Questions

The questions which we ask are:

- Can one fine tune LFL theory in such a way that it starts to look more like duals of black hole in AdS?
- Can one modify bulk side of the duality in such a way that generic LFL properties are reproduced?

To answer the first question we are going to look deeper into LFL theory
To answer the second question we are going to look at higher-derivative corrections to gravity

Dual of two-charge black hole

- We first explore the question of what is similar between Landau Fermi liquid and the field theory dual of the two-charge black hole
- The first similarity is the heat capacity, for two-charge black hole and LFL these are given by

$$c_V^{BH} = \frac{\pi L^3}{8G}\mu^2 T = \frac{N^2}{4}\mu^2 T \qquad c_V^{LFL} = \frac{k_F m^*}{3} T$$

- Charge densities are

$$\sigma^{BH} = \frac{L^3}{16\pi G}\mu^3 = \frac{N^2}{8\pi^2}\mu^3 \qquad \sigma^{LFL} = \alpha \frac{k_F^3}{6\pi^2}$$

- Matching these expression, we obtain

$$m_* = \alpha^{1/3}\left(\frac{3N^2}{4}\right)^{2/3}\mu \qquad v_F = \alpha^{-2/3}\left(\frac{3N^2}{4}\right)^{-1/3}$$

Dual of two-charge black hole: N-scaling

- Therefore if the dual of two-charge black hole is any sort of LFL, it is rather special LFL
- It has infinitely heavy quasiparticle effective mass $m_* = \mathcal{O}(N^{4/3})$ and infinitely small Fermi velocity, $v_F = \mathcal{O}(N^{-2/3})$
- The $v_F \ll 1$ implies that the particle-hole continuum (at small momenta and frequencies $k \simeq \omega/v_F$) of excitations is shifted away and is not visible
- Friedel oscillations ($2k_F$ zero-frequency singularity) are not visible, since $k_F = m_* v_F = \mathcal{O}(N^{2/3})$
- Log-violation of area law of entanglement entropy, characteristic for fermionic system in the presence of Fermi surface, $S \simeq L^2 k_F^2 \log L$ is $S \simeq k_F^2 = \mathcal{O}(N^{4/3})$, which is subleading relatively to $\mathcal{O}(N^2)$ tree-level result (Ryu, Takayanagi 2006; Kulaxizi, Parnachev, Schalm 2012)
- On top of it, LFL is the theory of weakly-coupled quasiparticles. Duals of gravity are strongly-coupled

Dual of two-charge black hole: Landau parameters

- We now switch to discussion of what are the values of Landau parameters F_l of the theory dual to two-charge BH
- Consider quasiparticles excited in the vicinity of Fermi surface, with the momentum $|k| \simeq k_F$. Weak interaction of quasiparticles is described by function $F(\vartheta)$ of the angle ϑ between vectors of momenta of two interacting quasiparticles.
- Quasiparticle energy ϵ and the function F are defined by the equation

$$\frac{1}{V}\delta E = \int d^3k \, \epsilon \, \delta n(k) + \int d^3k d^3k' \frac{k_F m_*}{\pi^2} F(\vartheta) \delta n(k) \delta n(k')$$

giving the change of the energy of the system when the distribution of quasiparticles $n(k)$ is varied.

Dual of two-charge black hole: Landau parameters

- Landau parameters are defined as

$$F(\theta) = \sum_{l=0}^{\infty} (2l+1) F_l P_l(\cos\theta)\,,$$

were P_l are Legendre polynomials.

- Quasiparticle effective mass and the speed of hydrodynamic sound depend on Landau parameters:

$$m^* = \mu \left(1 + \frac{F_1}{3}\right)$$

$$c_1 = \frac{v_F}{3}\left[(1+F_0)\left(1+\frac{F_1}{3}\right)\right]^{1/2}$$

- In CFT we have $c_1 = 1/\sqrt{3}$. Therefore $F_0 = \mathcal{O}(1)$ and $F_1 = \mathcal{O}(1/v_F^2) = \mathcal{O}(N^{4/3})$.

Dual of two-charge black hole: Landau parameters

- Finally we derive the value of F_2 paramater
- Recall that the two-charge black hole background at large density, $\mu/T \gg 1$, supports quasinormal sound mode described by hydrodynamics at any value of temperature
- In LFL crossover to hydrodynamics occurs only for sufficiently large temperature, $\omega\tau \simeq \omega\frac{\mu}{T^2} \ll 1$
- However one can fine-tune the Landau parameter $F_2 = -5$ (Pomeranchuk stability bound), so that generic finite-valued quasiparticle life-time $\tau = \mu/T^2$ becomes zero, $\tau = 0$
- Therefore hydrodynamic description of sound mode of Fermi liquid with $F_2 = -5$ persists all the way to $T = 0$

Dual of two-charge black hole: Landau parameters

- Let us show that the Landau parameters we found, $F_0 = \mathcal{O}(1)$, $F_1 = \mathcal{O}(N^{4/3})$, $F_2 = -5$, $F_n = \mathcal{O}(1)$, $n \geq 3$, are consistent with the speed of zero sound being equal to the speed of first sound
- This is actually clear from the fact that hydro description is valid at small temperatures. Therefore zero sound in such Fermi liquid is actually the usual (first) sound
- Zero sound is a collective fluctuation of shape of Fermi surface, described by the function $\nu(\theta, \varphi)$, satisfying equation

$$\left(\frac{u}{v_F} - \cos\theta\right)\nu(\theta,\varphi) = \cos\theta \int \frac{d^3p'}{(2\pi)^3} F(\vartheta)\nu(\theta',\phi')$$

Dual of two-charge black hole: Landau parameters

- Expand shape function $\nu(\theta, \phi)$ on Legendre polynomials,

$$\nu(\theta, \phi) = \sum_{l=0}^{\infty} (2l+1) \nu_l P_l(\cos\theta)$$

  Zero sound equation then gets rewritten as a system of linear equations

$$\nu_l + \sum_{l'} A_{ll'} \nu_{l'} = 0 \,, \;\; A_{ll'} = -\frac{1}{2} F_{l'} \int_{-1}^{1} P_l(y) P_{l'}(y) \frac{y \frac{v_F}{u}}{1 - y \frac{v_F}{u}}$$

  on the modes ν_l of the shape function
- Recall that Fermi velocity is small and we want to find when speed of zero sound is equal to speed of first sound, that is

$$\left(\frac{v_F}{u}\right)^2 = \frac{3}{(1+F_0)\left(1+\frac{F_1}{3}\right)} \ll 1$$

Dual of two-charge black hole: Landau parameters

- Expanding in powers of v_F/u, taking into account that $F_0 = \mathcal{O}(1)$ and $F_1 = \mathcal{O}((u/v_F)^2)$, and assuming $F_n = \mathcal{O}(1)$ when $n \geq 2$, we obtain

$$\det A = 1 - \left(\frac{v_F}{u}\right)^2 \left(\frac{F_0}{3} + \frac{F_1}{5} + \frac{F_0 F_1}{9} + \frac{4 F_1 F_2}{225}\right) + \mathcal{O}\left(\left(\frac{v_F}{u}\right)^2\right)$$
$$= -\frac{4 F_1}{25(1+F_0)(3+F_1)} \left(F_2 + 5 - \frac{75}{100 F_1} + o\left(\frac{1}{F_1}\right)\right)$$

- Therefore $u = c_1$ implies

$$F_2 = -5 + \mathcal{O}\left(\left(\frac{v_F}{u}\right)^2\right)$$

as expected

Dual of two-charge black hole: viscosity

- We have explained how one should fine-tune LFL so that it resembles the field theory dual of two-charge black hole
- Now the question is what can be done to the bulk side of the duality so that it resembles more generic, not fine-tuned, LFL
- One immediate feature of two-derivative gravity theories is a universal viscosity/entropy ratio,

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

- While in LFL

$$\frac{\eta}{s} \simeq (F_2 + 5)\frac{\mu^3}{T^3} + \cdots \simeq \frac{\mu^3}{T^3}$$

for generic F_2. Dots represent terms of the higher order in T/μ.

Higher derivative gravity: entropy and viscosity

- We are going to add HD terms to the bulk type-IIB reduced action

$$I_2 = \frac{1}{16\pi G}\int d^5x\sqrt{g}\left(R - \frac{1}{2}(\partial\phi)^2 - \frac{8}{L^2}e^{\phi/\sqrt{6}} - \frac{4}{L^2}e^{-2\phi/\sqrt{6}} + 2e^{2\phi/\sqrt{6}}F_{ab}F^{ab}\right)$$

- We need to retain low-temperature behavior of the entropy, $s \simeq T\mu^2$
- But we need higher-derivative terms to modify viscosity, so that the two-derivative (Einstein) result $\eta \simeq T\mu^2 \simeq s$ is replaced by the LFL expression $\eta \simeq \mu^5/T^2$.

Higher derivative gravity: entropy

- Entropy is determined by Wald formula

$$S = -2\pi \int d^3x \sqrt{g_3} \frac{\delta \mathcal{L}}{\delta R_{abcd}} E_{ab} E_{cd}$$

- We have found that the four-derivative action which doesn't contribute to entropy of two-charge black hole, is the Gauss-Bonnet action

$$I_4 = \frac{1}{16\pi G} \int d^5x \sqrt{g} (e^{7\phi/(2\sqrt{6})} + (1/2) e^{-7\phi/\sqrt{6}}) \times \left(R^2 - 4 R_{ab} R^{ab} + R_{abcd} R^{abcd} \right)$$

- Exponents of the dilaton are chosen so that in the low temperature limit $r_H/Q \ll 1$ the viscosity behaves as

$$\eta \simeq \frac{Q^5}{r_H^2} \quad \Rightarrow \quad \frac{\eta}{s} \simeq \frac{Q^3}{r_H^3}$$

Higher derivative gravity: subtleties

- It is important that the relations between thermodynamic parameters and two-charge black hole parameters are not messed up by higher-derivative corrections, $T \simeq r_H$ and $\mu \simeq Q$
- This requires additional action terms, correcting dilaton potential. We have verified numerically that one can control $T(r_H, Q)$ and $\mu(r_H, Q)$ by such terms while keeping $\eta(r_H, Q)$ and $s(r_H, Q)$ unchanged. Although dealing only with numerics it turned out to be hard to find precise dilaton potential

Higher derivative gravity: conclusions and further directions

- We see that by tuning higher-derivative corrections to the bulk action one can reproduce generic η/s behavior of LFL
- It's technically hard though, and getting precisely what is predicted by LFL is probably impossible
- One can study a sound mode in higher-derivative gravity tuned such that $\eta/s \simeq \mu^3/T^3$ and see whether the crossover to zero sound collisionless quantum/thermal regimes takes place

Thank you!